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Newton polyhedrons and a formal Gevrey space of double indices for linear partial differential operators

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Abstract

In this paper, we shall study a necessary condition to become that the below operator $L_m(t,x;\partial_t,\partial_x)$ is bijective on $\mathcal{G}^{\{s_t,s_x\}}$, where it is said that a formal power series $U(t,x)=\sum_{l\beta}U_{l\beta}t^l x^\beta/l!\beta!$ belongs to $\mathcal{G}^{\{s_t,s_x\}}$ when $U(t,x)=\sum_{l\beta}U_{l\beta}t^l x^\beta/(l!)^{s_t}(\beta!)^{s_x}$ is convergent near the origin for $s_t,s_x\geq 1$.

1 Introduction

Let us give the operator L_m that we shall study in this paper.

Let $t=(t_1,\cdots,t_q)\in\mathbf{C}^q$, $x=(x_1,\cdots,x_p)\in\mathbf{C}^p$ and $t\cdot\partial_t=(t_1\partial_{t_1},t_2\partial_{t_2},\cdots,t_q\partial_{t_q})$. Set

(1.1) $$(t\cdot\partial_t)^j=(t_1\partial_{t_1})^{j_1}(t_2\partial_{t_2})^{j_2}\cdots(t_q\partial_{t_q})^{j_q}$$

for $j=(j_1,\cdots,j_q)\in\mathbf{N}^q$ and

(1.2) $$P_m(t\cdot\partial_t)=\sum_{|j|\leq m}P_j(t\cdot\partial_t)^j,$$

where $P_\sigma\in\mathbf{C}$, and P_m is said to be of Fuchs type of order m in [M]. Then we consider the following operator:

(1.3) $$L_m=P_m(t\cdot\partial_t)+A(t,x;\partial_t;\partial_x)+B(t,x;\partial_t;\partial_x)$$

where

(1.4) $$A=\sum_{\substack{|\alpha|=0\\|\sigma'|=|\sigma|\leq m}}^{finite}a^{\alpha}_{\sigma,\sigma'}(t,x)t^{\sigma'}\partial_t^{\sigma}\partial_x^{\alpha},$$

and

(1.5) $$B=\sum_{|\sigma'|+|\alpha'|>|\sigma|+|\alpha|}^{finite}b^{\alpha,\alpha'}_{\sigma,\sigma'}(t,x)t^{\sigma'}x^{\alpha'}\partial_t^{\sigma}\partial_x^{\alpha},$$

where the coefficients $a^{\alpha}_{\sigma,\sigma'}(t,x)$ and $b^{\alpha,\alpha'}_{\sigma,\sigma'}(t,x)$ are holomorphic functions in a neighbourhood of the origin for any $(\sigma,\sigma',\alpha,\alpha')\in\mathbf{Z}^q\times\mathbf{N}^q\times\mathbf{Z}^p\times\mathbf{N}^p$.

Miyake and Hashimoto studied the unique solvability in $\mathcal{G}^{\{s_t,1\}}$ for such type operator. They characterized the Gevrey index s_t by Newton polygons in [M] and [MH].

Our motivation comes from the following facts. Put

$$L = (t\partial_t + 1) - 3t^3 x\partial_t^2\partial_x - (t\partial_t + 1)x^2\partial_x. \tag{1.6}$$

This operator is not bijective in $\mathcal{G}^{\{s_t,1\}}$ for any s_t, but is bijective in $\mathcal{G}^{\{s_t,s_x\}}$ for $s_t \geq 3$ and $s_x \geq 2$.

So it is our purpose that we shall consider $\mathcal{G}^{\{s_t,s_x\}}$ to obtain the unique solvability for this operator. We shall define a Newton polyhedrons to characterize double Gevrey indices.

In Section 2, we give our results after defining a function space and Newton polyhedron and listing some notations. In Section 3, we prove our theorems.

2 Statement of results

2.1 Notations.

We denote by **N**, **Z**, **R** and **C** the set of non negative integers, integers, real numbers and complex numbers, respectively.

$\mathbf{C}[[t,x]]$ denotes the set of formal power series in $t \in \mathbf{C}^q$ and $x \in \mathbf{C}^q$ with coefficients in **C**.

