数理解析研究所講究録
1036巻 1998年 110-124

Stability of equilibrium solution for free boundary problem for the motion of viscous compressible barotropic self-gravitating fluid.

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1. Introduction

We consider the motion of viscous compressible barotropic fluid in a domain $\Omega_t \subset \mathbb{R}^3$ bounded by a free surface S_t, $t \in \mathbb{R}_+$. The problem is described by equations:

$$
\begin{aligned}
(1.1)\quad & \varrho(v_t + v\cdot\nabla v) - \operatorname{div}\mathbb{T}(v,p) = \varrho\nabla U && \text{in } \tilde{\Omega}^T = \bigcup_{t\le T}\Omega_t\times\{t\},\\
& \varrho_t + \operatorname{div}(\varrho v) = 0 && \text{in } \tilde{\Omega}^T,\\
& \mathbb{T}\bar{n} = -p_0\bar{n} && \text{on } \tilde{S}^T = \bigcup_{t\le T} S_t\times\{t\},
\end{aligned}
$$

$$\varrho|_{t=0} = \varrho_0 > 0,\quad v|_{t=0} = v_0,\quad \Omega_t|_{t=0} = \Omega,\quad S_t|_{t=0} = S,$$

$$v\cdot\bar{n} = \text{velocity of } S_t,$$

where $v = v(x,t)$ is the velocity of the fluid, $\varrho = \varrho(x,t)$ the density, $p = p(\varrho) > 0$ the pressure, $\bar{n}$ is the unit outward vector normal to S_t, p_0 is the positive external constant pressure, $\mathbb{T}(v,p)$ is the stress tensor of the form

$$(1.2)\quad \mathbb{T}(v,p) = \mathbb{D}(v) - p\mathbb{I},$$

where $\mathbb{I}$ is the unit matrix and $\mathbb{D}(v)$ is the dilatation tensor of the form

$$(1.3)\quad \mathbb{D}(v) = \{\mu(\partial_{x_i}v_j + \partial_{x_j}v_i) + (\nu-\mu)\operatorname{div} v\,\delta_{ij}\}_{i,j=1,2,3},$$

and μ, ν are constant viscosity coefficients with $\nu > \frac{1}{3}\mu$. By $U(x,t)$ we denote the self-gravitation potential

$$(1.4) \qquad U(x,t) = k \int_{\Omega_t} \frac{\varrho(y,t)}{|x-y|} dy ,$$

where k is the gravitation constant.

The last equation $(1.1)_5$ is called the kinematic condition. In view of the equation of continuity $(1.1)_2$ and $(1.1)_5$ we have the conservation of mass

$$(1.5) \qquad \int_{\Omega} \varrho_0 dx = \int_{\Omega_t} \varrho dx = M ,$$

where M is a positive given constant.

Our aim is to prove global in time existence of some solutions to (1.1). Therefore we introduce some equilibrium solution and we prove existence of such global solution which initially is close to the equilibrium solution and remains close to it for all time. Since we consider a free boundary problem we have to have a mechanism which keeps the fluid together and control the shape of S_t. This is the gravitation force. Since the gravitation is rotationally invariant we expect that the equilibrium solution will be in a domain which is a ball. Therefore the global solution would be such that Ω_t would be close to the ball.

Solonnikov (see [1]) considered incompressible version of problem (1.1), but he considered the case with the surface tension. Hence instead of $(1.1)_3$ he has

$$(1.6) \qquad \mathbb{T}\bar{n} = -\sigma \Delta_{S_t} x - p_0 \bar{n} ,$$

where σ is the coefficient of the surface tension and Δ_{S_t} is the Laplace-Beltrami operator on S_t.
However the case with the surface tension is technically more complicated the surface tension is the second mechanism which keeps Ω_t close to a ball.
The results of this paper are more detaily described in [2,3,4].

