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Selected results on convergence properties in topological spaces, topological groups and function spaces

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We present a survey of selected results intended to demonstrate that the behaviour of convergence properties (such as Fréchet-Urysohn property, tightness and α_i-properties) tends to improve as one passes from general topological spaces to topological groups and function spaces. Related open questions are collected in a hope that this may generate an interest among set theorists leading to a possible solution of some of them.

1 Convergence properties in various classes of spaces

Let X be a topological space. For $A \subseteq X$ we use $\overline{A}$ to denote the closure of A in X. A space X is *first countable* if every point $x \in X$ has a countable local base $\mathcal{B}_x$ (i.e. a countable family $\mathcal{B}_x$ of open subsets of X such that, whenever U is an open subset of X containing x, there exists $B \in \mathcal{B}_x$ so that $x \in B \subseteq U$).

A *sequence converging to* $x \in X$ is a countable infinite set S such that $S \setminus U$ is finite for every open neighbourhood U of x. A space X is *Fréchet-Urysohn* provided that for each set $A \subseteq X$ if $x \in \overline{A}$, then there exists a sequence $S \subseteq A$ converging to x.

As usual, for a set A and a cardinal τ, $[A]^{\leq\tau}$ denotes the set of all subsets of A of size less or equal than τ.

Definition 1.1 (Arhangel'skiĭ [1]) The *tightness* $t(X)$ of a topological space X is defined as the smallest cardinal τ such that

$$\overline{A} = \bigcup\{\overline{B} : B \in [A]^{\leq\tau}\} \text{ for every } A \subseteq X.$$

It is easy to see that metric $\to$ first countable $\to$ Fréchet-Urysohn $\to$ $t(X) = \omega$.

Definition 1.2 (Arhangel'skiĭ [2, 3]) Let X be a topological space. For $i = 1, 2, 3$ and 4 we say that X is an *α_i-space*[1] if for every countable family $\{S_n : n \in \omega\}$ of sequences converging to some point $x \in X$ there exists a (kind of diagonal) sequence S converging to x such that:

(α_1) $S_n \setminus S$ is finite for all $n \in \omega$,

(α_2) $S_n \cap S$ is infinite for all $n \in \omega$,

(α_3) $S_n \cap S$ is infinite for infinitely many $n \in \omega$,

(α_4) $S_n \cap S \neq \emptyset$ for infinitely many $n \in \omega$.

Definition 1.3 (Nyikos [17]) We say that a space X is an *$\alpha_{3/2}$-space* if for every countable family $\{S_n : n \in \omega\}$ of sequences converging to some point $x \in X$ such that $S_n \cap S_m = \emptyset$ for $n \neq m$, there exists a sequence S converging to x such that $S_n \setminus S$ is finite for infinitely many $n \in \omega$.

The following implications hold:

$$\text{metric} \to \text{first countable} \to \alpha_1 \to \alpha_{3/2} \to \alpha_2 \to \alpha_3 \to \alpha_4. \tag{1}$$

The only nontrivial implication $\alpha_{3/2} \to \alpha_2$ is due to Nyikos [17].

Convergence properties in general topological spaces

Let us first mention that Fréchet-Urysohn spaces need not be α_4:

Example 1.4 Let X be the countable Fréchet-Urysohn fan. That is $X = (\omega \times \omega) \cup \{*\}$, where $* \notin \omega \times \omega$, all points of the set $\omega \times \omega$ are isolated and the family

$$\{\{\{*\} \cup \{(n, m) : m \leq f(n) \text{ for all } n \in \omega\} : f \in \omega^\omega\}$$

serves as the family of neighbourhoods of the only non-isolated point $*$. Then *X is Fréchet-Urysohn but is not an α_4.*

[1] In [2, 3] Arhangel'skiĭ used a different terminology. We adopt here the term α_i-space which is currently beeing used.

We now turn to a natural question of whether some arrows in (1) can be reversed.

