数理解析研究所講究録
1114巻 1999年 126-138

Nonsymmetric Indices of Power and their Application to the House of Councilors in Japan

Katsunori Ano, Susumu Seko and Takashi Suzuki
Department of Information Systems
and Quantitative Sciences
Nanzan University
Japan

1 Introduction

This paper deals with the Shapley-Shubik, Banzhaf and nonsymmetric Shapley-Owen indices of power and their application to the House of Councilors in Japan. We studied the following three areas.

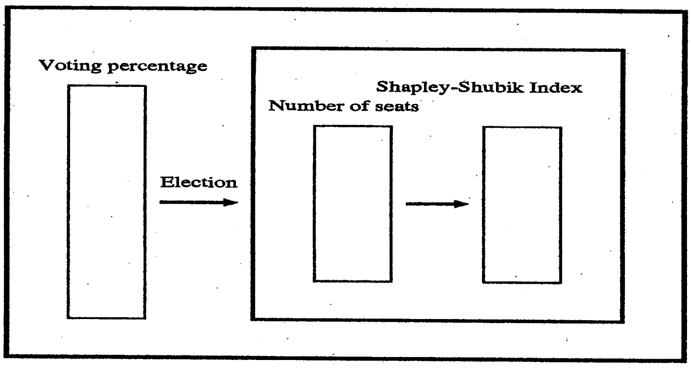
(1) We analyzed the party power of each party in the Japanese House of Councilors in 1998 using the nonsymmetric Shapley-Owen index. Ono and Muto [1] applied the nonsymmetric Shapley-Owen index to the Japanese House to evaluate the party power distribution in the Japanese House of Councilors during the period between 1989 and 1992. In the election of 1998, the Liberal Democratic Party (LDP) could not regain a majority in the House. They lost their dominant position in the election of 1989. The nonsymmetric Shapley-Owen index, which was originally developed by Owen [3] and later improved by Shapley [8], incorporated voter nonsymmetry into the original symmetric Shapley-Shubik index. This nonsymmetric index takes into account the ideological differences of the parties. Two different methods, factor analysis and quantification method III, were used to construct an ideology profile space of the pattern of voting among the parties observed between January and June of 1998 (before the election of 1998). The data showed that the distribution of bills submitted to the House was very strongly biased. This contradicts a criterion of the index which assumes random appearance of bills. Therefore, we employed Ono and Muto's method of evaluating the power of each party without assuming randomness of bills, to evaluate the current power of each party in the House. It is often said that small parties which are ideologically located between large parties have strong power in comparison with the number of seats they have. This was supported by our analysis as well as by the indices we obtained using quantification method III, which showed that the Liberal Party (LP) and the Social Democratic Party (SDP) have a certain power, although they hold only 12 and 13 seats, respectively.

(2) We investigated the power of each party in the Japanese House of Councilors after the 1998 election, taking into consideration the supporting rate for each party among eligible voters as well as the voting percentage, through the seat function which itself depended on the voting percentage and supporting rate. This index was named the Voting Shapley-Shubik (VSS) index with seat as a function of the voting percentage, to distinguish it from the original Shapley-Shubik index without the seat function. Figure 1 illustrates the VSS index. We also determined the Shapley-Shubik and Banzhaf indices of each party.

(3) We studied the power of each party in the House of Councilors after 1998 election , taking into consideration the supporting rate for each party among eligible voters and the percentage as a variable, with the nonsymmetric Shapley-Owen index. We demonstrate that the party power of the LDP decreases as the voting percentage in the election increases.

In Section 2, the nonsymmetric Shapley-Owen index of Ono and Muto [1] is reviewed. In Section 3, the nonsymmetric Shapley-Owen index of each party in the House of Councilors was calculated based on the 76 nonunanimous votes between January and June of 1998. An ideology profile spaces was constructed by two different methods, factor analysis and quantification method III. Section 4 deals with several symmetric indices of the seat function. In Section 5, we evaluate the power of each party after the 1998 election talking into account the supporting rate for each party among eligible

Figure 1 : Voting Shapley-Shubik Index



voters and the voting percentage as a variable,with the nonsymmetric Shapley-Owen index.

