数理解析研究所講究録
1102巻 1999年 35-48

Generalized non-linear Schrödinger equations and related systems with derivative non-linearity

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[1]Partially supported by NSF

We will first consider the non-linear generalized Schrödinger equations of the form

$$\text{(NLS)} \qquad \begin{cases} \frac{\partial u}{\partial t} = iLu + F(u, \overline{u}, \nabla_x u, \nabla_x \overline{u}), & x \in \mathbb{R}^n, \quad t \in [-T, T] \\ u\,|_{t=0} = u_0 \end{cases}$$

where $F : \mathbb{C}^{2n+2} \to \mathbb{C}$ is a polynomial with no constant or linear terms, and $L = \sum_{j=1}^{k} \frac{\partial^2}{\partial x_j^2} - \sum_{j=k+1}^{n} \frac{\partial^2}{\partial x_j^2}$. The reasons for considering this type of equation may become apparent later. We want to establish local and global (in time) well-posedness (existence, uniqueness, continuous dependence on the data) in Sobolev spaces (or weighted Sobolev spaces). When $F = G(u, \overline{u})$, the standard energy estimate applies, and we obtain local well-posedness in H^s, $s > n/2$. For power-like non-linearities, more refined results can be obtained by means of "mixed norm estimates" and their generalizations (Strichartz estimates, $X_{s,b}$ spaces, etc.), using the contracting mapping principle in suitable spaces. In the general case, the difficulty stems from trying to "recover" the derivative in the non-linear term, in order to apply the energy method. This can be done in some cases:

$$\begin{aligned} &n = 1 \quad F = \partial_x(|u|^k u) \\ &n \geq 1 \quad F = F(u, \overline{u}, \nabla_x \overline{u}) \\ &n \geq 1 \quad \partial_{\partial x_j u} F,\ \partial_{\partial x_j \overline{u}} F,\ j = 1, \dots, n \in \mathbb{R} \end{aligned}$$

(Tsutsumi-Fukuda [T-F1], [T-F2], Klainerman [K], Klainerman-Ponce [K-P], Shatah [Sh]), but not in general. In 1991, Kenig-Ponce-Vega [KPV1] developed a method for general F, using the "local smoothing" properties of the linear problem

$$\text{(LIVP)} \qquad \begin{cases} \partial_t w = iLw + f \\ w\,|_{t=0} = w_0, \end{cases}$$

namely, if $\mathbb{R}^n = \cup_\alpha Q_\alpha$, Q_α are non-overlapping unit cubes, and we introduce the norms $|||v|||_T = \sup_\alpha ||v||_{L^2(Q_\alpha \times [-T,T])}$, $|||v|||_T' = \sum_\alpha ||v||_{L^2(Q_\alpha \times [-T,T])}$, and we let $J = (I - \Delta_x)^{\frac{1}{2}}$, we have, for $w = e^{itL}w_0 + \int_0^t e^{i(t-t')L} f(t')\,dt'$,

$$\sup_{|t|\leq T} ||w(t)||_{H^s(\mathbb{R}^n} + |||J^{s+\frac{1}{2}}w|||_T \leq C\left\{||w_0||_{H^s(\mathbb{R}^n)} + |||J^{s-\frac{1}{2}}f|||_T\right\}.$$

Notice that, in the passage between f and w, a derivative is gained, which allows us, through the use of Duhanel's formula

$$u(t) = e^{itL}u_0 + \int_0^t e^{i(t-t')L} F(u, \overline{u}, \nabla u, \nabla \overline{u})\,dt',$$

to prove:

Theorem 1 [K-P-V1]: If F is cubic or higher, there is $s = s_n$ such that, if $s \geq s_n$, $||u_0||_{H^{s_n}} \leq \delta_n$, $\delta_n > 0$, (N.L.S.) is locally (in time) well-posed. If F is quadratic, we need in addition $||u_0||_{H^{s_n}} + ||u_0||_{L^2(\mathbb{R}^n, |x|^{m_n}\,dx)} \leq \delta_n$, then the same result holds.

Theorem 2 [K-P-V2]: If in addition $\partial^\alpha F(0) = 0$, $|\alpha| \leq 4$, for small data we actually have global well-posedness.