For multi indices $\sigma = (\sigma_1, \cdots, \sigma_q) \in \mathbf{Z}^q$ and $\alpha = (\alpha_1, \cdots, \alpha_p) \in \mathbf{Z}^p$, an integro-differential $\partial_t^\sigma \partial_x^\alpha U(t,x)$ of $U(t,x) = \sum_{\substack{l\in\mathbf{N}^q \\ \beta\in\mathbf{N}^p}} U_{l\beta}\frac{t^l x^\beta}{l!\beta!} \in \mathbf{C}[[t,x]]$ is defined as follows:

$$\partial_t^\sigma \partial_x^\alpha U(t,x) := \sum_{\substack{l\in\mathbf{N}^q, l-\sigma\in\mathbf{N}^q \\ \beta\in\mathbf{N}^p, \beta-\sigma\in\mathbf{N}^p}} U_{l\beta}\frac{t^{l-\sigma}x^{\beta-\alpha}}{(l-\sigma)!(\beta-\alpha)!}. \tag{2.1}$$

2.2 Formal Gevrey class $G_{\tau\rho}^{\{s_t,s_x\}}(T,X;m)$.

For $U(t,x) \in \mathbf{C}[[t,x]]$, we set $U(t,x) = \sum_{l,\beta} U_{l\beta}t^l x^\beta/l!\beta!$, where $U_{l\beta} \in \mathbf{C}$, $l \in \mathbf{N}^q$ and $\beta \in \mathbf{N}^p$ and $\mathbf{R}_+$ denotes the set of positive real numbers.

Let $s_t, s_x \geq 1, T > 0, X > 0, \tau = (\tau_1, \cdots, \tau_q) \in \mathbf{R}_+^q, \rho = (\rho_1, \cdots, \rho_p) \in \mathbf{R}_+^p$ and $m \in \mathbf{N}$. Then we define a space $G_{\tau\rho}^{\{s_t,s_x\}}(T,X;m) \subset \mathbf{C}[[t,x]]$ as follows.

$$G_{\tau\rho}^{\{s_t,s_x\}}(T,X;m) := \left\{U(t,x) \in \mathbf{C}[[t,x]]; ||U||_{T/\tau,X/\rho;m}^{\{s_t,s_x\}} < \infty\right\}, \tag{2.2}$$

where

$$(2.3)\qquad ||U||^{\{s_t,s_x\}}_{T/\tau,X/\rho;m} := \sup_{l,\beta} |U_{l\beta}| \frac{|l|!^m T^{|l|} X^{|\beta|}}{\{(s_t+m)|l|+s_x|\beta|\}!\tau^l\rho^\beta}$$

and $n! := \Gamma(n+1)$.
Hence there exist positive constants R_t,R_x and C such that

$$(2.4)\qquad |U_{l\beta}| \le C\frac{|l|!^{s_t}|\beta|!^{s_x}}{R_t^{|l|}R_x^{|\beta|}}$$

for any $l \in \mathbf{N}^q$ and $\beta \in \mathbf{N}^p$.
Here we define a formal Gevrey space as follows:

Definition 2.1

$$(2.5)\qquad G^{\{s_t,s_x\}} := \cup_{T,X>0} G^{\{s_t,s_x\}}_{\tau\rho}(T,X;m)$$

2.3 Newton polyhedron.

Here we define Newton polyhedron for a linear partial integro-differential operator and state some remarks L_m.

Let

$$(2.6)\qquad P = \sum^{finite} a^{\alpha,\alpha'}_{\sigma,\sigma'}(t,x) t^{\sigma'} x^{\alpha'} \partial_t^\sigma \partial_x^\alpha$$

be a linear partial integro-differential operator of finite order with holomorphic coefficients in a neighbourhood of the origin.