2 Nonsymmetric Shapley-Owen Index

In this section we review the nonsymmetric Shapley-Owen index presented by Ono and Muto [1]. Let $N = \{1, 2, \cdots, n\}$ be the set of voters in the House of Councilors who vote for or against particular bills, and let $v : 2^N \rightarrow R$ be a characteristic function, where 2^N is the set of all subsets $S \subseteq N$ and R is the set of real numbers. Denote the set of winning coalitions as W and the set of losing ones as L. Let us suppose that voters join a coalition one after another and eventually form the grand coalition. Then, there exists a unique voter who joins and thereby turns a losing coalition into a winning one. This voter is called a pivot. Each of $n!$ orderings of n voters has a unique pivot. The Shapley-Shubik index of a voter is the probability of his being a pivot when every ordering is equally likely. Therefore, the Shapley-Shubik index of a voting game (N, v) is given as the n-vector $\psi(v) = (\psi_1(v), \cdots, \psi_n(v))$ where $\psi_i(v) = \sum_{S\in L, S\cup\{i\}\in W}(s!(n-s-1)!/n!)$ for $i = 1, \cdots, n$, where s is the number of members in S. As shown in the definition, the Shapley-Shubik index assumes that each ordering of n voters forms with equal probability. However, in the actual political world, some orderings are more probable than others. For example, consider three voters 1, 2 and 3, who are a liberalist, a centrist and a conservative, respectively. Because the extreme voters 1 and 3 are opposed to each other, the ordering 132 is less likely to be formed than 123 or 321, where the ordering ijk implies that i is the first to join, j is the second to join, and k is the last to join, that is, the coalition is formed in the order of i, j, k. Owen [3] introduced an ideology profile space to consider the nonsymmetry of voters. If there is only one ideological axis, for example, a left-right axis, then the case of the three voters can be depicted in terms of a line as in Figure 2. For each $i = 1, 2, 3$, x^i denotes the position of voter i. The midpoints of the line segments x^1x^2, x^1x^3 and x^2x^3 serve as the borders which divide the line into four regions, E_1, E_2, E_3 and E_4. For any bill in region E_1, voter 1 is the closest, voter 2 is the next closest, and voter 3 is the most distant. Thus, the grand coalition is formed in the order 123. Similarly in regions E_2, E_3 and E_4, the orderings are 213, 231 and 321, respectively. It should be noted that regions E_1 and E_4 are unbounded intervals. This implies that if bills arise at random along the whole real line, orderings 123 and 321 would appear at an equal probability of 0.5. In a simple majority case, voter 2 is pivotal in both orderings. Thus, voter 2 has all of the power. In a unanimous case, voter 3 is pivotal in 123 and voter 1 is pivotal in 321. In this case, voters 1 and 3 have equal power of 0.5 and voter 2 is powerless.

Figure 3 depicts the case of three voters in a two-dimensional space. Each x^i denotes the position of voter i. Since there are three points, there are three perpendicular bisectors which intersect the midpoint of the lines which connect each pair of points. In Figure 3, the line $l_{ij} - l'_{ij}$ represents the perpendicular bisector of x^i and $x^j, j = 1, 2, 3, i \neq j$. For instance, bills in the sector formed by the half-lines Ol_{13} and Ol'_{12} produce the ordering 312, since for any bill in this region, voter 3 is the most enthusiastic supporter, voter 1 is the next enthusiastic, and voter 2 is the least enthusiastic. Assuming that issues appear at random in the two-dimensional space, the probability that ordering

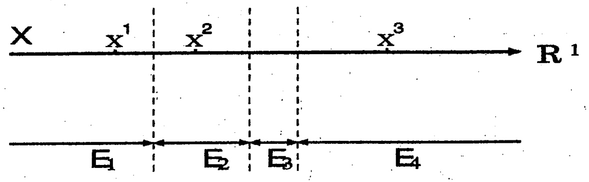


Figure 2 : The locations of three voters on a one-dimensional profile space

312 is produced is $\alpha/2\pi$ where α is the angle formed by the two lines Ol_{13} and Ol'_{12}. For each of the other regions, we can find which ordering is produced and its probability in a similar manner. The ordering in each region is given in Figure 3. Therefore, in the simple majority case where the second voter is always a pivot, the nonsymmetric Shapley-Owen index is given by $(\alpha/\pi, \beta/\pi, \gamma/\pi)$. In the unanimous case, it is given by $((\beta+\gamma)/2\pi, (\alpha+\gamma)/2\pi, (\alpha+\beta)/2\pi)$, since the last voter is the pivot.