2. Conservation laws

To prove global existence and existence of the equilibrium solution we need

Lemma 1
Assume $p = A\rho^{\varkappa}$, $\varkappa > 1$, A is a positive constant. Then for solutions of problem (1.1) the following conservation laws hold

$$(2.1) \quad \frac{d}{dt}\Big[\int_{\Omega_t} \tfrac{1}{2}\rho v^2 dx + E(\rho, \Omega_t)\Big] + \int_{\Omega_t} |\mathbb{D}_0(v)|^2 dx = 0,$$

where

$$(2.2) \quad E(\rho, \Omega_t) = p_0|\Omega_t| + \frac{A}{\varkappa - 1}\int_{\Omega_t} \rho^{\varkappa} dx - \frac{k}{2}\int_{\Omega_t}\int_{\Omega_t} \frac{\rho(x,t)\rho(y,t)}{|x-y|} dx\,dy,$$

$$|\mathbb{D}_0(v)|^2 = \frac{\mu}{2}(\partial_{x_i} v_j + \partial_{x_j} v_i)^2 + (\nu - \mu)|\operatorname{div} v|^2,$$

$|\Omega_t| = \operatorname{vol}\Omega_t$, and

$$(2.3) \quad \frac{d}{dt}\int_{\Omega_t} \rho v \cdot \eta \, dx = 0,$$

where $\eta = a + b \times x$, a, b are arbitrary constant vectors, and finally

$$(2.4)\quad \frac{d}{dt}\int_{\Omega_t} \rho x \, dx = \int_{\Omega_t} \rho v \, dx .$$

3. Equilibrium solution.

To determine the equilibrium solution we examine the free energy functional (2.2). To find the equilibrium solution we examine a minimum of $E(\rho, \Omega_t)$ with respect to some variations of ρ and Ω_t, where Ω_t is close to a ball B_R. Therefore we assume the existence of an orientation-preserving diffeomorphism $T_t : \overline{B}_R \to \overline{\Omega}_t$. Then we replace the density ρ on $\overline{\Omega}_t$ by a density $\hat{\rho}$ on $\overline{B}_R$ by the formula $\hat{\rho}(x) = J_{T_t}(x)\,\rho(T_t(x))$, where

$$\int_{B_R} \hat{\rho}(x)\,dx = \int_{B_R} J_{T_t}(x)\,\rho(T_t(x))\,dx = \int_{\Omega_t} \rho(x)\,dx = M,$$

and $J_{T_t}(x)$ is the Jacobian of the transformation T_t. Moreover we have

$$E(\rho, \Omega_t) = p_0 \int_{B_R} J_{T_t}\,dx + \frac{A}{\varkappa-1}\int_{B_R} \hat{\rho}^{\varkappa}(x,t)\, J_{T_t}^{-\varkappa+1}\,dx - \frac{k}{2}\int_{B_R}\int_{B_R} \frac{\hat{\rho}(x,t)\,\hat{\rho}(y,t)}{|T_t(x)-T_t(y)|}\,dx\,dy$$

$$\equiv I(\hat{\rho}, T_t),$$

and we consider $I(\hat{\rho}, T_t)$ on the set

$$D(I) = \{\hat{\rho} \in C^0(\overline{B}_R) : \hat{\rho}(x,t) > 0,\ x \in \overline{B}_R\} \times \{T_t \in C^1(\overline{B}_R) :$$

$$T_t \text{ is one-to-one and } J_{T_t}(x) > 0 \text{ for } x \in \overline{B}_R\}.$$

To examine variations of $I(\hat{\varrho}, T_t)$ we consider $I(\varrho + s\varphi, id_{B_R} + su)$, where s is a parameter, $\varrho > 0$, $\varphi, \varrho \in C^0(\bar{B}_R)$, $u \in C^1(\bar{B}_R)$ and φ, u are variations.

Lemma 2

Assume that $\varphi \in C^0(\bar{B}_R), u \in C^1(\bar{B}_R)$, $\int_{B_R} \varphi dx = 0$. Then

$$\frac{d}{ds} I(\varrho_e + s\varphi, id_{B_R} + su)\Big|_{s=0} = 0,$$

where $\varrho_e = \varrho(r)$ and r is the radius of spherical coordinates, is equivalent to the problem

$$(3.1) \qquad \varrho_e^{\varkappa-1}(x) = \frac{k(\varkappa-1)}{A\varkappa} \int_{B_{R_e}} \frac{\varrho_e(y)}{|x-y|} dy + C \qquad \text{in } \bar{B}_{R_e},$$

$$\varrho_e^{\varkappa}(x) = \frac{1}{A} p_0 \qquad \text{on } \partial\bar{B}_{R_e},$$

where C is any constant.