Theorem 1.5 (Simon [24]) *There exists a compact Fréchet-Urysohn α_4-space that is not α_3.*

Theorem 1.6 (Reznichenko [19], Gerlits and Nagy [10], Nyikos [16]) *There exists a compact Fréchet-Urysohn α_3-space that is not α_2.*

For $f, g \in \omega^\omega$ we write $f <^* g$ if $f(n) < g(n)$ for all but finitely many $n \in \omega$. A family $\mathcal{F} \subseteq \omega^\omega$ is *unbounded* if for every function $g \in \omega^\omega$ there exists $f \in \mathcal{F}$ such that $g <^* f$. We define $\mathfrak{b}$ to be the smallest cardinality of an unbounded family in $(\omega^\omega, <^*)$. The cardinality of the continuum is denoted by $\mathfrak{c}$.

Theorem 1.7 (Nyikos [17]) *If $\mathfrak{b} = \omega_1$ holds, then there exists a countable Fréchet-Urysohn α_2-space that is not α_1.*

Theorem 1.8 *The existence of the following spaces is consistent with ZFC:*
(i) a compact Fréchet-Urysohn α_2-space that is not $\alpha_{3/2}$,
(ii) a compact Fréchet-Urysohn $\alpha_{3/2}$-space that is not α_1.

Theorem 1.9 (Dow [8]) *α_2 implies α_1 in the Laver model for the Borel conjecture.*

It follows that α_2, $\alpha_{3/2}$ and α_1 properties coincide in the Laver model for the Borel conjecture.

Theorem 1.10 (folklore) *Let $G = \{f \in 2^{\omega_1} : |\{\beta \in \omega_1 : f(\beta) = 1\}| \leq \omega\}$. Then G is a Fréchet-Urysohn topological group that is α_1 but is not first countable.*

The group G in the previous example is not countable. In fact, all countable subsets of G are metrizable (and thus first countable). The question of the existence of a *countable* Fréchet-Urysohn α_1-space that is not first countable turns out to be delicate. We start with two results of independent interest.

Theorem 1.11 (Nyikos [16]) *Every space of character $< b$ is α_1.*

Theorem 1.12 (Malyhin and Shapirovskiǐ [11]) *If MA holds, then every countable space of character $< c$ is Fréchet-Urysohn.*

Let X be any countable dense subset of $\{0,1\}^{\omega_1}$, and let G be the subgroup of $\{0,1\}^{\omega_1}$ algebraically generated by X. Since G is countable, under $MA + \neg CH$ it will be Fréchet-Urysohn (Theorem 1.12), and since $\omega_1 < b = c$ under MA, G will be an α_1-space (Theorem 1.11). Thus one gets

Corollary 1.13 (Malyhin, 197?) *$MA + \neg CH$ implies the existense of a countable Fréchet-Urysohn group that is an α_1-space but is not first countable.*

Theorem 1.14 (Dow and Steprāns [9]) *There is a model of ZFC in which all countable Fréchet-Urysohn α_1-spaces are first countable.*

Corollary 1.15 *The existence of a countable Fréchet-Urysohn α_1 space that is not first countable is both consistent with and independent of ZFC.*

The next result demonstrates that one cannot get the natural strengthening of both Theorems 1.7 and 1.14 simultaneously:

Theorem 1.16 (Gerlits and Nagy [10], Nyikos [16]) *There exists a countable Fréchet-Urysohn α_2-space that is not first countable.*

Convergence properties in topological groups

Example 1.4 shows that our next theorem is specific for topological groups.

Theorem 1.17 (Nyikos [15]) *Every Fréchet-Urysohn topological group is α_4.*

As a natural corollary from Example 1.4 and Theorem 1.17 one obtains:

Corollary 1.18 (Nyikos [15]) *There exists a Fréchet-Urysohn space that cannot be be embedded as a subspace of a Fréchet-Urysohn topological group.*

We next examine whether some arrows in (1) can be reversed in the class of topological groups.

Theorem 1.19 (Shakhmatov [22]) *Let M be a model of ZFC obtained by adding ω_1 many Cohen reals to an arbitrary model of ZFC. Then M contains a countable Fréchet-Urysohn topological group G that is not α_3.* (Note that G is α_4 by Theorem 1.17.)

