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REMARKS ON SPLENDID TILTING COMPLEXES

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0. INTRODUCTION

In [1,Question 6.2] Broué proposed a question concerning derived equivalences between blocks of finite groups. Then the question has been much interested and studied by many people (see,also [2]and [3]).

The question has been answered in the case that groups are p-solvable in [5] and that blocks have cyclic defect groups in [9] (see also [13]). Recently it was answered for defect two blocks of Symmetric groups in [4]. And in [7] we checked the question for the principal 3-blocks of the groups, $A_6, A_7, A_8, S_6, PSL(3,4), M_{11}, M_{22}, M_{23}$ and HS.

Our aim in this talk is to show that the examples in [7] we checked are all splendid equivalent. In section 1 in this note we shall explain our method in [7]. And then we discuss how to construct twosided tilting complexes in section 2 to use to show that our above examples are splendid equivalent in section 3 .

We only give outlines of proofs of our results and the detailed and final version of this note will be submitted for publication elsewhere.

1. PRELIMINARIES

In this section we summarize some results from theory of derived equivalences of algebras which will be needed in our discussion.

Throughout this section let A be a finite dimensional symmetric algebra over an algebraically closed field k of characteristic $p>0$. By modules over an algebra we mean right modules of finite dimension.

Let P_0 be a projective A-module and set $\mathcal{P}=add(P_0)$. A right $\mathcal{P}$-approximation of an A-module X is a sequence ; $P \xrightarrow{f} X$ satisfying

(1) $P\in\mathcal{P}$

(2) $Hom_A(P',P) \xrightarrow{Hom_A(P',f)} Hom_A(P',X)\rightarrow 0$ (exact), for any $P'\in\mathcal{P}$

Every A-module X has a unique minimal right $\mathcal{P}$-approximation and any right $\mathcal{P}$-approximation has minimal one as direct summand.

Let $S_i, i\in I$ be the set of all simple A-modules and P_i be a projective cover of $S_i, i\in I$. Take a subset I_0 of I (and fix it). And set $\mathcal{P}(I_0)=add(\oplus\sum_{i\in I_0}P_i)$.

For each $i \in I$ we shall construct a compjex $P_i^* \in K^b(P_A)$, the homotopy category of bounded complexes of projective A-modules.

For $i \notin I_0$ let P_i^* be $\cdots \longrightarrow 0 \longrightarrow R_i \xrightarrow{\delta_i} P_i \longrightarrow 0 \longrightarrow \dots$, where $\delta_i : R_i \to P_i$ is a minimal right $\mathcal{P}(I_0)$-approximation of P_i (P_i is in degree 0 and R_i is in degee -1).

For $i \in I_0$ let P_i^* be $\cdots \longrightarrow 0 \longrightarrow P_i \longrightarrow 0 \longrightarrow \dots$ (P_i is in degree -1).

Now set $P^*\{I_0\} = \oplus \sum_{i \in I} P_i^*$. Let P^0 (resp. P^1) be the 0th (resp. -1st) term of $P^*\{I_0\}$ and $\delta_0 = \oplus \sum_{i \in I} \delta_i$.

Theorem 1(Rickard [10],see also [14]).
$P^*\{I_0\}$ is a tilting complex for A.

Put $C = End_{K^b(P_A)}(P^*\{I_0\})$. Let $\widehat{P}_i$ be an indecomposable projective C–module corresponding to an inecomposable direct summand P_i^* of $P^*\{I_0\}$ and $\widehat{S}_i$ be a simple C–module corresponding to $\widehat{P}_i$, $i \in I$.

For $i \notin I_0$ set $\rho_i = Hom_A(\delta_0, S_i) : Hom_A(P^0, S_i) \to Hom_A(P^1, S_i)$ ($\rho_i = 0$,in fact).

For $i \in I_0$, let $U_i/S_i \subset P_i/S_i$ be the largest submodule of P_i/S_i each of whose composition factors is isomorphic to S_k for some $k \notin I_0$. Set $\rho_i = Hom_A(\delta_0, U_i) : Hom_A(P^0, U_i) \to Hom_A(P^1, U_i)$.