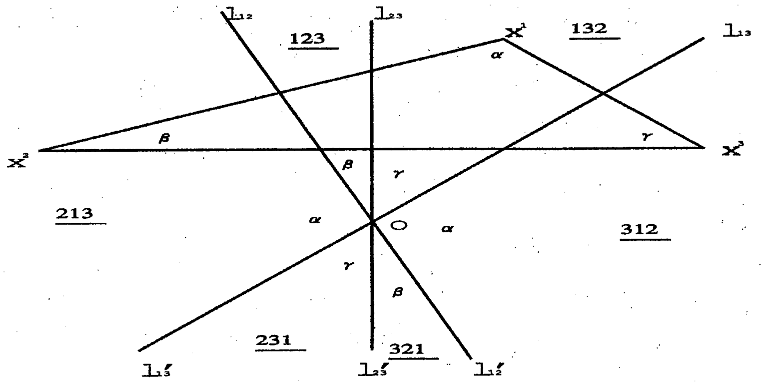


Figure 3 : The locations of three voters in a two-dimensional space

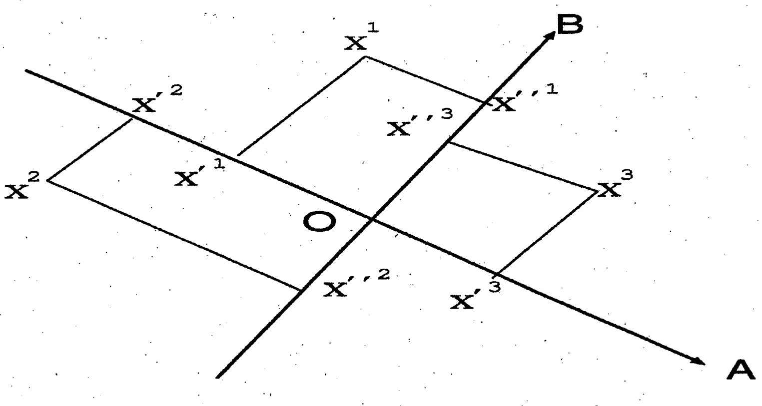


Figure 4 : Shapley's construction of orderings

Shapley [8] developed an alternative method of finding the nonsymmetric Owen index applied to the case with $(n-1)$-dimensional space where n is the number of voters. For a more detailed explanation, please refer to Owen and Shapley [4]. In his method, the voters are also represented as points in a space. Issues, however, are represented not by points but by vectors passing through the origin of the space. Consider two voters, i and j, and an issue A. The positions of the two voters are represented by points x^i and x^j. Drop perpendiculars from x^i and x^j to A, and denote their foots by x'^i and x'^j, respectively. In this model, voter i prefers A more than j does, or i precedes j with respect to A, if $||Ox'^i|| > ||Ox'^j||$ where O is the origin of the space and $||Ox'^i||$ or $||Ox'^j||$ is the distance along the arrow A between O and x'^i, or between O and x'^j, respectively. In Figure 4, for issue A We have $||Ox'^3|| > -||Ox'^1|| > -||Ox'^2||$, producing the ordering 312. Similarly, the ordering for issue B is 132. If we rotate the arrow around the origin assuming that issues arise at random, we find for each ordering the sector in which it is produced. For each ordering the proportion of the corresponding angle to 2π gives the probability at which the ordering appears. The nonsymmetric index is obtained by finding a pivotal voter in an arbitrary position. The same indices are obtained even if the origins are different. The nonsymmetric index produced by this method is often called the Shapley-Owen index.

3 Application of the nonsymmetric Shapley-Owen index to the voting behavior of the Japanese House of Councilors in January-June, 1998

We evaluated the power of each party in the Japanese House of Councilors in 1998. In the election of 1998, the LDP did not regain a majority of seats in the House. The LDP had held a majority until the election of 1989. After the election of 1989, they lost the majority, and this produced movement by many renewal parties such as the Liberal Party and the Democratic Party of Japan. Ono and Muto [1] analyzed the power of each party in the House during the period 1989-1992. In this study, we investigated the power of each party in the House during the period of January-June, 1998 by analyzing the data concerning the "yea/nay" voting pattern of the parties in nonunanimous votes. The number of seats held by each party after the election of 1998 is given in Table 1.

Table 1: Number of Seats in the House of Councilors held by each party before and after the election of 1998

Party	Before	After
Liberal Democratic Party (LDP)	118	105
Democratic Party of Japan (DPJ)	38	47
Japan Communist Party (JCP)	14	23
Komeito (Komei)	24	22
Social Democratic Party (SDP)	20	13
Liberal Party (LP)	11	12
Others	27	30
Total	252	252

The voting system in the House was formulated as a weighted majority game in which each party is represented as a voter, and the number of seats held by each party is represent as its weight. In the following analysis, we selected the six largest parties as voters in the game to simplify the analysis. Thus, the weighted majority game in the House before the election of 1998 is shown in Table 2.

Table 2: The weighted majority game in the House of 1998 before the election

quota	LDP	DPJ	JCP	Komei	SDP	LP
127	118	38	14	24	20	11

The symmetric Shapley-Shubik index and Banzhaf index of each party in the House before the election of 1998 are given in Table 3.