In the space $\mathbf{R}^3$, we define the following lower half line for $(\sigma,\sigma',\alpha,\alpha') \in \mathbf{Z}^q \times \mathbf{N}^q \times \mathbf{Z}^p \times \mathbf{N}^p$:

$$(2.7)\qquad \begin{aligned} &Q(\sigma,\sigma',\alpha,\alpha') \\ &:= \{(\mathcal{X},\mathcal{Y},\mathcal{Z}) \in \mathbf{R}^3; \mathcal{X} = |\sigma'|-|\sigma|, \mathcal{Y} = |\alpha'|-|\alpha|, \mathcal{Z} \le |\sigma|+|\alpha|\}. \end{aligned}$$

Definition 2.2 *Newton polyhedron $N(P)$ of the operator P is defined by*

$$(2.8)\qquad N(P) := Ch\{Q(\sigma,\sigma',\alpha,\alpha'); (\sigma,\sigma',\alpha,\alpha') \quad with \quad a^{\alpha,\alpha'}_{\sigma,\sigma'}(t,x) \not\equiv 0\},$$

where $Ch\{\cdot\}$ denotes the convex hull of sets in $\{\cdot\}$.

Let $N(L_m)$ be Newton polyhedron of L_m.

Remark 2.3 *By the form of L_m, the lower half line $\{(0,0,\mathcal{Z}); \mathcal{Z} \le m\}$ becomes a side of $N(L_m)$ and the point $(0,0,m)$ becomes a vertex of $N(L_m)$.*

Next let

$$\mathfrak{A} = \{(\mathcal{X},\mathcal{Y},\mathcal{Z}); a\mathcal{X} + b\mathcal{Y} - \mathcal{Z} + m \geq 0\}. \tag{2.9}$$

Then we define the following set of pairing indices:

$$\mathcal{S} = \{(s_t, s_x); s_t = a+1, s_x = b+1, N(L_m) \subseteq \mathfrak{A}\}. \tag{2.10}$$

Remark 2.4 *Since the boundary of $\mathcal{S}$ is a hyper plane which goes through the point $(0,0,m)$, there exists (s_t, s_x) such that $N(L_m) \subseteq \mathfrak{A}$ by Remark 2.3.*

Remark 2.5 *For any (s_t, s_x) belonging to $\mathcal{S}$, we obtain*

$$s_t(|\sigma| - |\sigma'|) + s_x(|\alpha| - |\alpha'|) + |\sigma'| + |\alpha'| - m \leq 0 \tag{2.11}$$

for $(\sigma,\sigma',\alpha,\alpha')$ with $Q(\sigma,\sigma',\alpha,\alpha') \in N(L_m)$.

2.4 Main results.

We assume the following additional condition.
(A.1) If $m(|\sigma'| - |\sigma|) + |\sigma'| < m$ for $(\sigma,\sigma',\alpha,\alpha')$ with $b^{\alpha\alpha'}_{\sigma\sigma'}(t,x) \not\equiv 0$, then

$$(m + s_t)(|\sigma| - |\sigma'|) + s_x(|\alpha| - |\alpha'|) + |\alpha'| \leq 0, \tag{2.12}$$

for (s_t, s_x) belonging to $\mathcal{S}$.
(S.C) For any $\epsilon > 0$,there exist $\tau \in \mathbf{R}^q_+$ and $\rho \in \mathbf{R}^n_+$ such that

$$\sum_{\substack{|\alpha|=0 \\ |\sigma'|=|\sigma|\leq m}}^{finite} |a^{\alpha}_{\sigma,\sigma'}|\tau^{\sigma-\sigma'}\rho^{\alpha} < \epsilon, \tag{2.13}$$

where $a^{\alpha}_{\sigma,\sigma'} = a^{\alpha}_{\sigma,\sigma'}(0,0)$.
It is said that the condition (S.C) is Spectral condition in [M].
Then we obtain the following results.

Theorem 2.6 *Assume that L_m satisfies the condition (A.1) and (S.C) and further assume that there exists a positive constant C such that*

$$|P_m(l)| \geq C(|l| + 1)^m \quad \textit{for all} \quad l \in \mathbf{N}^q. \tag{2.14}$$

Then the mapping

$$L_m : G^{\{s_t,s_x\}} \to G^{\{s_t,s_x\}} \tag{2.15}$$

is bijective for any (s_t, s_x) belonging to $\mathcal{S}$.

