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COUNTABLE STAR-COVERING PROPERTIES

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ABSTRACT. We introduce two new notions of topological spaces called a countably starcompact space and a countably absolutely countably compact (= countably acc) space. We clarify the relations between these spaces and other related spaces and investigate topological properties of countably starcompact spaces and countably acc spaces. Some examples showing the limits of our results are also given.

1. INTRODUCTION

By a space, we mean a topological space. Let us recall that a space X is *countably compact* if every countable open cover of X has a finite subcover. Fleischman defined in [4] a space X to be *starcompat* if for every open cover $\mathcal{U}$ of X, there exists a finite subset B of X such that $St(B,\mathcal{U}) = X$, where

$$St(B,\mathcal{U}) = \bigcup\{U \in \mathcal{U} : U \cap B \neq \emptyset\}.$$

He proved that every countably compact space X is starcompact. Conversely, van Douwen-Reed-Roscoe-Tree [2] proved that every starcompact T_2-space is countably compact, but this does not hold for T_1-spaces (see Example 2.5 below). Strengthening the definition of starcompactness, Matveev defined in [5] a space X to be *absolutely countably compact* (= *acc*) if for every open cover $\mathcal{U}$ of X and every dense subspace D of X, there exists a finite subset F of D such that $St(F,\mathcal{U}) = X$. Every acc T_2-space is countably compact ([5]), but an acc T_1-space need not be countably compact (see Example 2.4 below). These definitions motivate us to define the following spaces:

Definition 1.1. A space X is *countably starcompact* if for every countable open cover $\mathcal{U}$ of X, there exists a finite subset B of X such that $St(B,\mathcal{U}) = X$.

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Definition 1.2. A space X is *countably absolutely countably compact* (= *countably acc*) if for every countable open cover $\mathcal{U}$ of X and every dense subspace D of X, there exists a finite subset F of D such that $St(F, \mathcal{U}) = X$.

The purpose of this paper is to clarify the relationship among these spaces and to consider topological properties of a countably starcompact space and a countably acc space. From the definitions and above remarks, we have the following diagram, where $A \to B$ means that every A-space is a B-space:

$$\begin{tikzcd}
 & \text{countably compact} \arrow[d] & \\
\text{acc} \arrow[r] \arrow[d] & \text{starcompact} \arrow[r] \arrow[d, "T_2"] & \text{countably starcompact} \\
\text{countably acc} & \text{countably compact} &
\end{tikzcd}$$

DIAGRAM 1

The cardinality of a set A is denoted by $|A|$. For a cardinal κ, κ^+ denotes the smallest cardinal greater than κ. Let $\mathfrak{c}$ denote the cardinality of the continuum, ω the first infinite cardinal and $\omega_1 = \omega^+$. As usual, a cardinal is the initial ordinal and an ordinal is the set of smaller ordinals. For each ordinals α, β with $\alpha < \beta$, we write $(\alpha, \beta) = \{\gamma : \alpha < \gamma < \beta\}$, $[\alpha, \beta) = \{\gamma : \alpha \leq \gamma < \beta\}$ and $[\alpha, \beta] = \{\gamma : \alpha \leq \gamma \leq \beta\}$, Other terms and symbols will be used as in [3].

2. Relations among spaces

In this section, we consider the relations among countably acc spaces, countably starcompact spaces and other related spaces.

Proposition 2.1. *Every countably compact space is countably acc and every countably acc space is countably starcompact.*

Proof. Let X be a countably compact space. Let $\mathcal{U}$ be a countable open cover of X and let D be a dense subspace of X. Then, there exists a finite subcover $\{U_1, U_2, \ldots U_n\}$ of $\mathcal{U}$, since X is countably compact. Pick a point $x_i \in U_i \cap D$ for $i = 1, 2, \ldots n$. Then, $St(\{x_1, x_2, \cdots, x_n\}, \mathcal{U}) = X$, which shows that X is countably acc. Hence, every countably compact space is countably acc. It follows immediately from the definitions that every countably acc space is countably starcompact. □

Proposition 2.2. *For a T_2-space X, the following conditions are equivalent:*

(1) X *is countably compact*;
(2) X *is countably acc*;
(3) X *is countably starcompact.*