Using the procedures of Rabinowitz and Macdonald [6] and Ono and Muto [1], we constructed an ideology profile space by performing factor analysis of the 76 nonunanimous votes that occurred in the House of Councilors during the period January - June, 1998. Table 4 shows the patterns of yea/nay voting behavior of the parties and the number of times each pattern occurred in the 76 nonunanimous votes. We also constructed an ideology profile space by quantification method III and

Table 3: Shapley-Shubik index and Banzhaf index of each party

Index	LDP	DPJ	JCP	Komei	SDP	LP
Shapley-Shubik	.835	.033	.033	.033	.033	.033
Banzhaf	.968	.031	.031	.031	.031	.031

evaluated the indices of this profile space.

Table 4: Pattern of yes/no voting of the 6 parties in the 76 nonunanimous votes during January-June, 1998 (Sangiin Kaigiroku [7])

Type	LDP	DPJ	Komei	SDP	JCP	LP	Number
1	Y	Y	Y	Y	N	Y	50
2	Y	N	N	Y	N	N	12
3	Y	N	Y	Y	N	Y	6
4	Y	Y	Y	Y	N	N	2
5	Y	Y	N	Y	N	Y	1
6	Y	N	Y	Y	Y	Y	1
7	Y	Y	Y	N	N	N	1
8	Y	Y	N	Y	N	N	1
9	Y	Y	Y	N	N	Y	1
10	N	Y	Y	N	Y	Y	1

Indices of the profile space constructed using factor analysis

Factor analysis showed that the first main factor accounted for 41.011 percent of the variance in the 76 votes, and that the second factor accounted for 34.354 percent of the variance.

Table 5: Position of each party in the ideology profile space constructed using factor analysis

Party	1st factor	2nd factor	3rd factor	4th factor	5th factor
LDP	0.58120	1.08979	-0.21761	0.52755	1.92172
DPJ	0.24418	-1.04659	-1.71312	-0.27420	-0.04130
JCP	-2.02265	0.26989	0.26500	0.03890	0.02193
Komei	0.34389	-0.76896	0.76362	1.59576	-0.57231
SDP	0.54723	1.21513	-0.05561	-0.54636	-1.44536
LP	0.30615	-0.75921	1.11962	-1.34165	0.51532

Table 6: Direction of each type of issue in the ideology profile space constructed using factor analysis

Type	1st factor	2nd factor	3rd factor	4th factor	5th factor
1	0.991	-0.132	-0.013	-0.019	-0.011
2	0.437	0.893	-0.106	-0.007	-0.030
3	0.689	0.301	0.653	-0.091	0.008
4	0.665	0.190	-0.474	0.505	-0.208
5	0.656	0.193	-0.306	-0.633	0.213
6	-0.120	0.513	0.839	0.134	0.020
7	0.427	-0.265	-0.426	0.675	0.332
8	0.501	0.459	-0.725	-0.107	-0.013
9	0.571	-0.575	0.011	0.197	0.551
10	-0.437	-0.893	0.106	0.007	-0.030

Therefore, the first two factors accounted for 75.365 percent of the variance. Tables 5 and 6 show the position of each party and the direction of each type of issue, respectively, in the profile space. Figure 5 shows the two-dimensional profile space constructed using factor analysis.

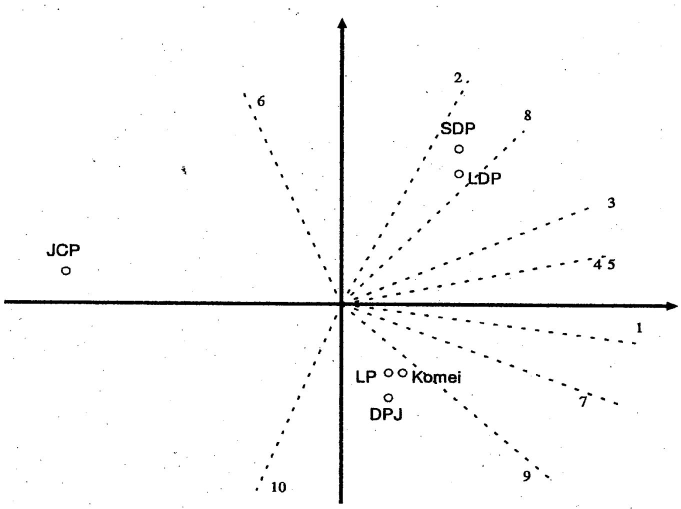


Figure 5 : Two-dimensional profile space constructed by factor analysis

Figure 5 shows that the observed issues are not uniformly distributed. The nonuniformity of issues is intrinsic to our study because bills are mainly submitted by a cabinet, in which most of the members belong to the governing party. The nonsymmetric Shapley-Owen index assumes randomness of the bills. The results of the factor analysis do not meet this assumption. Therefore, we employed the method of Ono and Muto [1] to calculate the nonsymmetric index using only the data. Thus, the obtained index shows only the power distribution during January-June, 1998, and does not predict future voting behavior. We need to find a pivotal party for each type of issue in the two-dimensional case. For all types of issues, the LDP is the pivot. For example, the order of type 5 is SDP $\rightarrow$ LDP $\rightarrow$ Komei$\rightarrow$ LP $\rightarrow$ DPJ $\rightarrow$ JCP, and the LDP is the pivot. Thus, the index for LDP is $76/76 = 1$. The indices of the other five parties are 0. The indices for the one-dimensional case are also noted in Table 7.