Theorem 1.20 (Shibakov [23]) *CH implies the existence of a countable Fréchet-Urysohn topological group that is α_3 but is not α_2.*

Theorem 1.21 (Shakhmatov [22]) *Let M be a model of ZFC obtained by adding ω_1 many Cohen reals to an arbitrary model of ZFC. Then M contains a countable Fréchet-Urysohn topological group G that is α_2 but is not $\alpha_{3/2}$.*

Theorem 1.22 (Shibakov [23]) *A Fréchet-Urysohn topological group that is an $\alpha_{3/2}$-space is α_1. Thus $\alpha_{3/2}$ and α_1 are equivalent for Fréchet-Urysohn topological groups.*

Theorem 1.23 (Birkhoff, Kakutani, 1936) *A topological group is metrizable if and only if it is first countable.*

Question 1.24 (Shakhmatov [22]) Is it consistent with ZFC that every Fréchet-Urysohn topological group is α_3? What about countable Fréchet-Urysohn topological groups?

Question 1.25 Is it consistent with ZFC that every Fréchet-Urysohn topological group that is an α_3-space is automatically α_2? What about countable Fréchet-Urysohn topological groups?

Question 1.26 (Shakhmatov [22]) Is it consistent with ZFC that every *countable* Fréchet-Urysohn topological group that is an α_2-space is first countable?

The reader may have noticed that all of the examples of Fréchet-Urysohn topological groups distinguishing α_i-properties presented so far are of consistency nature. This is because even the following fundamental problem is still open:

Question 1.27 (Malyhin, 197?) Without any additional set-theoretic assumptions beyond ZFC, does there exist a *countable* Fréchet-Urysohn topological group that is not first countable?

A consistent example was given in Theorem 1.13. The word "countable" is essential in view of Theorem 1.10.

We will now present a possible approach to constructing a ZFC example. First let us remind ourselves the following folklore construction of topological groups.

For $A, B \in [\omega]^{<\omega}$ define $A \cdot B = (A \setminus B) \cup (B \setminus A) \in [\omega]^{<\omega}$. This operation makes $[\omega]^{<\omega}$ into an Abelian group with $\emptyset$ as the identity element such that $A \cdot A = \emptyset$ (thus A coincides with its own inverse, and all elements of $[\omega]^{<\omega}$ have order 2). For a filter $\mathcal{F}$ on ω let $G(\mathcal{F})$ be the group $([\omega]^{<\omega}, \cdot, \emptyset)$ equipped with the topology whose base of open neighbourhoods of $\emptyset$ is given by the family $\{[F]^{<\omega} : F \in \mathcal{F}\}$.

Reznichenko and Sipacheva [20] say that a filter $\mathcal{F}$ on ω is a *FUF-filter* privided that the following property holds: if $\mathcal{K} \subseteq [\omega]^{<\omega}$ is a family of finite subsets of ω such that for every $F \in \mathcal{F}$ there exists $K \in \mathcal{K}$ with $K \subseteq F$, then there exists a sequence $\{K_n : n \in \omega\} \subseteq \mathcal{K}$ so that for every $F \in \mathcal{F}$ one can find $n \in \omega$ with $K_m \subseteq F$ for all $m \geq n$.

Theorem 1.28 (folklore) *Let $\mathcal{F}$ be a filter on ω. Then:*

(i) $G(\mathcal{F})$ is Hausdorff if and only if $\mathcal{F}$ is free (i.e. $\bigcap \mathcal{F} = \emptyset$),

(ii) $G(\mathcal{F})$ is Fréchet-Urysohn if and only if $\mathcal{F}$ is an FUF-filter,

(iii) $G(\mathcal{F})$ is first countable if and only if $\mathcal{F}$ is countably generated.

Corollary 1.29 (folklore) *If there exists a free FUF-filter on ω that is not* *countably generated, then there exists a countable Fréchet-Urysohn topological group that is not first countable.*

Question 1.30 (folklore) Is there, in ZFC only, a free FUF-filter on ω that is not countably generated?

Theorem 1.31 (Reznichenko and Sipacheva [20]) *Let $\mathcal{F}$ be a filter on ω, and let $\omega_{\mathcal{F}}$ be the space obtained by adding to the discrete copy of ω a single point $*$ whose filter of open neighbourhoods is $\{F \cup \{*\} : F \in \mathcal{F}\}$. If $\mathcal{F}$ is a FUF-filter, then the space $\omega_{\mathcal{F}}$ is α_2.*

This theorem implies that a positive answer to Question 1.30 would provide a strengthening of the example from Theorem 1.16.