Proof. The implications (1) $\Rightarrow$ (2) and (2) $\Rightarrow$ (3) are true by Proposition 2.1. It remains to show that (3) $\Rightarrow$ (1). Suppose that X is not countably compact. Then, there exists an infinite closed discrete subset $D = \{x_n : n \in \omega\}$ in X. For each $m \in \omega$, let $D_m = \{x_n : 2^m \leqslant n < 2^{m+1}\}$; then $|D_m| = 2^m$. Since X is a T_2-space, there exists a collection $\mathcal{U}_m = \{U_n : 2^m \leqslant n < 2^{m+1}\}$ of pairwise disjoint open sets in X such that $U_n \cap D = \{x_n\}$ for each $n \in \omega$. Take such a collection $\mathcal{U}_m$ for each $m \in \omega$ and let

$$\mathcal{U} = \{X \setminus D\} \cup \bigcup_{m \in \omega} \mathcal{U}_m.$$

Then, $\mathcal{U}$ is a countable open cover of X. Let B be any finite subset of X with $|B| = k$. Since $|B| < 2^k = |\mathcal{U}_k|$ and $\mathcal{U}_k$ is disjoint, some $U_n \in \mathcal{U}_k$ does not intersect B. Then, $x_n \notin St(B,\mathcal{U})$, because U_n is only member of $\mathcal{U}$ containing x_n. Hence, X is not countably starcompact. This proves that (3) $\Rightarrow$ (1). $\square$

Proposition 2.3. *Every countably starcompact space X is pseudocompact.*

Proof. Let f be a continuous real-valued function on X, and let $U_n = \{x \in X : n - 1 < f(x) < n + 1\}$ for each $n \in \mathbb{Z}$. Then, $\mathcal{U} = \{U_n : n \in \mathbb{Z}\}$ is a countable open cover of X. Since X is countably starcompact, there exists a finite subset B of X such that $St(B,\mathcal{U}) = X$. Since $\mathcal{U}$ is point-finite, the set $\{U \in \mathcal{U} : U \cap B \neq \emptyset\}$ is finite, say $\{U_{n_1}, U_{n_2}, \dots U_{n_k}\}$. If we put $M = \max\{|n_i| + 1 : i = 1, 2, \dots k\}$, then $|f(x)| \leq M$ for each $x \in X$. Hence, every continuous real-valued function on X is bounded, which means that X is pseudocompact. $\square$

Summing up the above results, we have the following diagram, where the implications (1)–(6) hold for arbitrary spaces and the inverses of implications (2)–(5) also hold for T_2-spaces:

$$\begin{tikzcd}
 & & \text{acc} \arrow[d, "(1)"] \\
 & \text{countably compact} \arrow[r, "(2)"] \arrow[d, "(3)"] & \text{countably acc} \arrow[d, "(4)"] \\
\text{acc} \arrow[r, "(6)"] & \text{starcompact} \arrow[r, "(5)"] \arrow[d, "T_2"] & \text{countably starcompact} \\
 & \text{countably compact} &
\end{tikzcd}$$

DIAGRAM 2

In the rest of this section, we give examples which show the implications (1)–(6) in Diagram 2 cannot be reversed in the realm of T_1-spaces. The first one shows that the inverses of the implications (2) and (3) do not hold for T_1-spaces.

Example 2.4. *There exists an acc T_1-space which is not countably compact.*

Proof. Let κ be an infinite cardinal and A a set of cardinality κ. Define $X = \kappa^+ \cup A$. We topologize X as follows: κ^+ has the usual order topology and is an open subspace of X, and a basic neighborhood of $a \in A$ takes the form

$$G_\beta(a) = (\beta, \kappa^+) \cup \{a\}, \quad \text{where } \beta < \kappa^+.$$

Then, X is a T_1-space which is not countably compact, because A is infinite discrete closed in X. To show that X is absolutely countably compact, let $\mathcal{U}$ be an open cover of X. Let D be the set of all isolated points of κ^+. Then, D is dense in X and every dense subspace of X includes D. Thus, it is suffices to show that there exists a finite subset $F \subseteq D$ such that $St(F,\mathcal{U}) = X$. Since κ^+ is absolutely countably compact, there is a finite subset $F' \subseteq D$ such that $\kappa^+ \subseteq St(F',\mathcal{U})$. For each $a \in A$, there is $\beta(a) < \kappa^+$ such that $G_{\beta(a)}(a)$ is included in some member of $\mathcal{U}$. If we choose $\beta \in D$ with $\beta > \sup\{\beta(a) : a \in A\}$, then $A \subseteq St(\beta,\mathcal{U})$. Let $F = F' \cup \{\beta\}$. Then, $St(F,\mathcal{U}) = X$. Hence, X is absolutely countably compact, which completes the proof. □

The second example shows that the inverses of the implications (3), (4) and (6) in Diagram 2 do not hold for T_1-spaces.

