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The Discrete-Time Opportunistic Replacement Models with Application to Scheduled Maintenance for Electric Switching Device

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Abstract: In this paper, we consider the discrete-time opportunistic replacement models with application to scheduled maintenance for electric switching devices. It is shown that a replacement model with three maintenance options can be classified into six kinds of models by the priority of maintenance options. Further, we develop the models with probabilistic priority to unify the six models with deterministic priority.

1. Introduction

In this paper, we consider the discrete-time opportunistic replacement models with application to scheduled maintenance for electric switching devices to distribute the electric power to other places. The electric switching devices equipped with telegraph poles have to be replaced preventively before they fail and the electric current is off over an extensive area. On the other hand, it can be replaced if the telegraph pole is removed for any construction before its age has elapsed a threshold level. This problem is reduced to a simple opportunity based age replacement model. In the earlier literature, many authors analyzed several opportunistic replacement models. Radner and Jorgenson [1] was the seminal work on the opportunistic replacement model for a single unit. Berg [2], Pullen and Thomas [3] and Zheng [4] discussed opportunity-triggered replacement policies for multiple-unit systems. Further, Dekker and Smeithink [5, 6], Dekker and Dijkstra [7], and Zheng and Fard [8] extended the models from a variety of standpoints. Recently, simple but somewhat different opportunity based age replacement models were considered by Iskandar and Sandoh [9, 10]. In fact, their model [10] is essentially same as ours in this paper except that it is considered in a discrete-time setting.

Ordinarily, the discrete-time models are considered as trivial analogies of the continuous-time ones. First, Nakagawa and Osaki [11] formulated a discrete-time model for the classical age replacement problem. Kaio and Osaki [12, 13] derived some discrete maintenance policies along the line of Nakagawa and Osaki [11]. Nakagawa [14-18] summarized and generalized the discrete-time maintenance models by taking account of the significant concept of minimal repair. For the details of discrete models, see Kaio and Osaki [19]. The main reasons to adopt the discrete-time model for the scheduled maintenance problem for electric switching devices are as follows. (i) In the electric power company under investigation, the failure time data of electric switching devices are recorded as group data (the number of failures per year). (ii) It is not easy to carry out the preventive replacement schedule at the unit of week or month, since the service team is engaged in other works, too. From our questionnaire, it is helpful for practitioners that the preventive replacement schedule should be determined roughly at the unit of year. These motivate our discrete-time opportunistic replacement model. In addition, we show in this paper that a replacement model with more than two maintenance options can be classified into some kinds of models by the priority of maintenance options. This implies that the discrete-time model has more delicate aspects for analysis than the continuous one.

The rest part of this paper is organized as follows. In Section 2, the discrete-time opportunistic replacement models under consideration are described with notation and assumptions. By the priority of maintenance options, we introduce six kinds of models. In Section 3, the optimal preventive replacement times which minimize the expected costs per unit time in the steady-state are derived for respective models. Section 4 develops the models with probabilistic priority to unify the six models with deterministic priority.

2. Model Description

First, we consider a discrete-time model corresponding to Iskandar and Sandoh [10]. Let us consider the single-unit system with a non-repairable item in a discrete-time setting. Suppose that the interval between opportunities for replacements X obeys the geometric distribution $\Pr\{X = x\} = g_X(x) = p(1-p)^{x-1}$ $(x = 1, 2, \cdots; 0 < p < 1)$ with survivor function $\Pr\{X \geq x\} = (1-p)^{x-1} = \overline{G}_X(x-1)$, mean $\mathrm{E}[X] = 1/p$ and variance $\mathrm{Var}[X] = (1-p)/p^2$, where in general $\overline{\phi}(\cdot) = 1 - \phi(\cdot)$. Then, the unit may be replaced at a first opportunity after elapsed time S (S is a non-negative integer) even if it does not fail. The failure time (lifetime) Y follows the common probability mass function $\Pr\{Y = y\} = f_Y(y)$ $(y = 1, 2, \cdots)$ with survivor function $\Pr\{Y \geq y\} = \overline{F}_Y(y-1)$ and failure rate $r_Y(y) = f_Y(y)/\overline{F}_Y(y-1)$. Without any loss of generality, we assume that $f_Y(0) = g_X(0) = 0$. If the failure occurs before a prespecified preventive replacement time T $(T = 1, 2, \cdots)$, the corrective replacement may be executed. On the other hand, if the unit does not fail up to the time T, the preventive replacement may be made at time T. The configuration of the opportunistic replacement model is depicted in Fig. 1.