Table 7: Nonsymmetric Shapley-Owen index of each party in the ideology profile space constructed by factor analysis

Dimension	LDP	DPJ	JCP	Komei	SDP	LP
1	0.026	0	0	0	0.974	0
2	1.0	0	0	0	0	0

Indices of the profile space constructed using quantification method III

Factor analysis assumes that the distribution of the data is normal, while quantification method III assumes categorical data. Thus, we can make a profile space in which the parties and issues are represented by points. The position of each party and the position of each issue in the ideology profile space constructed using quantification method III are given in Tables 8 and 9, respectively.

Table 8: Position of each party in the ideology profile space constructed using quantification method III

Party	1st axis	2nd axis	3rd axis	4th axis	5th axis
LDP	-0.29048	-1.00258	-0.19608	0.51956	1.41022
DPJ	-0.01951	1.05822	-1.85980	-0.43518	-0.05965
JCP	12.5135	-2.44468	-0.95954	-0.11216	0.03835
Komei	0.15059	0.84133	0.64291	1.67177	-0.60648
SDP	-0.29535	-1.12619	0.00666	-0.52647	-1.36737
LP	0.16825	0.83024	1.29852	-1.31924	0.58238

Table 9: Position of each issue in the ideology profile space constructed using quantification method III

Type	1st axis	2nd axis	3rd axis	4th axis	5th axis
1	-0.12057	0.37251	-0.05989	-0.18909	-0.15178
2	-0.61635	-3.29855	-0.40515	-0.03650	0.40341
3	-0.14045	-0.35422	2.83500	0.91206	0.08829
4	-0.23922	-0.17760	-2.10654	3.24510	-2.93087
5	-0.22993	-0.18619	-1.08076	-4.64814	2.66485
6	5.15380	-1.7986 0	1.06695	0.49285	0.21502
7	-0.11180	0.92656	-2.94767	6.17927	4.6734 0
8	-0.42458	-1.10589	-4.14991	-1.55556	-0.10174
9	0.00466	1.33816	-0.17908	1.15299	6.24620
10	6.74016	0.22088	-1.37359	-0.51410	-0.21094

As with factor analysis, we can obtain the index of each party from the profile space using quantification method III. In the two-dimensional case, the LDP is pivotal for Types 1, 2, 6, 7, 8, 9 and 10; the SDP is pivotal for Types 3 and 5; and the LP is pivotal for Type 4. Thus, the index is $(50+12+1+1+1+1+1)/76 = 0.882$ for the LDP; $(6+2+1)/76 = 0.118$ for the SDP. Table 10 gives the non-symmetric Shapley-Owen index of each party for the one-dimensional and two-dimensional cases. The indices obtained using quantification method III are more realistic than those obtained using factor analysis. The index values of each party show that the LDP has a large amount of power, even though they do not hold a majority of seats in the House of Councilors.

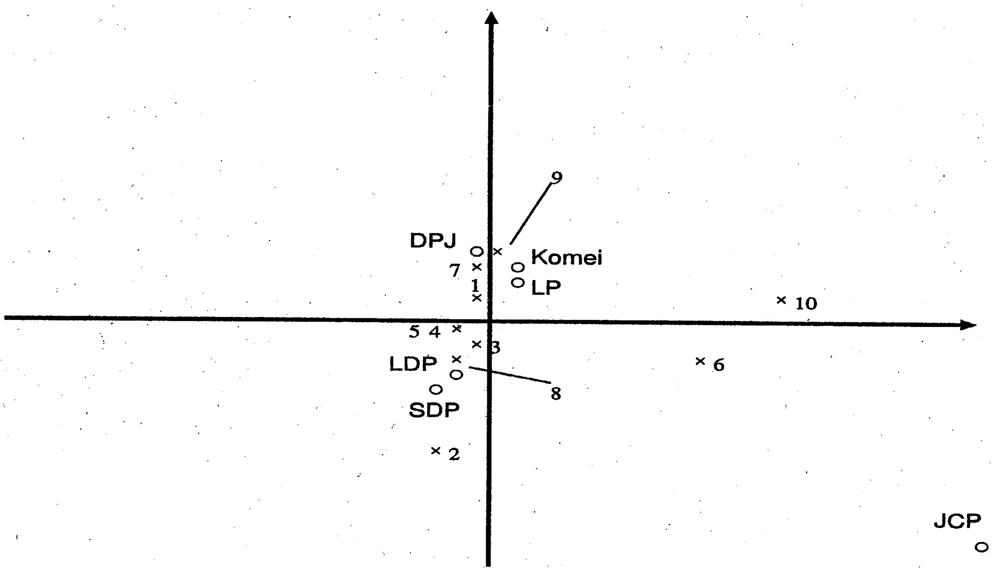


Figure 6 : Two-dimensional profile space constructed by quantification method III