Next set

(2.16)
$$\delta = \max\left\{\left\{\frac{|\alpha'| + \max\{|\sigma'| - m, m(|\sigma| - |\sigma'|)\}}{|\alpha'| + |\sigma'| - |\alpha| - |\sigma|}; b_{\sigma\sigma'}^{\alpha\alpha'}(t,x) \not\equiv 0\right\}, 1\right\}.$$

Then for $s_x \geq \delta$, there exist indices s_t with $s_t \geq 1$ such that if $m(|\sigma'| - |\sigma|) + |\sigma'| - m \geq 0$, then

$$s_t(|\sigma| - |\sigma'|) + s_x(|\alpha| - |\alpha'|) + |\sigma'| + |\alpha'| - m \leq 0, \tag{2.17}$$

and if $m(|\sigma'| - |\sigma|) + |\sigma'| - m < 0$, then

$$(s_t + m)(|\sigma| - |\sigma'|) + s_x(|\alpha| - |\alpha'|) + |\alpha'| \leq 0. \tag{2.18}$$

For example if $s_x = s_t \geq \delta$, then the above formulas are satisfied.
So we obtain the following Corollary.

Corollary 2.7 *Assume that L_m satisfies the condition (S.C) and further assume the inquation* (2.14). *Then for $s_x \geq \delta$, there exist indices s_t with $s_t \geq 1$ such that the mapping* (2.15) *is bijective.*

3 Proof of Theorem.

In this section first we estimate a operator of form $t^{\sigma'} x^{\alpha'} \partial_t^\sigma \partial_x^\alpha P_m^{-1}$ on $G_{\tau\rho}^{\{s_t,s_x\}}(T,X;m)$, next by using the estimate we show $L_m P_m^{-1}$ is bijective on same space, at last we give a proof of Theorem 2.6.

3.1 The estimate of operator $t^{\sigma'} x^{\alpha'} \partial_t^\sigma \partial_x^\alpha P_m^{-1}$.

Here we study a estimate of the operator $t^{\sigma'} x^{\alpha'} \partial_t^\sigma \partial_x^\alpha P_m^{-1}$ of the mapping

$$t^{\sigma'} x^{\alpha'} \partial_t^\sigma \partial_x^\alpha P_m^{-1} : G_{\tau\rho}^{\{s_t,s_x\}}(T,X;m) \to G_{\tau\rho}^{\{s_t,s_x\}}(T,X;m), \tag{3.1}$$

where

$$P_m^{-1} : \sum U_{l\beta} \frac{t^l x^\beta}{l!\beta!} \mapsto \sum P_m(l)^{-1} U_{l\beta} \frac{t^l x^\beta}{l!\beta!}. \tag{3.2}$$

Lemma 3.1 *Assume that the conditions in Theorem* 2.6 *are satisfied. Then for any (s_t, s_x) belonging to S, there exist a positive constant C such that the operator norm of the mapping (*3.1*) is estimated as follows:*

$$||t^{\sigma'} x^{\alpha'} \partial_t^\sigma \partial_x^\alpha P_m^{-1}|| \leq C T^{|\alpha'|-|\alpha|} X^{|\sigma'|-|\sigma|} \tau^{\sigma-\sigma'} \rho^{\alpha-\alpha'}, \tag{3.3}$$

where the constant C depends only on $m, s_t, s_x, \sigma, \sigma', \alpha$ and α', and $||\cdot||$ denotes the operator norm on $G_{\rho\tau}^{\{s_t,s_x\}}(T,X;m)$.

Proof. Let $t^{\sigma'}x^{\alpha'}\partial_t^{\sigma}\partial_x^{\alpha}P_m^{-1}U(t,x) = \sum_{\beta l} V_{l\beta}t^l x^\beta/l!\beta!$. Then we obtain

$$V_{l\beta} = \frac{l!}{(l-\sigma')!}\frac{\beta!}{(\beta-\alpha')!}P_m(l+\sigma-\sigma')^{-1}U_{l+\sigma-\sigma'\beta+\alpha-\alpha'}, \tag{3.4}$$

where $l+\sigma-\sigma' \in \mathbf{N}^q$ and $\beta+\alpha-\alpha' \in \mathbf{N}^p$.
Therefore we have

$$\begin{aligned} &|V_{l\beta}|\frac{|l|!^m T^{|l|}X^{|\beta|}}{\{(s_t+m)|l|+s_x|\beta|\}!\tau^l\rho^\beta} \\ &\le C_0|l|^{m(|\sigma'|-|\sigma|)+|\sigma'|-m}|\beta|^{|\alpha'|}T^{|\sigma'|-|\sigma|}X^{|\alpha'|-|\alpha|}\tau^{\sigma-\sigma'}\rho^{\alpha-\alpha'} \\ &\times\frac{\{(s_t+m)|l|+s_x|\beta|+(s_t+m)(|\sigma|-|\sigma'|)+s_x(|\alpha|-|\alpha'|)\}!}{\{(s_t+m)|l|+s_x|\beta|\}!}. \end{aligned} \tag{3.5}$$

By Remark 2.5 and the condition (A.1), we obtain the estimation (3.3). Q.E.D.