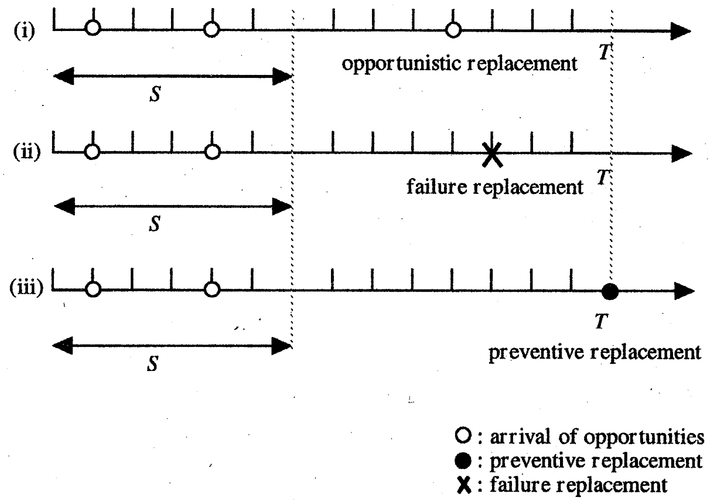

Figure 1: Configuration of the model.

The cost components under consideration are the following;

c_1 (> 0): corrective replacement cost per failure

c_2 (> 0): cost for each preventive replacement

c_3 (> 0): cost for each opportunistic replacement.

From the above notation, we make two types of assumptions;

Assumption (A-1): $c_1 > c_3 > c_2$

Assumption (A-2): $c_1 > c_2 > c_3$

It is valid to assume that the corrective replacement cost is most expensive. The relationship between the preventive replacement cost and the opportunistic replacement one has to be ordered taking account of the economic justification.

Note that the discrete-time model above has to be treated carefully. At an arbitrary discrete point of time, the decision maker has to select one decision among three options, failure (corrective) replacement $\mathrm{F_a}$, preventive replacement $\mathrm{S_c}$ and opportunistic replacement $\mathrm{O_p}$. We introduce the following symbol for the priority relationship;

Definition 2.1: The option P has a priority to the option Q if $\mathrm{P} \succ \mathrm{Q}$.

From Definition 2.1, if two options occur at the same time point, the option with higher priority is executed. In our model setting, consequently, it is possible to consider total six different models as follows.

Model 1: $\mathrm{S_c} \succ \mathrm{F_a} \succ \mathrm{O_p}$

Model 2: $\mathrm{F_a} \succ \mathrm{S_c} \succ \mathrm{O_p}$

Model 3: $\mathrm{S_c} \succ \mathrm{O_p} \succ \mathrm{F_a}$

Model 4: $\mathrm{O_p} \succ \mathrm{S_c} \succ \mathrm{F_a}$

Model 5: $\mathrm{F_a} \succ \mathrm{O_p} \succ \mathrm{S_c}$

Model 6: $\mathrm{O_p} \succ \mathrm{F_a} \succ \mathrm{S_c}$

For Model 1, Model 2 and Model 5, the probabilities that the system is replaced at time n $(n = 0, 1, 2, \cdots)$ are

$$h_1(n) = h_2(n) = h_5(n) = \begin{cases} f_Y(n) & (0 \le n \le S) \\ f_Y(n)\overline{G}_X(n-1-S) + \overline{F}_Y(n)g_X(n-S) & \\ & (S+1 \le n \le T-1) \\ \overline{F}_Y(T-1)\overline{G}_X(T-1-S) & (n = T) \\ 0 & (n \ge T+1), \end{cases} \tag{1}$$

respectively. In a fashion similar to Eq.(1), the probabilities that the system is replaced at time n $(n = 0, 1, 2, \cdots)$ for the other models are