Proposition 1(see [7]). *The followings hold.*

(1) *For $i \notin I_0, \widehat{S}_i \cong Ker\rho_i$*
(2) *For $i \in I_0, \widehat{S}_i \cong Cok\rho_i$*

Using the result due to Rickard in [11] we can costruct an (A, C)-bimodule ${}_AL_C$ which and ${}_CHom_k(L, k)_A$ give a stable equivalence of Morita type between A and C with the following property; for $i \notin I_0$, $S_i \otimes_A L \cong \widehat{S}_i$ and for $i \in I_0$, $W_i \otimes_A L \cong \widehat{S}_i$, where $W_i = \Omega^{-1}(U_i)$.

Now let B be a symmetric algebra which is stabley equivalent of Morita type to A and let bimodules ${}_BN_A$ and ${}_AM_B = Hom_k({}_BN_A, k)$ give a stably equivalence of Morita type between B and A. We assume that the set of simple B-modules $\{T_i; i \in I\}$ is also indexed by I (We also assume that A and B are connected and nonsimple).

Set A-modules $X_i = T_i \otimes_B N, i \in I$.

If $X_i, i \in I$ are all simple in the stable module category of A, then the result of Linckelmann in [6] says that B and A are Morita equivalent.

Otherwise, we do the following procedure.

(*) Take a "nice" subset I_0 of I and construct $P^*\{I_0\}$ and calculate
(*-1) $C = End(P^*\{I_0\})$ and
(*-2) C-modules $\widehat{X}_i = X_i \otimes_A L, i \in I$.

Then ${}_BN \otimes_A L_C$ gives a stably equivalence of Morita type between B and C and $\widehat{X}_i = T_i \otimes_B (N \otimes_A L)$.

If we have here that $\widehat{X}_i, i \in I$ are all simple , then use the result of Linckelmann to conclude that B and C are Morita equivalent and therefore B and A are derived equivalent.

So assume that we could find a sequece of "nice" subsets I_0 for A and then I_1 for C and ... of I such that the followings occur; we do the procedure (*) for A with respect to I_0 , then for C with respect to I_1, ... and the resulting module corresponding to X_i , $i \in I$ over the final algebra are all simple.

Then the final algebra is Morita equivalent to B by Linckelmann's result and we can conclude that B and A are derived equivalent (although our assumption is too strong !).

2. Twosided Tilting Complexes

We shall use the notations in section 1. So A and B are symmetric algebras over k which are stably equivalent of Morita type. Let $S_i, i \in I$ be the set of all simple A-modules and P_i be a projective cover of $S_i, i \in I$ as before. Let ${}_BN_A$ be a (B, A)-bimodule which gives a stable equivalence between A and B such that ${}_BN$ and N_A are projective. We assume that ${}_BN_A$ has no projective (B, A)-summand. Take a subset I_0 of I (and fix it). And set $\mathcal{P}(I_0) = add(\oplus \sum_{i \in I_0} P_i)$. U_i and W_i, $i \in I_0$, are A-modules defined in section 1.

Set $\mathcal{P}(B, I_0) = add(B \otimes_k \oplus \sum_{i \in I_0} P_i)$ (this is a subcategory of the category of (B, A)-modules). And let $P^*(N, I_0)$ be a complex of (B, A) -bimodules ;

$P^*(N, I_0) : \cdots \longrightarrow 0 \longrightarrow {}_BP_A \xrightarrow{\delta} {}_BN_A \longrightarrow 0 \longrightarrow \ldots$,

where $\delta : {}_BP_A \rightarrow {}_BN_A$ is a minimal right $\mathcal{P}(B, I_0)$-approximation of ${}_BN_A$.

Theorem 2. *As a complex of A-modules* $P^*(N, I_0)_A$ *is a tilting complex for A.*

Proof. As a map of A-modules, $\delta : P_A \rightarrow N_A$ is a right $\mathcal{P}(I_0)$- approximation (not necessarily minimal) of N_A. So as a complex of A-modules, $P^*(N, I_0)$ is a direct sum of complexes isomorphic to $P_i^*, i \in I$ (P_i^* is the complex defined in section 1). We shall show that for each $i \in I$, the multiplicity of P_i^* in $P^*(N, I_0)_A$ is nonzero.

