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A mathematical method of fuzzy reasoning

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Abstract

In this paper, we introduce in a new natural method of fuzzy reasoning by defining some fuzzy mathematical concepts. It is well-known that Mamdani's method never holds the property of Monotony. Our proposal method holds the monotony property.

1. Introduction

The monotony is the important point for applying fuzzy reasoning to "educational evaluation", "clinical decision making", etc. Then, by defining some fuzzy mathematical concepts, we can mathematically introduce a new fuzzy reasoning method that could make it clear the above point.

2. Fuzzy Mathematical Concepts

[Definition 2.1] Normal

A fuzzy set A is normal if its membership function attains 1, that is,

$$\sup_{x\in R} A(x)=1.$$

[Definition 2.2] Convex

A fuzzy set A is convex if its membership function is such that

$$A(\lambda x+(1-\lambda)y)\geqq A(x)\wedge A(y)$$

for any $x,y\in R$ and $\lambda\in[0,1]$.

Lemme2.1 Let A be a normal convex fuzzy set. If $A(x)$ is a membership function of A, we have

$$A(x)=\begin{cases} l(x), & x\in I_1 \\ 1, & x\in I_2 \\ r(x), & x\in I_3 \end{cases}$$

where $l(x)$ and $r(x)$ are a monotonous increasing function and a monotonous decreasing function, respectively, I_1, I_2, I_3 are intervals which satisfy $I_1 \cup I_2 \cup I_3 = \mathbf{R}$.

Lemme2.2 Let $A(x)$ be a membership function of normal convex fuzzy set A defined on a bounded closed interval $I \subset \mathbf{R}$. Then $A(x)$ is integrable over I.

[Definition 2.3] Fuzzy Partitioned Space

Let I be a bounded closed interval on $\mathbf{R}$ and let

$$FP(I)=\{A_i : i \in \Lambda\}$$

be a family of normal fuzzy sets on I. If $FP(I)$ satisfies the equation

$$\Sigma_{i \in \Lambda} A_i(x)=1, \ \forall x \in I,$$

then we call $FP(I)$ a fuzzy partition of I, I a fuzzy partitioned space and each A_i, which is a element of $FP(I)$, a fuzzy partition set.

[Definition 2.4] α-Cut

The α-cut of A, denoted by $[A]^\alpha$, is a set consisting of those elements of the universe $\mathbf{R}$ whose membership values exceed the threshold level α,

$$[A]^\alpha=\{x \mid A(x) \geqq x\}$$

for any $\alpha \in [0,1]$.

Lemme2.3 Let A be a normal convex fuzzy set in a closed interval I on $\mathbf{R}$. Then the closure of $[A]^\alpha$ is a closed interval.

[Definition 2.5] Ordered Fuzzy Partition Sets

For two normal convex fuzzy sets A, B in a closed interval I on $\mathbf{R}$ with

$$\overline{[A]}^\alpha=[a_\alpha, b_\alpha], \ \ \overline{[B]}^\alpha=[c_\alpha, d_\alpha],$$

we define an order relation "$<$" as follows:

$$A<B \Leftrightarrow \ a_\alpha \leqq c_\alpha, \ \ b_\alpha \leqq d_\alpha, \ \ \forall \alpha \in [0,1].$$

Lemme2.4 Let $\{A_1, A_2, \cdots, A_n\}$ be a fuzzy partition of a closed interval I on $\mathbf{R}$. After renumbering if necessary, this set can be well ordered such that

$$A_1<A_2<\cdots<A_n.$$

3. Product-Sum-Gravity Method

Let $FP(F_1)=\{L_1, L_2, \cdots, L_m \mid L_1<L_2<\cdots<L_m\}$, $FP(F_2)=\{M_1, M_2, \cdots, M_n \mid M_1<M_2<\cdots<M_n\}$ and $FP(E)=\{A_1, A_2, \cdots, A_r \mid A_1<A_2<\cdots<A_r\}$ be ordered

fuzzy partition sets of F_1, F_2 and E respectively. And let the fuzzy rule be given by an onto-mapping

$$h: F_1 \times F_2 \rightarrow E$$

which is called **rule mapping.** Then we call **numerical fuzzy reasoning** on fuzzy partitioned space to calculate the conclusion $E=\phi(x_1, x_2)$ concerning with the fact (x_1, x_2), where we call the function a **numerical reasoning function.** Especially the numerical reasoning function of product-sum-gravity method is as follows:

$$\begin{aligned} &\phi(x_1, x_2) \\ &=\{\text{abscissa of the center of gravity of } \{L_i(x_1)M_j(x_2)A_k \\ &\quad +L_i(x_1)M_{j+1}(x_2)A_{k'} + L_{i+1}(x_1)M_j(x_2)A_{k''} + L_{i+1}(x_1)M_{j+1}(x_2)A_{k'''}\} \end{aligned}$$

where $L_s(x_1)=0$ $(s \neq i, i+1)$, $M_t(x_2)=0$ $(t \neq j, j+1)$ and the following rules satisfy:

$$L_i, M_j \rightarrow A_k,\ \ L_i, M_{j+1} \rightarrow A_{k'},\ \ L_{i+1}, M_j, \rightarrow A_{k''}, L_{i+1}, M_{j+1} \rightarrow A_{k'''}.$$