Problem: In Theorem 1, to obtain a local results, we need smallness. Let me explain why: let us consider, for example, when $n = 1$,

$$\begin{cases} \frac{\partial u}{\partial t} = i\frac{\partial^2}{\partial x^2}u + u^2\frac{\partial u}{\partial x} \\ u\,|_{t=0} = u_0 \in H^s(\mathbb{R}). \end{cases}$$

By Duhamel's formula, we have

$$u(t) = e^{it\Delta}u_0 + \int_0^t e^{i(t-t')\Delta}u^2\frac{\partial u}{\partial x}\,dt' = e^{it\Delta}u_0 + \frac{\partial}{\partial x}\int_0^t e^{i(t-t')\Delta}\frac{u^3}{3}\,dt'$$

and we attempt to solve the integral equation by an appropriate fixed-point argument.

We start by estimating

$$\begin{aligned} |||J^{s+\frac{1}{2}}u|||_T &\leq C||u_0||_{H^s} + C|||J^{s+\frac{1}{2}}(u^3)|||'_T \\ &\leq C||u_0||_{H^s} + C|||u^2 J^{s+\frac{1}{2}}(u)|||'_T + \text{lower order terms.} \end{aligned}$$

Now,

$$\begin{aligned}
|||u^2 J^{s+\frac{1}{2}}(u)|||'_T &= \textstyle\sum_\alpha ||u^2 J^{s+\frac{1}{2}}(u)||_{L^2(Q_\alpha\times[-T,T])} \\
&\leq \left(\sup_\alpha ||J^{s+\frac{1}{2}}(u)||_{L^2(Q_\alpha\times[-T,T])}\right)\cdot\left(\textstyle\sum_\alpha ||u||^2_{L^\infty(Q_\alpha\times[-T,T])}\right) \\
&= |||J^{s+\frac{1}{2}}u|||_T \left(\textstyle\sum_\alpha ||u||^2_{L^\infty(Q_\alpha\times[-T,T])}\right).
\end{aligned}$$

This seems to be fine, but in order to map a ball into itself, one needs to have that

$$\sum_\alpha ||u||^2_{L^\infty(Q_\alpha\times[-T,T])}$$

is small. But since this is an L^∞ norm, it forces the initial data to be small. The method of proof also allowed us to obtain corresponding results for the Zakharov-Schulman systems [Z-Sc]:

$$\text{(Z-S)}\qquad \begin{cases} \partial_t u = iL_1 u - i\varphi u, & x\in\mathbb{R}^n,\quad t\in[-T,T] \\ L_2\varphi = L_3|u|^2 \\ u\,|_{t=0} = u_0 \end{cases}$$

where L_1, L_2 are non-degenerate second-order, not necessarily elliptic, and L_3 is of order 2. These systems model the interactions of small amplitude high frequency waves with acoustic waves. When $n = 2$ they coincide with the Davey-Stewartson [D-S] systems, for which Linares-Ponce [L-P] had obtained the analog of Theorem 1. In general, we have

Theorem 3 [K-P-V3]: Local well-posedness for (Z-S) with small data in weighted Sobolev spaces.

To see the connection between these problems, in (Z-S), we solve for φ, so that $\varphi = L_2^{-1}L_3|u|^2$, and thus, (Z-S) reduces to the single equation

$$\begin{cases} \partial_t u = iL_1 u - iL_2^{-1}L_3(|u|^2)\cdot u \\ u\,|_{t=0} = u_0. \end{cases}$$

If L_2 is elliptic, $L_2^{-1}L_3$ is of order 0, and this equation behaves like a cubic Schrödinger equation. If L_2 is not elliptic, L_2^{-1} "recovers" only one derivative, and hence $L_2^{-1}L_3$ is "of order 1," going back to the equations we started discussing.

Question: Can one remove smallness?

In 1992, Hayashi and Ozawa [H-O] were able to remove smallness in Theorem 1, when $n = 1$. Their idea was to eliminate the "bad" first-order term ("bad" from the point of view of the energy method) by using a change of the dependent variable u (by means of an "integrating factor"), setting

$$v = u(x,t)\exp\left(-\frac{1}{2}\int_{-\infty}^{x}\frac{\partial F}{\partial_{\partial_x u}}\right),$$

and then seeing that v verifies an equation that can be treated by the energy method. Then, in 1995, H. Chirara [C] succeeded, in the case when $L = \Delta$, in removing the smallness for all n in Theorem 1. I want to outline (briefly) his method, for future reference. It has two main steps:

Step 1: Diagonalization.