Example 2.5. *There exists a starcompact T_1-space which is not countably acc.*

Proof. Let $Y = (\omega+1)\times\omega_1$, where both $\omega+1$ and ω_1 have the usual order topologies and Y has the Tychonoff product topology. Let $X = \omega \cup Y$. We topologize X as follows: Y is an open subspace of X; a basic neighborhood of a point $n \in \omega$ takes the form

$$O_\alpha(n) = \{n\} \cup ((n,\omega] \times (\alpha,\omega_1)) \quad \text{where } \alpha < \omega_1.$$

Then, X is a T_1-space. To show that X is starcompact, let $\mathcal{U}$ be an open cover of X. Then, there exists finite subset F_1 of Y such that $Y \subseteq St(F_1,\mathcal{U})$, since Y is countably compact. For each $n \in \omega$, there is $\alpha_n < \omega_1$ such that $O_{\alpha_n}(n)$ is included in some member of $\mathcal{U}$. If we choose $\alpha_0 < \omega_1$ with $\alpha_0 > \sup\{\alpha_n : n \in \omega\}$, then $\omega \subseteq St(\langle\omega,\alpha_0\rangle,\mathcal{U})$. Let $F_0 = F_1 \cup \{\langle\omega,\alpha_0\rangle\}$. Then, $X = St(F_0,\mathcal{U})$, which shows that X is starcompact.

Next, we show that X is not countably acc. Let $D = \omega \times \omega_1$. Then, D is dense in X. Therefore, it is suffices to show that there exists a countable open cover $\mathcal{V}$ of X such that $St(A,\mathcal{V}) \neq X$ for any finite subset A of D. Let us consider the countable open cover

$$\mathcal{V} = \{Y\} \cup \{O_0(n) : n \in \omega\}.$$

Let A be any finite subset of D. Then, there exists $n \in \omega$ such that $([n,\omega]\times\omega_1)\cap A = \emptyset$. Hence, $n \notin St(A,\mathcal{V})$, since $O_0(n)$ is only element of $\mathcal{V}$ such that $n \in O_0(n)$. This shows that X is not countably acc. □

The third example shows that the inverses of the implications (1) and (5) in Diagram 2 do not hold for T_1-spaces.

Example 2.6. *There exists a countably acc T_1-space which is not a starcompact space.*

Proof. Let $X = \omega_1 \cup A$, where $A = \{a_\alpha \colon \alpha \in \omega_1\}$. We topologize X as follows: ω_1 has the usual order topology and is an open subspace of X; a basic neighborhood of a point $a_\alpha \in A$ takes the form

$$O_\beta(a_\alpha) = \{a_\alpha\} \cup (\beta, \omega_1) \quad \text{where } \beta < \omega_1.$$

Then, X is a T_1-space. To show that X is countably acc, let $\mathcal{U}$ be a countable open cover of X. Let D be the set of all isolated points of ω_1. Then, D is dense in X and every dense subspace of X includes D. Thus, it is suffices to show that there exists a finite subset $F \subseteq D$ such that $St(F,\mathcal{U}) = X$. Since ω_1 is absolutely countably compact, there is a finite subset $F' \subseteq D$ such that $\omega_1 \subseteq St(F',\mathcal{U})$. Let $\mathcal{V} = \{U \in \mathcal{U} : U \cap A \neq \emptyset\}$. For each $U \in \mathcal{V}$, there exists a $\beta_U < \omega_1$ such that $(\beta_U, \omega_1) \subseteq U$. Since $\mathcal{V}$ is countable, we can choose $\beta \in D$ with $\beta > \sup\{\beta_U : U \in \mathcal{V}\}$. Thus, $A \subseteq St(\beta, \mathcal{V}) \subseteq St(\beta,\mathcal{U})$, since $\beta \in U$ for each $U \in \mathcal{V}$. Let $F = F' \cup \{\beta\}$. Then, $X = St(F,\mathcal{U})$, which shows that X is countably acc.