Definition 1.32 A space X has the *Ramsey property* if, for every matrix $\mathcal{M} = \{x_{ij} : i, j \in \omega\}$ of points in X such that $\lim_{i\to\infty} \lim_{j\to\infty} x_{ij} = x$ for some point $x \in X$, there exists an infinite set $M \subseteq \omega$ with the following property: For every U, an open neighbourhood of x, one can find $k \in \omega$ such that $x_{mn} \in U$ whenever $m, n \in M$ and $k \leq m < n$.

A somewhat weaker property than in the above definition first appeared in the classical paper of Ramsey [18].

Theorem 1.33 (Nogura and Shakhmatov [14]) *(i) A space with the Ramsey property is α_3.*

(ii) There exists an α_1-space without the Ramsey property.

(iii) A topological group that is an $\alpha_{3/2}$-space has the Ramsey property.

Convergence properties in locally compact groups

Since locally compact groups have a nice structure, it is necessary to expect that many convergence properties coincide for them. It is indeed the case.

Theorem 1.34 (folklore) *A locally compact group G with $t(G) = \omega$ is metrizable.*

Theorem 1.35 (Nogura and Shakhmatov [14]) *All α_i properties for $i = 1, 3/2, 2, 3, 4$, as well as the Ramsey property, coincide for locally compact topological groups.*

Theorem 1.36 (Nogura and Shakhmatov [14]) *The following conditions are equivalent:*

(i) every compact group that is an α_1-space is metrizable,

(ii) every locally compact group that is an α_4-space is metrizable,

(iii) $b = \omega_1$.

Corollary 1.37 (Nogura and Shakhmatov [14]) *Under CH, a locally compact group is metrizable if and only if it is α_4.*

Convergence properties in functions spaces $C_p(X)$

For a topological space X let $C_p(X)$ be the set of all real-valued continuous functions on X equipped with the topology of pointwise convergence, i.e with the topology which the set $C_p(X)$ inherits from $\mathbf{R}^X$, the latter space having the Tychonoff product topology. For every space X, $C_p(X)$ is both a (locally convex) topological vector space and a topological ring.

Theorem 1.38 (Scheepers [21]) *Let X be a topological space. Then $C_p(X)$ is α_2 if and only if $C_p(X)$ is α_4. Therefore, all three properties α_4, α_3 and α_2 coincide for spaces of the form $C_p(X)$.*

Corollary 1.39. (Scheepers [21]) *If $C_p(X)$ is Fréchet-Urysohn, then $C_p(X)$ is α_2.*

Theorem 1.40 (Scheepers [21]) *It is consistent with ZFC that there exists a subset of real numbers $X \subseteq \mathbf{R}$ such that $C_p(X)$ is Fréchet-Urysohn (and thus α_2) but is not α_1.*

Note that the existence of the above space is not only consistent with ZFC but also independent of ZFC by Theorem 1.9.

Theorem 1.41 (Scheepers [21]) *It is consistent with ZFC that there exists a subset of real numbers $X \subseteq \mathbf{R}$ such that $C_p(X)$ is α_1 but is not Fréchet-Urysohn.*

A set $X \subseteq \mathbf{R}$ of real numbers is said to be *Sierpinski set* if it has cardinality c, and its intersection with any set of Lebesgue measure zero is uncountable.

Theorem 1.42 (Scheepers [21]) *If X is a Sierpinski set, then $C_p(X)$ is α_1.*

Definition 1.43 (Császár and Laczkovich [7], Bukovská [4]) A sequence of real-valued functions $\{f_n : n \in \omega\}$ definied on a set X *quasi-normally converges* to a real-valued function f provided that there exists a sequence $\{\varepsilon_n : n \in \omega\}$ of positive real numbers such that:

(i) $\lim_{n\to\infty} \varepsilon_n = 0$,

(ii) for each $x \in X$, $|f_n(x)| < \varepsilon_n$ for all but finitely many n.

Definition 1.44 (Bukovský, Recław and Repický [5]) A space X is a *QN-space* provided that, whenever a sequence $\{f_n : n \in \omega\}$ of continuous real-valued functions defined on X converges pointwise to the continuous function f, this convergence is automatically quasi-normal.