$$h_3(n) = h_4(n) = h_6(n) = \begin{cases} f_Y(n) & (0 \le n \le S) \\ f_Y(n)\overline{G}_X(n-S) + g_X(n-S)\overline{F}_Y(n-1) & \\ & (S+1 \le n \le T-1) \\ \overline{F}_Y(T-1)\overline{G}_X(T-1-S) & (n = T) \\ 0 & (n \ge T+1), \end{cases} \tag{2}$$

where $\sum_{n=0}^{\infty} h_j(n) = 1$ $(j = 1, \cdots, 6)$.

From Eqs.(1) and (2), the mean time length of one cycle $A_j(T)$ for Model j $(j = 1, \cdots, 6)$ are all same, that is, $A_1(T) = A_2(T) = A_3(T) = A_4(T) = A_5(T) = A_6(T)$, where

$$A_1(T) = \sum_{n=0}^{S} n f_Y(n) + \sum_{n=S+1}^{T-1} n\Big\{ f_Y(n)\overline{G}_X(n-1-S)$$

$$+\overline{F}_Y(n)g_X(n-S)\Big\} + T\overline{F}_Y(T-1)\overline{G}_X(T-1-S)$$

$$= \sum_{k=1}^{S}\overline{F}_Y(k-1) + \sum_{k=S+1}^{T}\overline{F}_Y(k-1)\overline{G}_X(k-S-1), \qquad (3)$$

and are independent of priorities.

On the other hand, the total expected costs during one cycle $B_j(T)$ for Model j ($j = 1, \cdots, 6$) are

$$\begin{aligned} B_1(T) &= c_1\sum_{n=0}^{S} f_Y(n) + c_1\sum_{n=S+1}^{T-1} f_Y(n)\overline{G}_X(n-1-S) \\ &\quad + c_2\overline{F}_Y(T-1)\overline{G}_X(T-1-S) + c_3\sum_{n=S+1}^{T-1}\overline{F}_Y(n)g_X(n-S), \end{aligned} \qquad (4)$$

$$\begin{aligned} B_2(T) &= c_1\sum_{n=0}^{S} f_Y(n) + c_1\sum_{n=S+1}^{T} f_Y(n)\overline{G}_X(n-1-S) \\ &\quad + c_2\overline{F}_Y(T)\overline{G}_X(T-1-S) + c_3\sum_{n=S+1}^{T-1}\overline{F}_Y(n)g_X(n-S), \end{aligned} \qquad (5)$$

$$\begin{aligned} B_3(T) &= c_1\sum_{n=0}^{S} f_Y(n) + c_1\sum_{n=S+1}^{T-1} f_Y(n)\overline{G}_X(n-S) \\ &\quad + c_2\overline{F}_Y(T-1)\overline{G}_X(T-1-S) + c_3\sum_{n=S+1}^{T-1}\overline{F}_Y(n-1)g_X(n-S), \end{aligned} \qquad (6)$$

$$\begin{aligned} B_4(T) &= c_1\sum_{n=0}^{S} f_Y(n) + c_1\sum_{n=S+1}^{T-1} f_Y(n)\overline{G}_X(n-S) \\ &\quad + c_2\overline{F}_Y(T-1)\overline{G}_X(T-S) + c_3\sum_{n=S+1}^{T}\overline{F}_Y(n-1)g_X(n-S), \end{aligned} \qquad (7)$$

$$\begin{aligned} B_5(T) &= c_1\sum_{n=0}^{S} f_Y(n) + c_1\sum_{n=S+1}^{T} f_Y(n)\overline{G}_X(n-1-S) \\ &\quad + c_2\overline{F}_Y(T)\overline{G}_X(T-S) + c_3\sum_{n=S+1}^{T}\overline{F}_Y(n)g_X(n-S) \end{aligned} \qquad (8)$$

and

$$\begin{aligned} B_6(T) &= c_1\sum_{n=0}^{S} f_Y(n) + c_1\sum_{n=S+1}^{T} f_Y(n)\overline{G}_X(n-S) \\ &\quad + c_2\overline{F}_Y(T)\overline{G}_X(T-S) + c_3\sum_{n=S+1}^{T}\overline{F}_Y(n-1)g_X(n-S), \end{aligned} \qquad (9)$$

respectively.