3.2 The estimate of operator $L_m P_m^{-1}$.

For $x \in \mathbf{C}^p$,we set $|x| = x_1 + \cdots + x_p$ and $||x|| = |x_1| + \cdots + |x_p|$. For a domain $\Omega \subset \mathbf{C}^p$, $\mathcal{O}(\Omega)$ denotes the set of holomorphic functions in Ω, $\mathcal{O}(\overline{\Omega}) := \mathcal{O}(\Omega) \cap \mathbf{C}(\overline{\Omega})$. Similar notations will be used frequently for functions defined in a domain $\mathbf{C}_{t,x}^{q+p}$.

Let $a(t,x) = \sum a_{l\beta}t^l x^\beta/l!\beta! \in \mathcal{O}(\{||t|| \le \kappa T\} \times \{||x|| \le \kappa X\})$ $(\kappa > 0)$ and put

$$||a||_{\kappa T,\kappa X} := \max_{\substack{||t||\le\kappa T \\ ||x||\le\kappa X}} |a(t,x)|. \tag{3.6}$$

By Cauchy's integral formula on a polycircle $\prod_{j=1}^q\{|t_j| = \eta_j\kappa T\} \times \prod_{i=1}^p\{|x_i| = \xi_i\kappa X\}$ $(\eta_j > 0, \eta_1+\cdots+\eta_q = 1)$ and $(\xi_i > 0, \xi_1+\cdots+\xi_p = 1)$, we have

$$|a_{l\beta}| \le C\frac{1}{(\kappa T)^{|l|}(\kappa X)^{|\beta|}}\frac{l!\beta!}{\eta^l\xi^\beta}. \tag{3.7}$$

Since η^l and ξ^β take its maximum on the above mentioned domain at a point $\eta = (l_1/|l|,\cdots,l_q/|l|)$ and $\xi = (\beta_1/|\beta|,\cdots,\beta_p/|\beta|)$, we have

$$|a_{l\beta}| \le ||a||_{\kappa T,\kappa X}\frac{1}{(\kappa T)^{|l|}(\kappa X)^{|\beta|}}\frac{|l|^{|l|}|\beta|^{|\beta|}l!\beta!}{l^l\beta^\beta}. \tag{3.8}$$

Hence by Stirling's formula, we have

$$|a_{l\beta}| \le C(q,p)||a||_{\kappa T,\kappa X}\frac{(|l|+[q/2])!(|\beta|+[p/2])!}{(\kappa T)^{|l|}(\kappa X)^{|\beta|}}, \tag{3.9}$$

for some positive constant $C(q,p)$ depending only on the dimension q of t and the dimension p of x. Here $[q/2]$ (resp.$[p/2]$) denotes the integral part of $q/2$ (resp.$p/2$).
Then we have the following lemma.

Lemma 3.2 *Let $U(t,x) \in G^{\{s_t,s_x\}}_{\tau\rho}(T,X;m)$ and $a(t,x) \in \mathcal{O}(\{||\tau t|| \le \kappa T\} \times \{||\rho x|| \le \kappa X\})$ $(\kappa > 0)$. Then $a(t,x)U(t,x) \in G^{\{s_t,s_x\}}_{\tau\rho}(T,X;m)$ for any $\kappa > 1$ and it holds*

$$(3.10)\qquad \begin{aligned} ||aU||^{\{s_t,s_x\}}_{T/\tau,X/\rho;m} &\le C(q,p)\frac{[q/2]!}{(1-1/\kappa)^{[q/2]+1}}\frac{[p/2]!}{(1-1/\kappa)^{[p/2]+1}} \\ &\times ||a||_{\kappa T,\kappa X}||U||^{\{s_t,s_x\}}_{T/\tau,X/\rho;m}. \end{aligned}$$