Next we have

Lemma 3

For solutions of (3.1) we have

$$\frac{d^2}{ds^2} I(\varrho_e + s\varphi, id_{B_{R_e}} + su)\Big|_{s=0} > 0.$$

Therefore the function $E(\varrho, \Omega_t)$ attains minimum at $\Omega_t = B_{R_e}$ and $\varrho = \varrho_e(r)$, where ϱ_e is a solution of (3.1).

Summarizing the above we obtain (see [2])

<u>Theorem 4</u>

Assume $M>0$ and $p_0>0$ are given. Then there exists exactly one number $R_e>0$ and one function $\varrho_e : [0,R_e] \to [0,\infty)$ with $\varrho_e \in C^\infty([0,R_e])$ such that $\int_{B_{R_e}} \varrho_e dx = M$ and

$$(3.2) \qquad \Delta \varrho_e^{\varkappa-1}(r) = -\frac{k(\varkappa-1)}{4\pi A \varkappa}\varrho_e(r), \quad A\varrho_e^{\varkappa}(R_e) = p_0 .$$

This solution has the property that there is an $\varepsilon>0$ and an $\eta>0$ such that if

$$(3.3) \qquad \begin{aligned} &\|T_t - id_{B_{R_e}}\|_{C^1(\bar{B}_{R_e})} + \|T_t - id_{B_{R_e}}\|_{H^2(B_{R_e})} \leq \varepsilon, \\ &\int_{B_{R_e}} \hat{\varrho}\, dx = M, \qquad \|\hat{\varrho} - \varrho_e\|_{C^0(\bar{B}_{R_e})} \leq \varepsilon, \end{aligned}$$

then

$$(3.4) \qquad I(\hat{\varrho}, T_t) - I(\varrho_e, id_{B_{R_e}}) \geq \eta\Big[\|\hat{\varrho} - \varrho_e\|^2_{L_2(B_{R_e})} + \|T_t - id_{B_{R_e}}\|_{L_2(B_{R_e})}\Big].$$

4. Stability of equilibrium solution.

First we have to prove existence of local solutions. To do it we introduce the Lagrangian coordinates ξ by

$$(4.1) \qquad \frac{dx}{dt} = v(x,t), \quad x|_{t=0} = \xi,$$

so integrating (4.1) we obtain the relation

$$(4.2) \qquad x = \xi + \int_0^t u(\xi,\tau)d\tau \equiv x(\xi,t),$$

where $u(\xi,t) = v(x(\xi,t),t)$.

In view of the kinematic condition $(1.1)_5$ we have
$\Omega_t = \{x \in \mathbb{R}^3 : x = x(\xi,t), \xi \in \Omega\}$, $S_t = \{x \in \mathbb{R}^3 : x = x(\xi,t), \xi \in S\}$.
Then problem (1.1) takes the form

$$
\begin{aligned}
(4.3)\quad & \eta u_t - \operatorname{div}_u \mathbb{T}_u(u,q) = \eta \nabla_u U_u(\eta) && \text{in } \Omega^T = \Omega \times (0,T),\\
& \eta_t + \eta \operatorname{div}_u u = 0 && \text{in } \Omega^T,\\
& \mathbb{T}_u(u,q)\bar{n}_u = -p_0 \bar{n}_u && \text{on } S^T = S \times (0,T),\\
& \eta|_{t=0} = \varrho_0, \quad u|_{t=0} = v_0 && \text{in } \Omega,
\end{aligned}
$$

where $\eta(\xi,t) = \varrho(x(\xi,t),t)$, $q(\xi,t) = p(\eta(\xi,t))$, $\nabla_u = \frac{\partial \xi_i}{\partial x} \nabla_{\xi_i}$,
$\bar{n}_u = \bar{n}(x(\xi,t),t)$, $U_u(\eta)(\xi,t) = \int_\Omega \frac{\eta(\xi',t)}{|x(\xi,t) - x(\xi',t)|} J_{x(\xi',t)} d\xi'$, and
the operators div_u, $\mathbb{T}_u$ are obtained from div, $\mathbb{T}$ by replacing ∇ by ∇_u.