Then the expected costs per unit time in the steady-state $C_j(T)$ for Model j $(j = 1, 2, \cdots, 6)$ are, from the familiar renewal reward argument,

$$\begin{aligned} C_j(T) &= \lim_{n\to\infty} \frac{\text{E[total cost on } (0,n]]}{n} \\ &= B_j(T)/A_j(T) \quad (j = 1, \cdots, 6), \end{aligned} \tag{10}$$

and the problem is to determine the optimal preventive replacement time T^* which minimizes the expected cost $C_j(T)$ for a fixed S.

Remark: When the scheduled maintenance problem for electric switching devices is considered, it is meaning to assume that the variable S is determined in advance. Because the threshold age to start the opportunistic replacement should be estimated from the efficiency and price of an electric switching device. Hence, throughout the paper, we suppose that the variable S is fixed from any physical or economical reason.

3. Optimal Replacement Policies

In this section, we consider six models, Model 1 $\sim$ Model 6, and derive the respective optimal preventive replacement policies which minimize the expected costs per unit time in the steady-state. Define the non-linear functions;

$$q_1(T) \equiv \frac{1}{1-p}\Big\{(c_1 - c_2)R_Y(T) + p(c_3 - c_2)\Big\}A_1(T) - B_1(T), \tag{11}$$

$$q_2(T) \equiv \Big\{(c_1 - c_2)r_Y(T+1) + \frac{p(c_3 - c_2)}{1-p}\Big\}A_2(T) - B_2(T), \tag{12}$$

$$q_3(T) \equiv \Big\{\Big[(c_1 - c_2) + \frac{p}{1-p}(c_3 - c_2)\Big]R_Y(T) + \frac{p}{1-p}(c_3 - c_2)\Big\}A_3(T) - B_3(T), \tag{13}$$

$$q_4(T) \equiv \Big\{(c_1 - c_2)R_Y(T) + p(c_3 - c_2)\Big\}A_4(T) - B_4(T), \tag{14}$$

$$q_5(T) \equiv \Big\{\Big[(c_1 - c_2) + p(c_2 - c_3)\Big]r_Y(T+1) + p(c_3 - c_2)\Big\}A_5(T) - B_5(T) \tag{15}$$

and

$$q_6(T) \equiv \Big\{(1-p)(c_1 - c_2)r_Y(T+1) + p(c_3 - c_2)\Big\}A_6(T) - B_6(T), \tag{16}$$

where

$$R_Y(T) \equiv f_Y(T)/\overline{F}_Y(T). \tag{17}$$

Lemma 3.1: The function $R_Y(T)$ is strictly increasing [decreasing] if the failure time distribution is strictly IFR (Increasing Failure Rate) [DFR (Deceasing Failure Rate)].

Theorem 3.2: (1) For Model j $(j = 1, 2, 3)$, suppose that the failure time distribution is strictly IFR and the assumption (A-1) holds.

(i) If $q_j(S+1) < 0$ and $q_j(\infty) > 0$ $(j = 1,2,3)$, then there exists a finite and unique optimal preventive replacement time T^* $(S+1 < T^* < \infty)$ which satisfies $q_j(T^*-1) < 0$ and $q_j(T^*) \geq 0$.

(ii) If $q_j(\infty) \leq 0$ $(j = 1,2,3)$, then the optimal preventive replacement time is $T^* \to \infty$ and it is optimal to carry out either the failure replacement or the opportunistic one.

(iii) If $q_j(S+1) \geq 0$ $(j = 1,2,3)$, then the optimal preventive replacement time is $T^* = S+1$ and it is optimal to carry out either the failure replacement or the preventive one.

(2) For Models j $(j = 4,5,6)$, suppose that the failure time distribution is DFR and the assumption (A-1) holds. Then the optimal preventive replacement time is $T^* \to \infty$ or $T^* = S+1$.