The idea here is to use the method used in the theory of symmetric hyperbolic systems. One writes the equation as a system in $\vec{w} = \binom{u}{\overline{u}}$. Then, one "eliminates the terms in $\frac{\partial}{\partial x}\overline{u}$ ", by diagonalizing this system. This involves a transformation $\vec{v} = \Lambda\vec{w}$, where $\Lambda = I - S$, $S = \begin{pmatrix} 0 & s_1 \\ s_2 & 0 \end{pmatrix}$, and s_i are classical pseudodifferential operators of order -1. It is at this point that the ellipticity of Δ is crucial.

Step 2: Energy estimates via the "sharp Gårding inequality."

After step 1 and "linearization" one is reduced to considering single equations of the form

$$\begin{cases} \frac{\partial v}{\partial t} &= i\Delta v + \vec{b}_1(x)\cdot\nabla v + Cv \\ v\,|_{t=0} &= v_0 \end{cases}$$

where C is a zero'th order, classical pseudodifferential operator in the x variable.

Problems of this kind had been considered for a long time. For instance, when $C = 0$, Mizohata [M] showed that a necessary condition for the estimate

$$\sup_{|t|\leq T} ||v(t)||_{L^2(\mathbb{R}^n)} \leq C_T ||v_0||_{L^2(\mathbb{R}^n)}$$

is

$$\sup_{x\in\mathbb{R}^n,\,\omega\in S^{n-1}} \left| \operatorname{Im} \int_0^R \vec{b}_1(x+r\omega)\cdot\omega\, dr \right| < \infty,$$

and Mizohata [M] also showed that

$$\sup_{x\in\mathbb{R}^n,\,\omega\in S^{n-1}} \int_0^\infty \left| D^\alpha \vec{b}_1(x+r\omega)\right| dr \leq C_\alpha,$$

for all α, is a sufficient condition. Mizohata's proof involved the use of the (exotic) pseudodifferential class $S^{0,0}$ of Calderón-Vaillancourt [C-V]. An alternative approach to this problem, using only classical pseudodifferential operators, was found by S. Doi [D], who, by introducing an appropriate classical zero'th order, positive pseudodifferential Ψ, and letting $w = \Psi v$, writing the system in w, using the "positivity of the commutator $i[\,\Delta, \Psi\,]$" and the sharp Gårding inequality, succeeded in implementing the energy method in this context. It was the approach of Doi that Chihara used, thus finishing the proof. We will see more explicit details of all this later on. Unfortunately, this elegant approach does not seem to be applicable to the case of general L.

As far as the smallness in Theorem 1, we now have:

Theorem 4 [K-P-V4]: Theorem 1 holds without the smallness condition.

I will now try to sketch the ideas used in the proof of Theorem 4. Let us illustrate our reduction to a linear problem, through the example used before:

$$\begin{cases} \frac{\partial u}{\partial t} = i\frac{\partial^2}{\partial x^2}u + u^2\frac{\partial u}{\partial x} \\ u\,|_{t=0} = u_0. \end{cases}$$

We rewrite the equation as

$$\frac{\partial u}{\partial t} = i\frac{\partial^2}{\partial x^2}u + u_0^2\frac{\partial u}{\partial x} + (u^2 - u_0^2)\cdot\frac{\partial u}{\partial x}.$$