By the Galerkin method and a fixed point argument

Lemma 5 (see [3])

Assume $\varrho_0, v_0 \in H^2(\Omega)$, Ω is a given bounded domain with the boundary $S \in H^{5/2}$. Then there exists a time $T > 0$ which depends on $F = \|v_0\|^2_{H^2(\Omega)} + \|\varrho_0\|^2_{H^2(\Omega)}$ such that for $t \le T$ there exists a solution to (1.1) such that
$u \in L_2(0,T;H^3(\Omega)) \cap L_\infty(0,T;H^1(\Omega))$, $u_t \in L_2(0,T;H^1(\Omega)) \cap L_\infty(0,T;L_2(\Omega))$,
$S_t = \partial\Omega_t \in H^{5/2}$, $\eta \in L_\infty(0,T;H^2(\Omega))$, $\eta_t \in L_\infty(0,T;L_2(\Omega)) \cap L_2(0,T;H^2(\Omega))$,
$\eta_{tt} \in L_2(\Omega^T)$, $\frac{1}{\eta} \in L_\infty(\Omega^T)$ and

$$
\begin{aligned}
(4.4)\quad & \|u\|_{L_\infty(0,T;H^1(\Omega))} + \|u\|_{L_2(0,T;H^3(\Omega))} + \|u_t\|_{L_\infty(0,T;L_2(\Omega))} + \|u_t\|_{L_2(0,T;H^1(\Omega))}\\
& + \|\eta\|_{L_\infty(0,T;H^2(\Omega))} + \|\eta_t\|_{L_\infty(0,T;L_2(\Omega))} + \|\eta_t\|_{L_2(0,T;H^2(\Omega))} +
\end{aligned}
$$

$$+\|\eta_{tt}\|_{L_2(\Omega^T)} + \left\|\frac{1}{\eta}\right\|_{L_\infty(\Omega^T)} \le \Phi(F),$$

where Φ is an increasing positive function.

To prove global existence of solutions to (1.1) we need more regular local solution than the one obtained in Lemma 5. Moreover, since we are able to prove global existence of solutions to (1.1) which are close to the equilibrium solution (3.2) we have to formulate a system for quantities which describe variations near the equilibrium solution.

Assume that the initial domain Ω is close to the ball B_{R_e}. Then in view of the local solution the domain Ω_t, $t \le T$, where the time T of local existence is sufficiently small, will be close to the ball B_{R_e} too. Hence we can introduce an orientation-preserving diffeomorphism

$$(4.5) \qquad T_t : \bar{B}_{R_e} \to \bar{\Omega}_t .$$

Therefore the equilibrium solution ϱ_e, p_e can be transformed from B_{R_e} on Ω_t by

$$(4.6) \qquad \bar{\varrho}_e = \varrho_e \circ T_t^{-1}, \quad \bar{p}_e = p_e \circ T_t^{-1} .$$

The transformation T_t is a composition of two transformations, $T_t = T_{1t} \circ T_2$, where $T_2 : \bar{B}_{R_e} \to \bar{\Omega}$ which is given by initial data and $T_{1t} : \bar{\Omega} \to \bar{\Omega}_t$ which is determined by the relation between the Lagrangian and the Eulerian coordinates. Hence $T_{1t}(\xi) \equiv \xi + \int_0^t u(\xi,\tau)\,d\tau = x$.

Now we formulate a problem for the equilibrium solution $\bar{\varrho}_e, \bar{p}_e$. Let $x \in B_{R_e}$ and $x = T_t^{-1}(x')$, $x' \in \Omega_t$. Moreover, $T_t^{-1} : S_t \to S_{R_e}$. Then the equilibrium solution determined by

(4.7) $$\nabla_x p(\varrho_e(x)) = k\varrho_e(x)\nabla_x \int_{B_{R_e}} \frac{\varrho_e(y)}{|x-y|}\,dy \quad , x \in B_{R_e},$$

$$p(\varrho_e(x))\big|_{x\in S_{R_e}} = p_0 ,$$

becomes a solution of the problem

(4.8) $$\nabla_{x'} p(\bar{\varrho}_e(x')) = k\bar{\varrho}_e(x')\nabla_{x'} \int_{\Omega_t} \frac{\bar{\varrho}_e(y')}{|T_t^{-1}(x') - T_t^{-1}(y')|}\, J_{T_t^{-1}}(y')\,dy',$$

$$\bar{p}_e(x') \equiv p_e(T_t^{-1}(x')) = p_0 \qquad x' \in S_t .$$

Therefore we introduce the quantities

(4.9) $$\varrho_\sigma = \varrho - \bar{\varrho}_e , \; p_\sigma = p - \bar{p}_e ,$$