Theorem 3.3: (1) For Models 4, 5 and 6, suppose that the failure time distribution is strictly IFR and the assumption (A-2) holds.

(i) If $q_j(S+1) < 0$ and $q_j(\infty) > 0$ $(j = 4,5,6)$, then there exists a finite and unique optimal preventive replacement time T^* $(S+1 < T^* < \infty)$ which satisfies $q_j(T^*-1) < 0$ and $q_j(T^*) \geq 0$.

(ii) If $q_j(\infty) \leq 0$ $(j = 4,5,6)$, then the optimal preventive replacement time is $T^* \to \infty$.

(iii) If $q_j(S+1) \geq 0$ $(j = 4,5,6)$, then the optimal preventive replacement time is $T^* = S+1$.

(2) For Models 4, 5 and 6, suppose that the failure time distribution is DFR and the assumption (A-2) holds. Then the optimal preventive replacement time is $T^* \to \infty$ or $T^* = S+1$.

From these results, it is found that the optimal preventive replacement schedule for each model should be generated under different cost assumptions.

4. The Unified Model with Probabilistic Priority

In this section, we unify six replacement models proposed in Section 2. Now suppose that the multiple maintenance options at any time may be selected with random priority. Under the assumption (A-1), define the probabilities p_a, p_b and p_c to select the priorities $\mathrm{S_c} \succ \mathrm{F_a} \succ \mathrm{O_p}$, $\mathrm{F_a} \succ \mathrm{S_c} \succ \mathrm{O_p}$ and $\mathrm{S_c} \succ \mathrm{O_p} \succ \mathrm{F_a}$, respectively, where $0 \leq p_a \leq 1$, $0 \leq p_b \leq 1$, $0 \leq p_c \leq 1$ and $p_a + p_b + p_c = 1$. Also, under the assumption (A-2), we define p_d, p_e and p_f to select the priorities $\mathrm{O_p} \succ \mathrm{S_c} \succ \mathrm{F_a}$, $\mathrm{F_a} \succ \mathrm{O_p} \succ \mathrm{S_c}$ and $\mathrm{O_p} \succ \mathrm{F_a} \succ \mathrm{S_c}$, respectively, where $0 \leq p_d \leq 1$, $0 \leq p_e \leq 1$, $0 \leq p_f \leq 1$ and $p_d + p_e + p_f = 1$. We call these two models with triplets (p_a, p_b, p_c) and (p_d, p_e, p_f) Model 7 and Model 8, respectively.

For Model 7 and Model 8, the probabilities that the system is replaced at time n $(n = 0,1,2,\cdots)$ are

$$h_7(n) = \begin{cases} f_Y(n) & (0 \leq n \leq S) \\ (p_a+p_b)\{f_Y(n)\overline{G}_X(n-S-1) + g_X(n-S)\overline{F}_Y(n)\} & \\ +p_c\{f_Y(n)\overline{G}_X(n-S) + g_X(n-S)\overline{F}_Y(n-1)\} & \\ & (S+1 \leq n \leq T-1) \\ \overline{F}_Y(T-1)\overline{G}_X(T-1-S) & (n = T) \\ 0 & (n \geq T+1) \end{cases} \tag{18}$$

and

$$h_8(n) = \begin{cases} f_Y(n) & (0 \leq n \leq S) \\ (p_d+p_f)\{f_Y(n)\overline{G}_X(n-S) + g_X(n-S)\overline{F}_Y(n-1)\} & \\ +p_e\{f_Y(n)\overline{G}_X(n-1-S) + g_X(n-S)\overline{F}_Y(n)\} & \\ & (S+1 \leq n \leq T-1) \\ \overline{F}_Y(T-1)\overline{G}_X(T-1-S) & (n = T) \\ 0 & (n \geq T+1), \end{cases} \tag{19}$$

where $\sum_{n=0}^{\infty} h_7(n) = p_a + p_b + p_c = 1$ and $\sum_{n=0}^{\infty} h_8(n) = p_d + p_e + p_f = 1$.