Next, we show that X is not starcompact. Let us consider the open cover

$$\mathcal{V} = \{\omega_1\} \cup \{O_\alpha(a_\alpha) : \alpha < \omega_1\}.$$

Let A be any finite subset of X. Then, there exists $\alpha < \omega_1$ such that $A \cap ((\alpha, \omega_1) \cup \{a_\beta : \beta > \alpha\}) = \emptyset$. Choose $\beta > \alpha$. Then $a_\beta \notin St(A, \mathcal{V})$, since $O_\alpha(a_\alpha)$ is only element of $\mathcal{V}$ containing a_α for each $\alpha \in \omega_1$. This shows that X is not starcompact. □

Remark 1. Pavlov [8] proved that a countably compact space need not be acc even if it is a normal T_2-space.

3. Discrete sum and subspaces

We begin with a proposition which follows immediately from the definitions of a countably starcompact space and a countably acc space:

Proposition 3.1. *The discrete sum of a finite collection of countably starcompact (resp. countably acc) spaces is countably starcompact (resp. countably acc).*

It is well-known that a closed subspace of a countably compact space is countably compact. However, a similar result does not hold for starcompactness, countable starcompactness and countable acc property. In fact, the following example shows that these properties do not preserved by taking regular closed subspaces.

Example 3.2. *There exists an acc T_1-space having a regular-closed subspace which is not countably starcompact.*

Proof. Let $S_1 = \omega \cup \mathcal{R}$ be the Isbell-Mrówka space [7], where $\mathcal{R}$ is a maximal almost disjoint family of infinite subsets of ω such that $|\mathcal{R}| = \mathfrak{c}$. Since$S_1$ is not countably compact, S_1 is not countably starcompact by Proposition 2.2.

Let $S_2 = \mathfrak{c}^+ \cup A$, where A is a set of cardinality $\mathfrak{c}$. We topologize S_2 as follows: $\mathfrak{c}^+$ has the usual order topology and is an open subspace of S_2, and a basic neighborhood of $a \in A$ takes the form

$$G_\beta(a) = (\beta, \mathfrak{c}^+) \cup \{a\}, \quad \text{where } \beta < \mathfrak{c}^+.$$

We assume that $S_1 \cap S_2 = \emptyset$. Let $\varphi : \mathcal{R} \to A$ be a bijection. Let X be the quotient space obtained from the discrete sum $S_1 \oplus S_2$ by identifying r with $\varphi(r)$ for each $r \in \mathcal{R}$. Let $\pi : S_1 \oplus S_2 \to X$ be the quotient map. It is easy to check that $\pi(S_1)$ is a regular-closed subset of X, however, it is not countably starcompact, since it is homeomorphic to S_1.

Next, we show that X is acc. For this end, let $\mathcal{U}$ be an open cover of X. Let S be the set of all isolated points of $\mathfrak{c}^+$ and let $D = \pi(S \cup \omega)$. Then, D is dense in X and every dense subspace of X includes D. Thus, it is suffices to show that there exists a finite subset F of D such that $X = St(F,\mathcal{U})$. By the proof of Example 2.4, S_2 is acc. Since $\pi(S_2)$ is homeomorphic to the space S_2, $\pi(S_2)$ is acc, hence, there exists a finite subset F_0 of $\pi(S)$ such that $\pi(S_2) \subseteq St(F_0,\mathcal{U})$. On the other hand, since $\pi(S_1)$ is homeomorphic to S_1, every infinite subset of $\pi(\omega)$ has an accumulation point in $\pi(S_1)$. Hence, there exists a finite subset F_1 of $\pi(\omega)$ such that $\pi(\omega) \subseteq St(F_1,\mathcal{U})$. For, if $\pi(\omega) \not\subseteq St(B,\mathcal{U})$ for any finite subset $B \subseteq \pi(\omega)$, then, by induction, we can define a sequence $\{x_n : n \in \omega\}$ in $\pi(\omega)$ such that $x_n \notin St(\{x_i : i < n\},\mathcal{U})$ for each $n \in \omega$. By the property of $\pi(S_1)$ mentioned above, the sequence $\{x_n : n \in \omega\}$ has a limit point x_0 in $\pi(S_1)$. Pick $U \in \mathcal{U}$ such that $x_0 \in U$. Choose $n < m < \omega$ such that $x_n \in U$ and $x_m \in U$. Then, $x_m \in St(\{x_i : i < m\},\mathcal{U})$, which contradicts the definition of the sequence $\{x_n : n \in \omega\}$. Let $F = F_0 \cup F_1$. Then, $X = St(F,\mathcal{U})$. Hence, X is acc, which completes the proof. □

4. Mappings

It is well-known that a continuous image of a countably compact space is countably compact (see [3]) and a continuous image of a starcompact space is starcompact (see [2]). Similarly, we have the following proposition.