and the following problem

(4.10) $$\begin{aligned} &\varrho(v_t + v\cdot\nabla v) - \operatorname{div}\mathbb{T}(v, p_\sigma) = \varrho\nabla U(\varrho) - \bar{\varrho}_e \nabla U_{T_t}(\bar{\varrho}_e) && \text{in } \Omega_t, \\ &\varrho_{\sigma t} + v\cdot\nabla\varrho_\sigma + \varrho \operatorname{div} v = 0 && \text{in } \Omega_t, \\ &\varrho_\sigma|_{t=0} = \varrho_0 - \bar{\varrho}_e|_{t=0} \equiv \varrho_{\sigma 0} ,\; v|_{t=0} = v_0 && \text{in } \Omega, \\ &\mathbb{T}(v, p_\sigma)\cdot \bar{n} = 0 && \text{on } S_t, \end{aligned}$$

where $t \in (0,T)$ and

(4.11) $$U_{T_t}(\bar{\varrho}_e(x)) = k\int_{\Omega_t} \frac{\bar{\varrho}_e(y)}{|T_t^{-1}(x) - T_t^{-1}(y)|}\, J_{T_t^{-1}}(y)\,dy .$$

We introduce the quantities

(4.12) $$\varphi_0(t) = \sum_{i=0}^{2}\left(\|\partial_t^i v\|^2_{H^{2-i}(\Omega_t)} + \|\partial_t^i p_\sigma\|^2_{H^{2-i}(\Omega_t)} \right),$$

$$\Phi(t) = \sum_{i=0}^{2} \left(\|\partial_t^i v\|^2_{H^{3-i}(\Omega_t)} + \|\partial_t^i p_\sigma\|^2_{H^{2-i}(\Omega_t)} \right),$$

and the spaces

(4.13) $$\mathfrak{N}(t) = \{(v, p_\sigma) : \varphi_0(t) < \infty\}$$

$$\mathfrak{M}(t) = \{(v, p_\sigma) : \varphi_0(t) + \int_0^t \Phi(\tau)d\tau < \infty\}.$$

Then we have

<u>Lemma 6</u> (see [4])

Let the initial data $v_0, p_{\sigma 0}, S$ of (1.1) be such that $(v(0), p_\sigma(0)) \in \mathfrak{N}(0)$ and $S \in H^{5/2}$. Let $\varepsilon_1 > 0$, $\varepsilon_2 > 0$. Let $v_0, p_{\sigma 0}$ be such that

(4.14) $$\varphi(0) \le \varepsilon_1,$$

where $\tilde{c}_1 \varphi_0(t) \le \varphi(t) \le \tilde{c}_2 \varphi_0(t)$ with constants $\tilde{c}_1, \tilde{c}_2$ dependent on data of the local solution from Lemma 5, and

(4.15) $$\|T_2^{-1}(x) - x\|^2_{L_2(\Omega)} \le \varepsilon_2.$$

Then for ε_1 and ε_2 sufficiently small there exists a local solution (v, p) of (1.1) such that $(v(t), p_\sigma(t)) \in \mathfrak{M}(t)$, $t \le T$, T is the time of local existence determined by Lemma 5 and there exists a constant c_1 such that

(4.16) $$\varphi(t) + \int_0^t \Phi(\tau)d\tau \le c_1(\varepsilon_1 + \varepsilon_2), \quad t \le T.$$

Next

<u>Lemma 7</u> (see [4])

For the local solution determined by Lemmas 5 and 6 we have

(4.17) $$\frac{d}{dt}\varphi + \Phi \le c_1 \varphi \Phi + c_2 \psi(t) + \varepsilon_3 \|\nabla_x^3 T_t^{-1}(x)\|^2_{L_2(\Omega_t)}$$
$$+ c_3\left(\frac{1}{\varepsilon_3}\right) \|T_t^{-1}(x) - x\|^2_{L_2(\Omega_t)}, \quad t \le T,$$

where $\psi(t) = \|v\|^2_{L_2(\Omega_t)} + \|\rho_\sigma\|^2_{L_2(\Omega_t)}$, $\varepsilon_3 \in (0,1)$ and c_3 is an increasing.