Table 10: Nonsymmetric Shapley-Owen index of each party in the ideology profile space constructed by quantification method III

Dimension	LDP	DPJ	JCP	Komei	SDP	LP
1	0.961	0	0	0	0.039	0
2	0.882	0	0	0	0.118	0

4 Shapley-Shubik and Banzhaf indices taking into consideration the supporting rate for each party and the voting percentage as a variable

The number of seats held by each party is determined by the results of the election, which depend on the voting percentage, i.e., the percentage of eligible voters who vote in the election. We consider the number of seats as a function, and combine the Shapley-Shubik index of a party with the voting percentage and supporting rate for that party. Table 11 shows the number of seats held by the top 6 parties after the election and the voting percentage in the elections of 1989, 1992, 1995 and 1998 of the House of Councilors. In the House, fifty percent of the members are up for election every three years.

In Table 11, the SDPJ (Social Democratic Party of Japan) of 1989 and 1992 and the SDP of 1995 and 1998, are different parties. However, all of the members of the SDP had been members of the SDPJ. The Rengo and the Minsha disappeared before the election of 1995, and almost all of the members of these two parties joined the new DPJ party. By regression analysis of the number of seats, where the independent variable is the number of seats held by a particular party after an election and the dependent variable is the voting percentage in the 4 elections, only the Komei had a high $R^2 = 0.921$. The R^2 of the other parties was at most 0.559, which indicates that the number of seats held by of each party cannot be estimated by the voting percentage. Thus, we arbitrarily made the seat function, $\phi_i(x), i = 1, \cdots, 6$ of the number of seats held by party after an i as follows, where x is the voting percentage in the election of members of the House. It is said in Japan that at higher voting percentages, the number of seats held by the LDP after an election decreases, and the number of seats held by the DPJ increases. The numbers in parentheses in Table 11 correspond to the number of seats held by each party after the election as calculated from the seat function,$\phi_i(x)$.

Table 11: Number of seats held by each party after the election and the voting percentage in the elections of 1989, 1992, 1995 and 1998

Year of Election	1989	1992	Year of Election	1995	1998
Voting percentage	65.2	50.7	Voting percentage	44.5	58.83
LDP	109	108	LDP	118(112)*	105(106)*
SDPJ	74	69	SDP	20(16) *	13(14) *
JCP	14	11	JCP	14(16) *	23(21) *
Komei	21	24	Komei	24(23) *	22(23) *
Rengo	12	12	DPJ	38(43) *	47(49) *
Minsha	10	7	LP	11(12)*	12(13)*

* The numbers in parentheses indicate the number of seats held by each party as calculated by the seat function, $\phi_i(x)$

Table 12: The seat function $\phi_i(x)$

Party(i)	LDP(1)	DPJ(2)	JCP(3)	Komei(4)	SDP(5)	LP(6)
$\phi_i(x)$	$130-40x$	$25+40x$	$3+30x$	23	$23-15x$	$9+7x$

The number of seats held by each party after an election when the voting percentage is 40, 50, 60, or 70 percent, as calculated by the seat function ϕ, is given in Table 13. Tables 14 and 15 give the Voting-Shapley-Shubik index and the Voting-Banzhaf index, respectively, of each party when the voting percentage varies between 40 and 70 percent. The indices suggest that as the voting percentage increases, the LDP loses power but would still have the largest amount of power among the parties.

Table 13: Number of seats held by each party after the election according to the seat function $\phi_i(x)$

Voting percentage (%)	LDP	DPJ	JCP	Komei	SDP	LP
40	114	41	15	23	17	12
50	110	45	18	23	16	13
60	106	49	21	23	14	13
70	102	53	24	23	12	14

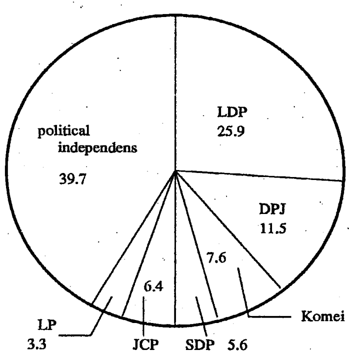
Table 14: Voting Shapley-Shubik indices using the seat function $\phi_i(x)$