Theorem 1.45 (Scheepers [21]) *If $C_p(X)$ is an α_1-space, then X is a QN-space.*

Question 1.46 (Scheepers [21]) Does the converse hold? I.e. is $C_p(X)$ an α_1-space if and only if X is a QN-space?

2 Convergence properties in products

Products of general spaces

The countable Fréchet-Urysohn fan from Example 1.4 demonstrates that the square of Fréchet-Urysohn space need not be Fréchet-Urysohn. Moreover, Simon gave even stronger counter-example:

Theorem 2.1 (Simon [24]) *There exists a compact Fréchet-Urysohn space X such that $X \times X$ is not Fréchet-Urysohn.*

It is this failure of preservation of the Fréchet-Urysohn property that was the primary motivation for Arhangel'skiĭ when he introduced α_i-spaces. He also proved the following:

Theorem 2.2 (Arhangel'skiĭ [3]) *If X is a Fréchet-Urysohn α_3-space and Y is a (countably) compact Fréchet-Urysohn space, then $X \times Y$ is Fréchet-Urysohn.*

Note that Theorems 2.1 and 2.1 imply Theorem 1.5.

Theorem 2.3 (Nogura [13])

(i) For $i = 1, 2, 3$, if X and Y are α_i-spaces, then $X \times Y$ is also an α_i-space.

(ii) There exist compact Fréchet-Urysohn α_4-spaces X and Y such that $X \times Y$ is neither Fréchet-Urysohn nor α_4.

Theorem 2.4 (Costantini and Simon [6]) *There exist two countable Fréchet-Urysohn α_4-spaces X and Y such that $X \times Y$ is α_4 but fails to be Fréchet-Urysohn.*

Theorem 2.5 (Simon [25]) *Under CH, there exist two countable Fréchet-Urysohn α_4-spaces X and Y such that $X \times Y$ is Fréchet-Urysohn but is not α_4.*

Question 2.6 (Simon [25]) Is there such an example in ZFC?

Products of topological groups

Recall that a topological group G is *compactly generated* provided that there exists a compact set $K \subseteq G$ such that the smallest subgroup of G that contains K coincides with G.

Theorem 2.7 (Todorčević [27]) *There exist two (compactly generated) Fréchet-Urysohn groups G and H such that $t(G \times H) > \omega$ (in particular, $G \times H$ is not Fréchet-Urysohn). Moreover, every countable subset of G and H is metrizable, and so both G and H are α_1.*

Theorem 2.8 (Malyhin and Shakhmatov [12]) *Add a single Cohen real to a model of $MA + \neg CH$. Then, in the generic extension, the exists a (hereditarily separable) Fréchet-Urysohn topological group G such that $t(G \times G) > \omega$ (in particular, $G \times G$ is not Fréchet-Urysohn). Moreover, G is an α_1-space.*

Theorem 2.9 (Shibakov [23]) *Under CH, there exists a countable Fréchet-Urysohn topological group G such that $G \times G$ is not Fréchet-Urysohn.*

Question 2.10 Is there such an example in ZFC only?

Question 2.11 In ZFC only, does there exist two *countable* Fréchet-Urysohn topological groups G and H such that $G \times H$ is not Fréchet-Urysohn?

Question 2.12 In ZFC only, is there a Fréchet-Urysohn topological group G such that G is α_1 but $G \times G$ is not Fréchet-Urysohn?

Products of function spaces $C_p(X)$

Theorem 2.13 (Tkačuk [26]) *If $C_p(X)$ is Fréchet-Urysohn, then even its countable power $C_p(X)^\omega$ is Fréchet-Urysohn.*

Theorem 2.14 (Todorčević [27]) *There exist two spaces X and Y such that both $C_p(X)$ and $C_p(Y)$ are Fréchet-Urysohn but $t(C_p(X) \times C_p(Y)) > \omega$ (in particular, $C_p(X) \times C_p(Y)$ is not Fréchet-Urysohn). Moreover, every countable subset of $C_p(X)$ and $C_p(Y)$ is metrizable, and so both $C_p(X)$ and $C_p(Y)$ are α_1.*

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