Proposition 4.1. *A continuous image of a countably starcompact space is countably starcompact.*

Proof. Suppose that X is a countably starcompact space and $f : X \to Y$ a continuous onto map. Let $\mathcal{U}$ be a countable open cover of Y. Then, $\mathcal{V} = \{f^{-1}(U) : U \in \mathcal{U}\}$ is a countable open cover of X. Since X is countable starcompact, there exists a finite set $B \subseteq X$ such that $St(B, \mathcal{V}) = X$. Let $F = f(B)$. Then, F is a finite set of Y and $St(F,\mathcal{U}) = Y$. Hence, Y is countably starcompact. □

Matveev showed in [5, Example 3.1] that a continuous image of an acc space need not be acc. Now, we give an example showing that a continuous image of an acc T_1-space need not be countably acc.

Example 4.2. *There exist an acc T_1-space X and a continuous map $f : X \to Y$ onto a space Y which is not countably acc.*

Proof. Let $X_1 = (\omega + 1) \times \omega_1$ with the Tychonoff product topology, where both $\omega + 1$ and ω_1 have the usual order topologies. Then, X_1 is acc by [5, Theorem 2.3], since $\omega + 1$ is a first countable, compact space and ω_1 is acc by [5, Theorem 1.8].

Let $X_2 = \omega_1 \cup \omega$. We topologize X_2 as follows: ω_1 has the usual order topology and is an open subspace of X_2, and a basic neighborhood of $n \in \omega$ takes the form

$$G_\beta(n) = (\beta, \omega_1) \cup \{n\}, \quad \text{where } \beta < \omega_1.$$

By the proof of Example 2.4, X_2 is acc.

Let $X = X_1 \oplus X_2$ be the discrete sum of X_1 and X_2. Then, X is acc by Proposition 1.3 [5].

Let $Y = X_1 \cup X_2$. We topologize Y as follows: X_1 is an open subspace of Y; a basic neighborhood of a point $\beta < \omega_1 \subseteq X_2$ takes the form

$$O_{\gamma,m}(\beta) = ([m, \omega] \times \omega_1) \cup (\gamma, \beta], \quad \text{where } \gamma < \beta \text{ and } m \in \omega.$$

a basic neighborhood of a point $n \in \omega$ takes the form

$$O_\alpha(n) = ([n, \omega] \times \omega_1) \cup (\alpha, \omega_1) \cup \{n\}, \quad \text{where } \alpha < \omega_1;$$

To show that Y is not countably acc. Let $D = \omega \times \omega_1$. Then, D is dense in Y. Therefore, it is suffices to show that there exists a countable open cover $\mathcal{V}$ of Y such that $St(A, \mathcal{V}) \neq Y$ for any finite subset A of D. Let us consider the countable open cover

$$\mathcal{V} = \{X_1 \cup \omega_1\} \cup \{O_0(n) : n \in \omega\}.$$

Let A be any finite subset of D. Then, there exists a $n \in \omega$ such that $([n, \omega] \times \omega_1) \cap A = \emptyset$. Hence, $n \notin St(A, \mathcal{V})$, since $O_0(n)$ is only element of $\mathcal{V}$ such that $n \in O_0(n)$ for each $n \in \omega$, which shows that Y is not countably acc.

Let $f : X \to Y$ be the identity map. Then, f is continuous. This completes the proof. $\square$

Recall from [5 or 6] that a continuous mapping $f : X \to Y$ is *varpseudoopen* provided $\mathrm{int}_Y f(U) \neq \emptyset$ for every nonempty open set U of X. In [5], it was proved that a continuous varpseudoopen image of an acc space is acc. Similarly, we prove the following proposition.