Voting percentage (%)	LDP	DPJ	JCP	Komei	SDP	LP
40	0.800	0.0500	0.0500	0.0500	0.0500	0
50	0.767	0.0667	0.0667	0.0667	0.0167	0.0167
60	0.767	0.0667	0.0667	0.0667	0.0167	0.0167
70	0.533	0.133	0.0833	0.0833	0.0833	0.0833

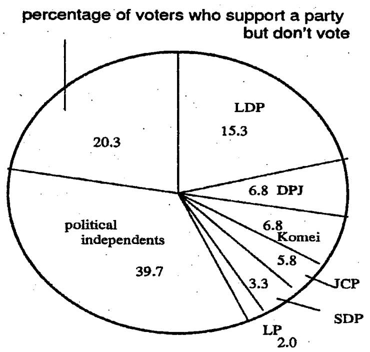
Table 15: Voting Banzhaf indices using the seat function $\phi_i(x)$

Voting percentage (%)	LDP	DPJ	JCP	Komei	SDP	LP
40	0.938	0.0625	0.0625	0.0625	0.0625	0
50	0.906	0.0938	0.0938	0.0938	0.0313	0.0313
60	0.906	0.0938	0.0938	0.0938	0.0313	0.0313
70	0.688	0.188	0.125	0.125	0.125	0.125

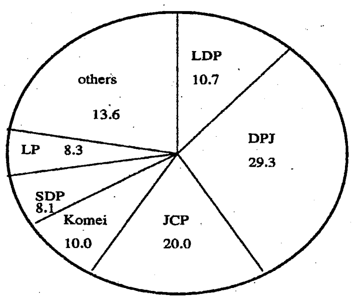
As the next step, we considered both the voting percentage and the supporting rate for each party, i.e., the percentage of eligible voters who support a particular party, and made the seat function dependent on these two factors. The supporting rates for each party in 1998 are given in Graph 1. There are, however, voters who support a party but don't vote. We assume that they comprise 20.3 percent of all eligible voters. We also assume that 40 percent of all voters support a party and vote for that party. This implies that the lowest voting percentage is 40 percent and the highest is 79.7 percent (see Graph 2). The JCP and Komei have well-organized, strong systems of voting among their members. Therefore, 90 percent of the supporters of the JCP and Komei are assumed to vote. Political independents comprise 39.7 percent of all voters. Their voting behavior is important for all parties. We assumed that the political independents voted for the respective parties at the rates indicated in Graph 3; these data were obtained from a public opinion poll conducted by Kyoudoutuusinsya on July 15, 1998.



Graph 1 : Supporting rates for each party in 1998



Graph 2 : Voting rates in an election



Imidas 1999 p.259

Graph 3 : Voting behavior of political independents in 1S

Therefore, when the voting percentage is x and the supporting rate is given by the graphs, the number of seats, $\phi_1'(x)$, held by the LDP after the election of 1998 can be determined by the equation, $\phi_1'(x) = 252((15.3/x) + (10.7/100)(x-40)/40)$, where $15.3/x$ represents the percentage of voters who support the LDP in the voting percentage x; $(10.7/100)(x-40)/x$ represents the percentage of voters who don't support the LDP but vote for the LDP in the voting percentage x; and 252 represents the total number of seats in the House. Similarly, the number of seats held by the DPJ after the election, $\phi_2'(x)$, is given by $\phi_2'(x) = 252((6.8/x) + (29.3/100)(x - 40/x))$, and for the JCP, Komei, SDP and LP, it is given by $\phi_3'(x) = 252((5.8/x) + (20/100)(x - 40/x)), \phi_4'(x) = 252((6.8/x) + (10/100)(x - 40/x)), \phi_5'(x) = 252((3.3/x) + (8.1/100)(x - 40/x))$, and $\phi_6'(x) = 252((2/x) + (8.3/100)(x - 40/x))$, respectively. The number of seats held by each party after the 1998 election as calculated by these formulas, is given in Table 16, and the Voting Shapley-Shubik index and the Voting Banzhaf index of each party are given in Tables 17 and 18, respectively. We can see that as the voting percentage increases, the power of the DPJ increases and the power of the LDP decreases. The results of this analysis indicate that if the voting percentage is 70 percent, all 6 parties after the election would have approximately the same amount of power.