The mean time lengthes of one cycle and the total expected costs during one cycle for Models 7 and 8 are

$$A_7(T) = A_8(T) = \sum_{j=1}^{S} \overline{F}_Y(j-1) + \sum_{j=S+1}^{T} \overline{F}_Y(j-1)\overline{G}_X(j-S-1), \tag{20}$$

$$\begin{aligned} B_7(T) = {} & c_1 \sum_{n=0}^{S} f_Y(n) + p_a\Big\{c_1 \sum_{n=S+1}^{T-1} f_Y(n)\overline{G}_X(n-S-1) \\ & + c_2\overline{F}_Y(T-1)\overline{G}_X(T-S-1) + c_3 \sum_{n=S+1}^{T-1} \overline{F}_Y(n)g_X(n-S)\Big\} \\ & + p_b\Big\{c_1 \sum_{n=S+1}^{T} f_Y(n)\overline{G}_X(n-S-1) + c_2\overline{F}_Y(T)\overline{G}_X(T-S-1) \\ & + c_3 \sum_{n=S+1}^{T-1} \overline{F}_Y(n)g_X(n-S)\Big\} + p_c\Big\{c_1 \sum_{n=S+1}^{T-1} f_Y(n)\overline{G}_X(n-S) \\ & + c_2\overline{F}_Y(T-1)\overline{G}_X(T-S-1) + c_3 \sum_{n=S+1}^{T-1} \overline{F}_Y(n-1)g_X(n-S)\Big\} \end{aligned} \tag{21}$$

and

$$\begin{aligned} B_8(T) = {} & c_1 \sum_{n=0}^{S} f_Y(n) + p_d\Big\{c_1 \sum_{n=S+1}^{T-1} f_Y(n)\overline{G}_X(n-S) \\ & + c_2\overline{F}_Y(T-1)\overline{G}_X(T-S) + c_3 \sum_{n=S+1}^{T} \overline{F}_Y(n-1)g_X(n-S)\Big\} \\ & + p_e\Big\{c_1 \sum_{n=S+1}^{T} f_Y(n)\overline{G}_X(n-S-1) + c_2\overline{F}_Y(T)\overline{G}_X(T-S) \\ & + c_3 \sum_{n=S+1}^{T} \overline{F}_Y(n)g_X(n-S)\Big\} + p_f\Big\{c_1 \sum_{n=S+1}^{T} f_Y(n)\overline{G}_X(n-S) \\ & + c_2\overline{F}_Y(T)\overline{G}_X(T-S) + c_3 \sum_{n=S+1}^{T} \overline{F}_Y(n-1)g_X(n-S)\Big\}, \end{aligned} \tag{22}$$

respectively.

Then the problem is to determine the optimal preventive replacement time T^* which minimizes the expected cost $TC_j(T)(j = 7, 8)$ for a fixed S, where

$$TC_j(T) = B_j(T)/A_j(T) \quad (j = 7, 8). \tag{23}$$

Define the following non-linear functions;

$$\begin{aligned} q_7(T) \equiv {} & \Big\{\Big[p_a(c_1-c_2)/(1-p) + p_c\{(c_1-c_2) + p(c_3-c_2)/(1-p)\}\Big]R_Y(T) \\ & + p_b(c_1-c_2)r_Y(T+1) + p(c_3-c_2)/(1-p)\Big\}A_7(T) - B_7(T) \end{aligned} \tag{24}$$

and

$$q_8(T) \equiv \Big\{p_d(c_1-c_2)R_Y(T) + \Big[p_e\{(c_1-c_2) + p(c_2-c_3)\} + p_f(1-p)(c_1-c_2)\Big]r_Y(T+1)$$

$$+p(c_3 - c_2)\Big\}A_8(T) - B_8(T). \quad (25)$$

Theorem 4.1: (1) For Models 7 and 8, suppose that the failure time distribution is strictly IFR and the assumptions (A-1) and (A-2) hold, respectively.

(i) If $q_j(S+1) < 0$ and $q_j(\infty) > 0$ $(j = 7, 8)$, then there exists a finite and unique optimal preventive replacement time T^* $(S + 1 < T^* < \infty)$ which satisfies $q_j(T^* - 1) < 0$ and $q_j(T^*) \geq 0$.