Proposition 4.3. *A continuous varpseudoopen image of a countably acc space is countably acc.*

Proof. Suppose that X is a countably acc space and $f : X \to Y$ is a continuous varpseudoopen onto map. Let $\mathcal{U}$ be a countable open cover of Y and D a dense subspace of Y. Then, $\mathcal{V} = \{f^{-1}(U) : U \in \mathcal{U}\}$ is a countable open cover of X, and $f^{-1}(D)$ is a dense subspace of X since f is a varpseudoopen map. Hence, there exists a finite set $B \subseteq f^{-1}(D)$ such that $St(B, \mathcal{V}) = X$. Let $F = f(B)$. Then, F is a finite set of D and $St(F, \mathcal{U}) = Y$, which shows that Y is a countably acc space. $\square$

Now, we consider preimages. It is well-known that a perfect preimage of a countably compact space is countably compact (see [3, Theorem 3.10.10]) but a perfect preimage of an acc space need not be acc (see [1, Example 3.2]). Now, we give an example showing that

(1) a perfect preimage of a starcompact space need not be starcompact,
(2) a perfect preimage of a countably starcompact space need not be countably starcompact, and
(3) a perfect preimage of a countably acc space need not be countably acc.

Our example uses the Alexandorff duplicate $A(X)$ of a space X: The underlying set of $A(X)$ is $X\times\{0,1\}$; each point of $X\times\{1\}$ is isolated and a basic open neighborhood of $\langle x,0\rangle \in X\times\{0\}$ is a set of the from $(U\times\{0\})\cup((U\times\{1\})\setminus\{\langle x,1\rangle\})$, where U is an open neighborhood of x in X.

Example 4.4. *There exists a perfect onto map $f: X\to Y$ such that Y is an acc T_1-space, but X is not countably starcompact.*

Proof. Let $Y=\omega_1\cup\omega$. We topologize Y as follows: ω_1 has the usual order topology and is an open subspace of Y, and a basic neighborhood of $n\in\omega$ takes the form

$$G_\beta(n)=(\beta,\omega_1)\cup\{n\},\quad \text{where } \beta<\omega_1.$$

By the proof of Example 2.4, Y is an acc space.

Let $X=A(Y)$. Then, X is not countably starcompact, since $\omega\times\{1\}$ is countable discrete, open and closed in X and countable starcompactness is preserved by open and closed set.

Let $f: X\to Y$ be the projection. Then, f is a perfect onto map. This completes the proof. □

5. Products

It is well-known that the product of a countably compact space and a compact space is countably compact. However, the product of an acc Tychonoff space with a compact T_2-space need not be acc (see [5, Example 2.2]). Also, in [4, Example 3], an example was given showing that the product of a starcompact T_1-space with a compact metric space need not be starcompact. Now, we show that the same example also shows that the product of a countably starcompact (resp. countably acc) T_1-space with a compact metric space need not be countably starcompact (resp. countably acc).

Example 5.1 (Fleischman). *There exist an acc T_1-space X and a compact metric space Y such that $X\times Y$ is not countably starcompact.*

Proof. Let $X=\omega_1\cup A$, where $A=\{a_n: n\in\omega\}$. We topologize X as follows: ω_1 has the usual order topology and is an open subspace of X, and a basic neighborhood of each $a_n\in A$ takes the form

$$G_\beta(a_n)=(\beta,\omega_1)\cup\{a_n\},\quad \text{where } \beta<\omega_1.$$

Then, X is an acc T_1-space By the proof of Example 2.4. Let $Y=\omega+1$ with the usual order topology. Then, Y is a compact metric space.

Next, we prove that $X\times Y$ is not countably starcompact. Let $U_n=[n,\omega_1)\cup\{a_n\}$ and $V_n=(n,\omega]$ for each $n\in\omega$. Let

$$\mathcal{U}=\{U_n\times V_n : n\in\omega\}\cup\{X\times\{n\}: n\in\omega\}.$$

Then, $\mathcal{U}$ is a countable open cover of $X\times Y$. Let F be a finite subset of $X\times Y$. Then, there exists a $n\in\omega$ such that $(X\times\{n\})\cap F=\emptyset$. Hence, $\langle a_n,n\rangle\notin St(F,\mathcal{U})$, since $X\times\{n\}$ is only element of $\mathcal{U}$ such that $\langle a_n,n\rangle\in X\times\{n\}$ for each $n\in\omega$. This completes the proof. □

Remark 2. By Example 5.1, we can see that

(1) an open perfect preimage of a starcompact space need not be starcompact,
(2) an open perfect preimage of a countably starcompact space need not be countably starcompact, and
(3) an open perfect preimage of a countably acc space need not be countably acc.

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