Note that now $(u^2 - u_0^2)$ is zero at $t = 0$, and thus, it is small for small T. Thus, if we had the same estimates for the variable coefficient linear equation $\frac{\partial u}{\partial t} = i\frac{\partial^2 u}{\partial x^2} + u_0^2 \frac{\partial u}{\partial x}$, with constants depending on appropriate norms of u_0^2, the previous method would apply. We were thus led to studying the following class of linear problems:

$$\text{(IVP)} \quad \begin{cases} \partial_t u = iLu + \vec{b}_1(x) \cdot \nabla u + \vec{b}_2(x) \cdot \nabla \overline{u} + a_1(x)u + a_2(x)\overline{u} + f \\ u \,|_{t=0} = u_0 \end{cases}$$

and try to establish the estimate

$$(*) \qquad \sup_{|t| \le T} ||u(t)||_{H^s} + |||J^{s+\frac{1}{2}}u|||_T \le A_T \left\{ ||u_0||_{H^s} + |||J^{s-\frac{1}{2}}f|||'_T \right\},$$

where A_T depends on suitable norms of $\vec{b}_i$, a_i and T. As we mentioned before, the work of Mizohata shows that even when $L = \Delta$, $\vec{b}_1$ must decay. Also, when $n = 2$, $L = \partial^2_{xy}$, if $\vec{b}_2$ does not decay, it is shown in [K-P-V5] that the full "one half derivative" gain (when $f \equiv 0$) in $(*)$ may fail. In [K-P-V4], it is shown that, if $\vec{b}_i$ decay, a_i, $\vec{b}_i$ are smooth enough, then $(*)$ holds. I will now sketch a proof of this. For simplicity, assume $a_i \equiv 0$, $f \equiv 0$, $\vec{b}_i \in C_0^\infty$.

Step 1: Eliminate $\vec{b}_1$ without "spoiling" $\vec{b}_2$.

We introduce a pseudodifferential C "of order zero" with symbol $C(x, \xi)$ and let $v = Cu$. The equation for v is:

$$\partial_t Cu = iLCu + i\,[\,CL - LC\,]u + C\vec{b}_1(x)\nabla(u) + C\vec{b}_2(x)\nabla\overline{u}.$$

We want to choose C so that, modulo errors "of order zero,"

$$i\,[\,CL - LC\,] + C\vec{b}_1(x)\nabla = 0, \quad \text{and}$$

$$C\vec{b}_2(x)\nabla\overline{u} = \vec{b}_2(x)\nabla\overline{Cu}.$$

If $L = \sum_{j=1}^k \frac{\partial^2}{\partial x_j^2} - \sum_{j=k+1}^n \frac{\partial^2}{\partial x_j^2}$, then $q(\xi) = -(\xi_1^2 + \cdots + \xi_k^2) + (\xi_{k+1}^2 + \cdots + \xi_n^2)$ is its symbol, and if we let $\tilde{\xi} = (\xi_1, \ldots, \xi_{k+1}, -\xi_{k+1}, \ldots, -\xi_n)$, at the symbol level, modulo error "of order zero," we need

$$-2\tilde{\xi}\nabla_x C(x, \xi) = C(x, \xi)\, i\vec{b}_1(x) \cdot \xi, \quad \text{and}$$

$$C\overline{u} = \overline{Cu}, \text{ so that } \; C(x, -\xi) = C(x, \xi).$$

If we write $C(x,\xi) = \exp\gamma(x,\xi)$, we want

$$-2\tilde{\xi}\cdot\nabla_x\gamma(x,\xi) = i\vec{b}_1(x)\cdot\xi,$$

γ even in ξ. The equation is odd in ξ, so if $\gamma_0(x,\xi)$ is a solution, so is $\gamma(x,\xi) = \frac{1}{2}\{\gamma_0(x,\xi)+\gamma_0(x,-\xi)\}$. To find γ_0, we integrate the ODE, and obtain

$$\gamma_0(x,\xi) = \frac{1}{2}\int_0^\infty i\vec{b}_1(x+s\tilde{\xi})\cdot\xi\,dx.$$

We thus see the point of Mizohata's condition.

Problem: γ_0 is not in any "reasonable" symbol class.