(ii) If $q_j(\infty) \leq 0$ $(j = 7, 8)$, then the optimal preventive replacement time is $T^* \to \infty$.

(iii) If $q_j(S+1) \geq 0$ $(j = 7, 8)$, then the optimal preventive replacement time is $T^* = S + 1$.

(2) For Models 7 and 8, suppose that the failure time distribution is DFR and the assumptions (A-1) and (A-2) hold, respectively. Then the optimal preventive replacement time is $T^* \to \infty$ or $T^* = S + 1$.

From Theorem 4.1, the models with probabilistic priority involve the deterministic priority models as special cases. For instance, it is seen that Model 7 is reduced to Model 1 if $(p_a, p_b, p_c) = (1, 0, 0)$. Although the earlier models in the literature [11-18] assumed the priority unconsciously in accordance with the order of costs, the rigorous treatment for modeling will be needed if the priority is uncertain.

References

[1] R. Radner and D. W. Jorgenson, "Opportunistic replacement of a single part in the presence of several monitored parts", *Management Science*, **10** (1963) 70-84.

[2] M. Berg, "General trigger-off replacement procedures for two-unit systems", *Naval Research Logistics*, **25** (1978) 15-29.

[3] K. Pullen and M. Thomas, "Evaluation of an opportunistic replacement policy for a 2-unit system", *IEEE Transactions on Reliability*, **R-53** (1986) 320-323.

[4] X. Zheng, "All opportunity-triggered replacement policy for multiple-unit systems", *IEEE Transactions on Reliability*, **R-44** (1995) 648-652.

[5] R. Dekker and E. Smeithink, "Opportunity-based block replacement", *European Journal of Operational Research*, **53** (1991) 46-63.

[6] R. Dekker and E. Smeithink, "Preventive maintenance at opportunities of restricted duration", *Naval Research Logistics*, **41** (1994) 335-353.

[7] R. Dekker and M. C. Dijkstra, "Opportunity based age replacement: exponentially distributed times between opportunities", *Naval Research Logistics*, **39** (1992) 175-190.

[8] X. Zheng and N. Fard, "A maintenance policy for repairable systems based on opportunistic failure-rate tolerance", *IEEE Transactions on Reliability*, **R-40** (1991) 237-244.

[9] B. P. Iskandar and H. Sandoh, "An opportunity-based age replacement policy considering warranty", *Proceedings of International Workshop on Reliability Modelling and Analysis: From Theory to Practice*, pp. 131-136 (1998).

[10] B. P. Iskandar and H. Sandoh, "An extended opportunity-based age replacement policy", to appear in *Revue Francaise d'Automatique, Informatique et Recherche Operationnelle (Recherche operationnelle/Operations Research)*.

[11] T. Nakagawa and S. Osaki, "Discrete time age replacement policies", *Operational Research Quarterly.* **28** (1977) 881-885.

[12] N. Kaio and S. Osaki, "Discrete-time ordering policies", *IEEE Transactions on Reliability,* **R-28** (1979) 405-406.

[13] N. Kaio and S. Osaki, "Discrete time ordering policies with minimal repair", *Revue Francaise d'Automatique, Informatique et Recherche Operationnelle* (*Recherche operationnelle/ Operations Research*), **14** (1980) 257-263.

[14] N. Kaio and S. Osaki, "A discrete repair limit policy", *Advances in Management Science,* **1** (1982) 157-160.

[15] T. Nakagawa, "A summary of discrete replacement policies", *European Journal of Operational Research,* **17** (1984) 382-392.

[16] T. Nakagawa, "Continuous and discrete age replacement policies", *Journal of Operational Research Society,* **36** (1985) 147-154.

[17] T. Nakagawa, "Optimal policy of continuous and discrete replacement with minimal repair at failure", *Naval Research Logistics Quarterly,* **31** (1984) 543-550.

[18] T. Nakagawa, "Modified discrete preventive maintenance policies", *Naval Research Logistics Quarterly,* **33** (1986) 703-715.

[19] N. Kaio and S. Osaki, "Review of discrete and continuous distributions in replacement models", *International Journal of Systems Science,* **19** (1988) 171-177.