Theorem3.1 If the rule mapping $h: F_1 \times F_2 \rightarrow E$ satisfies the following conditions, the numerical reasoning function of product-sum-gravity method on fuzzy partition space $E=\phi(x_1, x_2)$ is monotonous increasing.

i) Boundary Conditions

$$L_1, M_1 \rightarrow A_1,\ \ L_m, M_n \rightarrow A_r$$

ii) Monotonicity Condition

If $L_{i1}, M_{j1} \rightarrow A_p$, $L_{i2}, M_{j2} \rightarrow A_q$ $(i_1 \leqq i_2, j_1 \leqq j_2)$, then $p \leqq q$.

From i), ii) it follows that (L_i, M_j) is a complete lattice.

iii) Continuity Condition

If $L_{i1}, M_{ji} \rightarrow A_p$, $L_{i2}, M_{j2} \rightarrow A_q$ $(i_1 \leqq i_2, j_1 \leqq j_2, i_1+j_1+1=i_2+j_2)$, then $|p-q| \leqq 1$.

iv) $|A_1| \geqq |A_2| = \cdots = |A_{r-1}| \leqq |A_r|$

where

$$|A_i| = \int A_i(x)dx, \quad i=1, 2, \cdots, r.$$

Let $FP(F_1)=\{L_1, L_2, \cdots, L_m | L_1<L_2<\cdots<L_m\}$, $FP(F_2)=\{M_1, M_2, \cdots, M_n | M_1<M_2<\cdots<M_n\}$, $FP(F_3)=\{N_1, N_2, \cdots, N_u | N_1<N_2<\cdots<N_u\}$ and $FP(E)=\{A_1, A_2, \cdots, A_r | A_1<A_2<\cdots<A_r\}$ be respectively ordered fuzzy partition sets of F_1, F_2, F_3 and E.

Theorem3.2 If the rule mapping $h: F_1 \times F_2 \times F_3 \rightarrow E$ satisfies the following conditions, the numerical reasoning function of product-sum

-gravity method on fuzzy partition space $E=\phi(x_1, x_2, x_3)$ is monotonous increasing.

i) Boundary Conditions

$$L_1, M_1, N_1 \to A_1,\quad L_m, M_n, N_u \to A_r$$

ii) Monotonicity Condition

If $L_{i1}, M_{j1}, N_{k1} \to A_p$, $L_{i2}, M_{j2}, N_{k3} \to A_q$ $(i_1 \leqq i_2, j_1 \leqq j_2, k_1 \leqq k_2)$, then $p \leqq q$.

iii) Continuity Condition

If $L_{i1}, M_{j1}, N_{k1} \to A_p$, $L_{i2}, M_{j2}, N_{k3} \to A_q$ $(i_1 \leqq i_2, j_1 \leqq j_2, k_1 \leqq k_2, i_1+j_1+k_1+1=i_2+j_2+k_2)$, then $|p-q| \leqq 1$.

iv) $|A_1| \geqq |A_2| = \cdots = |A_{r-1}| \leqq |A_r|$.

Remark The same result holds if the rule mapping $h: F_1 \times F_2 \times F_3 \to E$ satisfies the Okuda's condition[4)]

$$|P_k(\lambda)| \cdot |Q_k(\lambda+1)| \geqq |P_k(\lambda+1)| \cdot |Q_k(\lambda)|$$

instead of the condition iv), where

$$P_k(\lambda)=\{(\tau_1, \tau_2, \tau_3): h(\cdots, H_{\tau k}', \cdots)=A_\lambda, \tau_i=\tau_i', \tau_i'+1\}$$
$$Q_k(\lambda)=\{(\tau_1, \tau_2, \tau_3): h(\cdots, H_{\tau k}'{}_{+1}, \cdots)=A_\lambda, \tau_i=\tau_i', \tau_i'+1\}.$$

References

1)Lotfy A.Zadeh:Fuzzy Logic and the Calculus of Fuzzy Rules and Fuzzy Graphs A Precise: Multi Val Logic,1996,Vol.1,pp1-38.

2)Lotfy A.Zadeh:Toward a theory of fuzzy information granulation and its centrality in human reasoning and fuzzy logic: Fuzzy Sets and Systems 90,1997,pp111-127.

3)Y.Okuda,H.Yamashita and J.Inaida:Fuzzy Reasoning using Fuzzy Partition and its Application: NAFIPS Congress XII,1998.

4)Y.Okuda,H.Yamashita and J.Inaida:Multiple Fuzzy Reasoning on the Fuzzy Partitioned Space: IFSA Congress XIII,1999.

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