In fact, the estimates for γ_0 are

$$\left|\partial_x^\beta\partial_\xi^\alpha\gamma_0(x,\xi)\right| \le C_{\alpha,\beta}\left(\frac{\langle x\rangle}{|\xi|}\right)^{|\alpha|},$$

where $\langle x\rangle = (1+|x|^2)^{\frac{1}{2}}$. Unfortunately, symbols with these bounds need not give L^2-bounded operators. When $L=\Delta$, γ_0 falls in a class of symbols considered by [C-K-S], and one can proceed with this program (see [K-P-V6]). This cannot be done using the results in [C-K-S], when L is not elliptic. In [K-P-V4], the way out was to consider

$$\gamma_{0,R}(x,\xi) = \gamma_0(x,\xi)\,\theta\left(\frac{R\langle x\rangle}{\langle\xi\rangle}\right)\cdot\psi\left(\frac{\langle\xi\rangle}{R}\right),$$

where $\theta\in C_0^\infty$, $\theta\equiv 1$ near 0, $\psi\in C^\infty$, $\psi\equiv 1$ at infinity, and $R>1$. Then $\gamma_{0,R}$ is in the class $S^0_{0,0}$, and we obtain L^2-boundedness by the Calderón-Vaillancourt theorem [C-V]. (This idea originates in [T].) Unfortunately, the class $S^0_{0,0}$ does not have a very good calculus, but this is overcome in [K-P-V4] by taking R large.

Step 2: "Energy method."

After step 1, we have

$$\partial_t v = iLv + \vec{b}_2(x)\nabla\overline{v} + Av,$$

where A is "order zero." The energy method then applies to give H^s estimates. Once they are obtained, Doi's approach [D] gives the "local smoothing" estimate.

There is also an alternative approach, which is developed in [K-P-R-V], and which shows that $\gamma_0(x,\xi)$, $\exp(\gamma_0(x,\xi))$ give rise to L^2 bounded operators for all L. The point is that the redeeming feature of $\gamma_0(x,\xi)$ is that its ξ support is contained in a cone, with opening angle of size $1/|x|$, and is homogeneous of degree 0 in ξ. The "bad ξ" are those for which $\xi \cdot \tilde{\xi} = 0$ (the characteristic directions). This is a "small set." An almost orthogonality argument then gives the L^2-boundedness, and a "partial calculus" (they are not an algebra), where everything is done composing with smooth cut-off functions. This allows us to extend the results just mentioned to variable coefficient L, where the multiplication by $\theta\left(\frac{R\langle x\rangle}{\langle\xi\rangle}\right)$ does not work. We thus have:

Theorem 5 [K-P-V-R]: The IVP

$$\begin{cases} \partial_t u = iLu + F(x,t,u,\overline{u},\nabla_x u,\nabla_x \overline{u}), & t\in[-T,T],\ x\in\mathbb{R}^n \\ u\,|_{t=0} = u_0 \end{cases}$$

is locally well-posed in appropriate Sobolev spaces, where

$$Lu = \sum_{j,k} \frac{\partial}{\partial x_j}\left(a_{jk}(x)\frac{\partial}{\partial x_k}u\right) + \vec{b}_1(x)\nabla_x u + \vec{b}_2(x)\cdot\nabla_x\overline{u} + C_1(x)u + C_2(x)\overline{u},$$

where the $\vec{b}_1$ are smooth and decay, the C_i are smooth and bounded, and $A(x) = (a_{jk}(x))$ is real, smooth, symmetric, invertible, with non-trapped bicharacteristics, and such that, outside of a compact set,

$$A(x) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}.$$

Then F is smooth, of polynomial growth, and the Taylor coefficients of $F(x,t,-,-,-,-)$ at the origin vanish for order ≤ 1.

We now turn to a different problem, where this circle of ideas has proved local well-posedness for large data, for the first time. It is the system introduced by Ishimori [I], as a two-dimensional generalization of the Heisenberg equation in ferromagnetism. It is the system (when $c_0 = 1$, $c_1 = 0$, Heisenberg system)

$$\begin{cases} \partial_t S = S \wedge (\partial_x^2 S + c_0\, \partial_y^2 S) + c_1\, (\partial_x \varphi \partial_y S + \partial_y \varphi \partial_x S) \\ \partial_x^2 \varphi + c_2\, \partial_y^2 \varphi = c_3\, S(\partial_x S \wedge \partial_y S) \\ S(x, y, 0) = S_0(x, y), \quad (x, y) \in \mathbb{R}^2, \end{cases}$$