Table 16: Number of seats held by each party after the 1998 election according to the seat function $\phi_i'(x)$

Voting percentage (%)	LDP	DPJ	JCP	Komei	SDP	LP
40	96	43	36	43	21	13
50	83	49	39	39	21	14
60	73	53	41	37	21	15
70	67	56	42	35	21	16

Table 17: Voting Shapley-Shubik indices using the seat function $\phi_i'(x)$, at various voting percentages

Voting percentage (%)	LDP	DPJ	JCP	Komei	SDP	LP
40	0.533	0.133	0.133	0.133	0.0333	0.0333
50	0.433	0.2	0.15	0.15	0.0333	0.0333
60	0.350	0.2	0.2	0.133	0.0833	0.0333
70	0.333	0.25	0.167	0.133	0.1	0.0667

Table 18: Voting Banzhaf indices using the seat function $\phi_i'(x)$, at various voting percentages

Voting percentage (%)	LDP	DPJ	JCP	Komei	SDP	LP
40	0.813	0.188	0.188	0.188	0.0625	0.0625
50	0.688	0.313	0.25	0.25	0.0625	0.0625
60	0.594	0.344	0.344	0.25	0.156	0.0625
70	0.5	0.438	0.313	0.25	0.188	0.125

5 Nonsymmetric Shapley-Owen index taking into consideration the supporting rate for each party at various voting percentage

In this section, using the nonsymmetric Shapley-Owen index and the seat functions, $\phi_i(x)$, and $\phi_i'(x)$, we analyze the power of each party. For the index, we use the same data on the voting behavior of the parties and the same ideology profile spaces constructed by factor analysis and quantification method III. Note that we use data on the voting behavior among the parties before the election of 1998, and the number of seats held by each party in the after the election of 1998. However, the derived indices may suggest the future power of each party. The method of calculation is the same as that in Section 3. Thus, we present only the results of the two-dimensional case in Tables 19-22. The indices using factor analysis seem to be unrealistic. We can see that the Komei has a certain power. This may be the reason that since January, 1999, the LDP has been exploring the possibility of making a coalition cabinet with the Komei.

Table 19: Nonsymmetric Shapley-Owen index of each party in the profile space constructed by factor analysis and the function $\phi_i(x)$,at various voting percentages

Voting percentage (%)	LDP	DPJ	JCP	Komei	SDP	LP
40	1	0	0	0	0	0
50	0.697	0	0.013	0.211	0	0.079
60	0.697	0	0.013	0.211	0	0.079
70	0.039	0	0.013	0.211	0	0.737

Table 20: Nonsymmetric Shapley-Owen index of each party in the profile space constructed by quantification method III and the function $\phi_i(x)$, at various voting percentages

Voting percentage (%)	LDP	DPJ	JCP	Komei	SDP	LP
40	0.882	0	0	0.026	0.092	0
50	0.697	0	0	0.026	0	0.276
60	0.697	0	0	0.026	0	0.276
70	0.697	0	0	0.026	0	0.276

Table 21: Nonsymmetric Shapley-Owen index of each party in the profile space constructed by factor analysis and the function $\phi_i'(x)$, at various voting percentages

Voting percentage (%)	LDP	DPJ	JCP	Komei	SDP	LP
40	0.684	0	0.013	0.224	0	0.079
50	0.026	0	0.013	0.303	0	0.658
60	0.026	0	0.013	0.303	0.658	0
70	0	0	0.013	0.092	0.684	0.211

Table 22: Nonsymmetric Shapley-Owen index of each party in the profile space constructed by quantification method III and the function $\phi_i'(x)$, at various voting percentages

Voting percentage (%)	LDP	DPJ	JCP	Komei	SDP	LP
40	0.684	0.013	0	0.026	0	0.276
50	0.684	0.013	0	0.303	0	0
60	0.684	0.013	0	0.276	0.026	0
70	0.684	0.013	0	0.276	0.026	0

6 Conclusion

In conclusion, we mention that when the voting percentage varies and the supporting rates for each party differ, analysis of power indices may be helpful in estimating the power of each party after an election, if we can make a reliable seat function.

Acknowledgments

This work was supported by a grant from Pache Research Subsidy I-A (Special Research) for 1999.

REFERENCES

1. Ono, R. and Muto, S., *Party Power in the House of Councilors in Japan: An Application of the Nonsymmetric Shapley-Owen Index*, Journal of the Operations Research Society of Japan, **40** (1997), 21-33.

2. Owen, G., *Political Games*, Naval Research Logistics Quarterly, **18** (1971), 345-355.

3. Owen, G., *Game Theory*, Third edition, Academic Press, 1995.

4. Owen, G. and Shapley, L. S., *Optimal Location of Candidates in Ideological Space*, International Journal of Game Theory, **18** (1989), 339-356.

5. *Imidas (in Japanese)*, (1998) Syueisya.

6. Rabinowitz, G. and Macdonald, S. E., *The Power of the States in U.S. Presidential Elections*, American Political Science Review, **80** (1986), 65-87.

7. *Sangiin Kaigiroku (in Japanese)*, (1998). Okurasyo-Insatukyoku.

8. Shapley, L. S., *A Comparison of Power Indices and a Nonsymmetric Generalization*, P-5872, (1977) The Rand Corporation, Santa Monica, California.