From the energy inequality (2.1) we have

$$(4.18)\quad \frac{1}{2}\int_{\Omega_t} \rho v^2 dx + E(\rho, \Omega_t) - E(\rho_e, B_{R_e}) + \int_0^t dt \int_{\Omega_t} |\mathbb{D}_0(v)|^2 dx$$

$$= \frac{1}{2}\int_{\Omega} \rho_0 v_0^2 dx + E(\rho_0, \Omega) - E(\rho_e, B_{R_e}).$$

From the variational considerations we have

$$(4.19)\quad E(\rho_e, B_{R_e}) = \min_{\rho, \Omega_t} E(\rho, \Omega_t).$$

Therefore

Lemma 8 (see [4])

Let $\varepsilon_0 > 0$ and

$$(4.20)\quad \frac{1}{2}\int_{\Omega} \rho_0 v_0^2 dx + E(\rho_0, \Omega) - E(\rho_e, B_{R_e}) \le \varepsilon_0 .$$

Then

$$(4.21)\quad \|v\|^2_{L_2(\Omega_t)} \le c_4 \varepsilon_0 .$$

From Theorem 4, (4.18) and (4.20) we have

$$(4.22)\quad \|\rho_\sigma\|^2_{L_2(\Omega_t)} \le c_5 \varepsilon_0 ,$$

and

$$(4.23)\quad \|T_t^{-1}(x) - x\|^2_{L_2(\Omega_t)} \le c_6 \varepsilon_0 .$$

In view of the above we have the lemma of prolongation

Lemma 9 (see [4])

Assume that there exists a local solution in $\mathfrak{M}(t)$, $t \le T$, satisfying assumptions of Lemmas 5, 6, with ε_1 and ε_2 so small that Lemma 7 holds. Assume that

(4.24) $\varphi(0) \leq \gamma$,

where γ is sufficiently small. Let ε_0 be sufficiently small. Then the local solution belongs to $\mathfrak{N}(t), t \leq T$, and

(4.25) $\varphi(t) \leq \gamma, \qquad t \leq T.$

To prove global existence we have to control the shape of the considered domain for all time and to guarantee that it is close to a ball. To this aim we introduce a system of coordinates with the origin in the barycentre of Ω_t, $t \geq 0$. For the equilibrium solution the barycentre coincides with the center of B_{R_e}, which we locate in the barycentre of Ω_t too. Finally we introduce a unit sphere S_1 with the same centre.

Lemma 10

Assume

(4.26) $\int_{\Omega} \varrho_0 v_0 \, d\xi = 0,$

then the barycentre of Ω_t is fixed in some external system of coordinates.

Assume

(4.27) $\int_{\Omega} \varrho_0 \xi \, d\xi = 0,$

then the barycentre is located in the origin of the system of coordinates.

Theorem 11 (global existence) (see [4])

Assume $p_0 > 0$, $M > 0$. Assume that $v = 0$, ϱ_e, B_{R_e} is the equilibrium solution. Assume that the initial data for (1.1) are such that $v_0, \varrho_{\sigma 0} \in \mathfrak{M}(0)$,

(4.28) $\varphi(0) \le \gamma \le \varepsilon_1$,

where γ is sufficiently small, and

(4.29) $$\int_\Omega \varrho_0 v_0 \, d\xi = 0, \quad \int_\Omega \varrho_0 \xi \, d\xi = 0, \quad \int_\Omega \varrho_0 \, d\xi = M.$$

Assume that $S \in H^{5/2}$ and the barycentre of Ω coincides with the centre of B_{R_e}. Assume that $\gamma_1 > 0$ is given and

(4.30) $$|\xi(s) - \xi_{R_e}(s)| \le \gamma_1 \le \varepsilon_2, \quad s \in S_1, \quad \xi(s) \in S, \quad \xi_{R_e}(s) \in \partial B_{R_e},$$

and S_1 is the unit sphere with the same centre as B_{R_e}. Assume that the transformation T_2 is defined by $T_2(\xi_{R_e}(s)) = \xi(s)$, $s \in S_1$. Assume $\varrho_e \in H^3(B_{R_e})$. Let $\varepsilon_0 > 0$ be given sufficiently small and let

$$E(\varrho_0, \Omega) - E(\varrho_e, B_{R_e}) \le \varepsilon_0.$$

Let ε_1 and ε_2 be sufficiently small.

Then there exists a global solution to (1.1) which is close to the equilibrium solution for all $t > 0$ such that

$$\varphi(t) \le \gamma, \quad |x(\xi(s),t) - \xi_{R_e}(s)| \le \gamma_1 \le \varepsilon_2, \quad s \in S_1 \text{ and}$$

$$\int_{\Omega_t} \varrho \, dx = M.$$

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