and $S(-,-,t) : \mathbb{R}^2 \to \mathbb{R}^3$, $|S|^2 = 1$, $S \to (0,0,1)$ as $(x, y) \to \infty$ and $\wedge$ denotes the wedge product in $\mathbb{R}^3$. The constants verify $(c_0, c_1, c_2, c_3) = (1, c_1, -1, -2)$ (elliptic-hyperbolic) or $(c_0, c_1, c_2, c_3) = (-1, c_1, 1, -2)$ (hyperbolic-elliptic). When $c_1 = 1$, it can be studied by inverse scattering [Su]. By using sterographic projection, we can eliminate the constraint $|S|^2 = 1$. Thus let $u : \mathbb{R}^2 \to \mathbb{C}$, and let $S = (S_1, S_2, S_3) = \frac{1}{1+|u|^2}\,(u + \overline{u}, -i(u - \overline{u}), 1 - |u|^2)$. We then rewrite the Ishimori system in u:

$$\text{(IS)} \quad \begin{cases} i\partial_t u + \partial_x^2 u + c_0\, \partial_y^2 u &= \frac{2\overline{u}}{1+|u|^2}\,((\partial_x u)^2 - c_0\,(\partial_y u)^2) \\ &+ \; ic_1\,(\partial_x u \partial_y \varphi + \partial_y u \partial_x \varphi) \\ \partial_x^2 \varphi + c_2\, \partial_y^2 \varphi &= 2ic_3\, \frac{\partial_x u \partial_y \overline{u} - \partial_x \overline{u} \partial_y u}{(1+|u|^2)^2} \\ u(x, y, 0) &= u_0(x, y). \end{cases}$$

The "hyperbolic-elliptic" case is easier, since we can solve for φ, and, since Δ^{-1} recovers two derivatives, we are left with an equation with "no derivatives in $\overline{u}$." One can then use a version of the method of Doi [D] to obtain local well-posedness. This was carried out by Souyer [S]. The elliptic-hyperbolic case is much more involved. In 1997, Hayashi [H] showed local well-posedness for small data in weighted Sobolev spaces. We now have:

Theorem 6 [K-P-V7]: The elliptic-hyperbolic (IS) is locally well-posed in weighted Sobolev spaces, for large data.

The strategy is to reduce things once more, to studying a linear system.

When $c_0 = 1$, $c_2 = -1$, after a rotation in the (x, y) plane, we obtain

$$\text{(IS')} \quad \begin{cases} i\partial_t u + \Delta u &= \frac{2\overline{u}}{1+|u|^2}\partial_x u \partial_y u + ic_1 \left\{\partial_x\varphi\partial_x u - \partial_y\varphi\partial_y u\right\}\partial_x\partial_y\varphi \\ &+ \; ic_3 \frac{\partial_x u\partial_y\overline{u} - \partial_x\overline{u}\partial_y u}{(1+|u|^2)^2} \\ u(x,y,0) &= u_0(x,y). \end{cases}$$

We then reduce this to a single equation

$$\text{(IE')} \quad \begin{cases} i\partial_t u + \Delta u &= \frac{2\overline{u}}{1+|u|^2}\partial_x u\, \partial_y u + ic_1\, \partial_x u \int_{-\infty}^{y} \frac{\partial_x u\, \partial_y\overline{u} - \partial_x\overline{u}\, \partial_y u}{(1+|u|^2)^2} \\ &- \; ic_1 c_3\, \partial_y u \int_{-\infty}^{x} \frac{\partial_x u\, \partial_y\overline{u} - \partial_x\overline{u}\, \partial_y u}{(1+|u|^2)^2} \\ u(x,y,0) &= u_0(x,y). \end{cases}$$

We wish to apply related ideas in our previous methods.

Problems: (1) $\int_{-\infty}^{x}$ does not decay in x, only in y. (2) $\int_{-\infty}^{x}$ is not a -1 order pseudodifferential operator (not L^2-bounded).

Way out: For (2) we observe that we have a "cubic" non-linearity, which gives rise, when we linearize, to terms like $\varphi_1 \int_{-\infty}^{x} \varphi_2$, where the φ_i decay in x. This actually is an order -1 pseudodifferential operator in x! For (1), we use pseudodifferential operators in each variable separately, viewed as Hilbert space valued pseudodifferential operators, and use vector valued sharp Gårding inequalities. After many calculations, this method works.

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