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Upper and Lower Bounds Computations of Drag Coefficients

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1 Introduction

In the recent paper we have derived an error estimation for drag coefficients and presented a numerical method by an extrapolation for a precise computation of drag coefficients [1]. In this paper we show another numerical method having upper and lower bounds. Our idea is to control a parameter appearing in the stabilized finite element method. Here we consider only two-dimensional problems for the simplicity of the notation. For the details of the method we refer to the paper [2].

2 A numerical method for drag coefficients

Let G be a two-dimensional body in a velocity field. Let U be the representative velocity and ρ be the density of the fluid. The drag coefficient of G is defined by

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 \ell},$$

where D is the component parallel to U of the total force exerted on G by the fluid and ℓ is the length of G orthogonal to U.

We suppose the velocity field is governed by the stationary Navier-Stokes equation,

$$(u \cdot \mathrm{grad})u + \frac{1}{Re}\Delta u + \mathrm{grad}\ p = 0,$$
$$\mathrm{div}\ u = 0,$$

where $u = (u_1, u_2)^T$ is the velocity, p is the pressure, and Re is the Reynolds number.

We consider the stabilized finite element method [3]. Let $V_h(g)$ be an affine finite element space satisfying the velocity boundary condition $u = g$, and Q_h be the finite element space for the pressure. We set $V_h = V_h(0)$. We seek the finite element solution $(u_h, p_h) \in V_h(g) \times Q_h$ such that

$$\mathcal{A}_h(u_h)((u_h, p_h), (v_h, q_h)) = 0 \quad (\forall (v_h, q_h) \in \mathcal{V}_h),$$

Table 1: Drag coefficients of the circle

Re	10	20	30	40
C_D	3.074	2.190	1.832	1.626

where $\mathcal{V}_h$ is the product space

$$\mathcal{V}_h = V_h \times Q_h,$$

and $\mathcal{A}_h(u_h)$ is a bilinear form in $\mathcal{V}_h$ defined by

$$\begin{aligned} &\mathcal{A}_h(w_h)((u_h,p_h),(v_h,q_h)) \\ = \quad & a_1(w_h,u_h,v_h) + a(u_h,v_h) + b(v_h,p_h) + b(u_h,q_h) + \mathcal{C}_h(w_h)((u_h,p_h),(v_h,q_h)), \end{aligned}$$

$$\begin{aligned} &\mathcal{C}_h(w)((u_h,p_h),(v_h,q_h)) = \sum_K \tau_K \\ \times \quad & \int_K \{(w\cdot \mathrm{grad})u_h + \frac{1}{Re}Lu_h + \mathrm{grad}\ p_h\}\{(w\cdot\mathrm{grad})v_h + \frac{1}{Re}Lv_h - \mathrm{grad}\ q_h\}dx. \end{aligned}$$

Here a_1, a, b are the trilinear form, the bilinear forms derived from the nonlinear convection term, the diffusion term and the divergence term, respectively. The summation is taken for all elements K and τ_K is the stabilization parameter defined by

$$\tau_K = \begin{cases} h_K^2 Re/(4c_0^2) & \text{when } Re_K < 1, \\ h_K/(2|w_K|) & \text{when } Re_K \geq 1, \end{cases}$$

where Re_K is an element Reynolds number

$$Re_K = \frac{h_K|w_K|Re}{2c_0^2}, \tag{1}$$

h_K is the diameter of element K, w_K is a representative velocity of w in K, e.g., the value of w at the centroid of K, and c_0 is a positive constant independent of h.

We define an approximate drag coefficient C_D^{hs} [2] by

$$C_D^{hs} = -\frac{2}{\rho U^2 \ell}\{a_1(u_h,u_h,\phi_h) + a(u_h,\phi_h) + b(\phi_h,p_h) + \mathcal{C}_h(u_h)((u_h,p_h),(\phi_h,0))\}.$$

where $\phi_h = (0, \phi_{2h})$ be a function in the velocity finite element space defined by

$$\phi_{2h} = \begin{cases} 1 & \text{at all nodal points on the boundary of } G, \\ 0 & \text{at the other nodal points.} \end{cases}$$

3 Upper and lower bounds computations

In (1) we have a parameter c_0. We can control it to obtain upper and lower bounds of drag coefficients. Table 1 shows drag coefficients of the unit circle obatined by this method [2].

References

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