Proof. We may be $\rho = (1,\cdots,1) \in \mathbf{R}^p_+$ and $\tau = (1,\cdots,1) \in \mathbf{R}^q_+$.
Set $a(t,x)U(t,x) = \sum V_{l\beta}t^l x^\beta/l!\beta!$, where

$$(3.11)\qquad V_{l\beta} = \sum_{\substack{0\le n\le l\\ 0\le\gamma\le\beta}} a_{n\gamma}U_{l-n\beta-\gamma}\frac{l!}{n!(l-n)!}\frac{\beta!}{\gamma!(\beta-\gamma)!}.$$

Then we have

$$(3.12)\qquad \begin{aligned} |V_{l\beta}| &\le C(q,p)||a||_{\kappa T,\kappa X}||U||^{\{s_t,s_x\}}_{T/\tau,X/\rho;m}\sum_{\substack{0\le n\le l\\ 0\le\gamma\le\beta}}\frac{(|n|+[q/2])!(|\gamma|+[p/2])!}{(\kappa T)^{|n|}(\kappa X)^{|\gamma|}} \\ &\times \frac{\{(s_t+m)(|l|-|n|)+s_x(|\beta|-|\gamma|)\}!}{|l-n|!^m T^{|l|-|n|}X^{|\beta|-|\gamma|}} \\ &\times \frac{l!}{n!(l-n)!}\frac{\beta!}{\gamma!(\beta-\gamma)!}. \end{aligned}$$

Hence we have

$$(3.13)\qquad \begin{aligned} ||aU||^{\{s_t,s_x\}}_{T/\tau,X/\rho;m} &\le C(q,p)||a||_{\kappa T,\kappa X}||U||^{\{s_t,s_x\}}_{T/\tau,X/\rho;m} \\ &\quad \frac{[q/2]!}{(1-1/\kappa)^{[q/2]+1}}\frac{[p/2]!}{(1-1/\kappa)^{[p/2]+1}}. \end{aligned}$$

Q.E.D
By Lemma 3.1 and Lemma 3.2, we obtain the following essential proposition.

Proposition 3.3 *Assume that the conditions of Theorem 2.6 are satisfied. Then for any (s_t,s_x) belonging to S, there exist a positive constant R_0, $\tau \in \mathbf{R}^q_+$ and $\rho \in \mathbf{R}^p_+$ such that the mapping*

$$(3.14)\qquad L_mP_m^{-1} : G^{\{s_t,s_x\}}_{\tau\rho}(R,R;m) \to G^{\{s_t,s_x\}}_{\tau\rho}(R,R;m)$$

is bijective for any R with $0 < R < R_0$.

Proof. Since $P_mP_m^{-1} = I$ on $G_{\tau\rho}^{\{s_t,s_x\}}(R,R;m)$, it is sufficiently that we show that $Q = (A+B)P_m^{-1}$ is a contraction mapping. By Lemma 3.1, Lemma 3.2 and the condition (A.1), we obtain a estimation $||AP_m^{-1}|| = O(\epsilon) + O(R)$, and by Lemma 3.1, Lemma 3.2 and the condition of the operator B ($|\sigma'| + |\alpha'| > |\sigma| + |\alpha|$), we obtain a estimation $||BP_m^{-1}|| = O(R)$. Hence the operator Q is a construction mapping on $G_{\tau\rho}^{\{s_t,s_x\}}(R,R;m)$ for sufficiently small ϵ and R. Q.E.D.

Proof of Theorem 2.6.
Let $P_m^{-1}U(t,x) = \sum P_m(l)^{-1}U_{l\beta}t^lx^\beta/l!\beta!$ for $U(t,x) = \sum U_{l\beta}t^lx^\beta/l!\beta!$. By Proposition 3.3, $L_mP_m^{-1}$ is bijective on $G_{\tau\rho}^{\{s_t,s_x\}}(R,R;m)$, and since $P_m^{-1}P_m = P_mP_m^{-1} = I$ (identity) holds on $G_{\tau\rho}^{\{s_t,s_x\}}(R,R;m)$, $L_m$is bijective on $G_{\tau\rho}^{\{s_t,s_x\}}(R,R;m)$. This completes the proof.

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