For $i \notin I_0$, the multiplicity of P_i^* in $P^*(N, I_0)_A$ is $dimKer\delta_i^*$ by Proposition 1, where $\delta_i^* = Hom_A(\delta, S_i) : Hom_A(N_A, S_i) \rightarrow Hom_A(P_A, S_i)$. $Ker\delta_i^* = Hom_A(N_A, S_i) \neq 0$ as δ_i^* is actually a zero map.

For $i \in I_0$, the multiplicity of P_i^* in $P^*(N, I_0)_A$ is $dimCok\delta_i^*$ by Proposition 1, where $\delta_i^* = Hom_A(\delta, U_i) : Hom_A(N_A, U_i) \rightarrow Hom_A(P_A, U_i)$. We shall show that $Cok\delta_i^* \cong Hom_A({}_BN_A, W_i)^\circ$, a (unique) non projective indecomposable B-summand of $Hom_A({}_BN_A, W_i)$, in the following steps.

Step 1. *The map* $Hom_A(N_A, U_i) \rightarrow Hom_A(Im\delta, U_i)$ *induced by the inclusion* $Im\delta \hookrightarrow N$ *is an isomorphism. In particular,* δ_i^* *is injective.*

Proof. Apply $Hom_A(-, U_i)$ for $0 \to Im\delta \to N \to Cok\delta \to 0$ to obtain the exact sequence;

$$0 \to Hom_A(Cok\delta, U_i) \to Hom_A(N, U_i) \to \\ Hom_A(Im\delta, U_i) \to \underline{Hom}_A(Cok\delta, W_i) \to 0$$

$(W_i = \Omega^{-1}(U_i))$. As $SocU_i = S_i$ and $SocW_i$ is a direct sum of simple modules isomorphic to S_k $,k \in I_0$, it holds that $Hom_A(Cok\delta, U_i) = 0 = Hom_A(Cok\delta, W_i)$.

Step 2. *For any simple B-module T , the map $Hom_B({}_BP_A, Hom_k(T_B, k) \otimes_k U_i)_A \to Hom_B({}_BKer\delta_A, Hom_k(T_B, k) \otimes_k U_i)_A$ induced by the inclusion ${}_BKer\delta_A \hookrightarrow {}_BP_A$ is a zero map.*

Proof. P_A is a direct sum of modules isomorphic to $P_k, k \in I_0$ and $S_i = SocU_i \subset U_i$ is a unique composition factor of U_i which is in $\{S_k; k \in I_0\}$. So for any (B, A)-map $f : {}_BP_A \to Hom_k(T_B, k) \otimes_k U_i$, Imf is contained in $Hom_k(T_B, k) \otimes_k S_i$. If the map in the statement of Step 2 was nonzero, then there exists a projective map from ${}_BKer\delta_A$ to a simple (B, A)- mdule $Hom_k(T_B, k) \otimes_k S_i$. This is not the case because ${}_BKer\delta_A$ has no projective summand (δ is minimal).

Step 3. *$\delta_i^* : Hom_A({}_BN_A, U_i) \to Hom_A({}_BP_A, U_i)$ is a minimal injective hull of a B-module $Hom_A({}_BN_A, U_i)$. In particular, $Cok\delta_i^* = \Omega^{-1}(Hom_A({}_BN_A, U_i))$ as B-modules.*

Proof. From the exact sequence $0 \to Ker\delta \to {}_BP_A \to Im\delta \to 0$ we have the following exact sequences of B-modules ;

$$0 \to Hom_A(Im\delta, U_i) \xrightarrow{\zeta} Hom_A({}_BP_A, U_i) \xrightarrow{\eta} Hom_A(Ker\delta, U_i)$$

We claim that $Soc_B(Hom_A({}_BP_A, U_i)) \subset Ker\eta$. For a simple B-module T , apply $Hom_B(T, -)$ for the above sequence to obtain $Hom_B(T_B, Hom_A({}_BP_A, U_i)) \xrightarrow{\eta_*} Hom_B(T_B, Hom_A({}_BKer\delta_A, U_i))$. By Step 2 and the isomorphism ;

$$Hom_B(T_B, Hom_A({}_B?_A, U_i)) \cong Hom_B({}_B?_A, Hom_k(T_B, k) \otimes_k U_i)_A$$

, we see that η_* is a zero map. So our claim follows. As $Hom_A({}_BP_A, U_i)$ is a projective B-module , ζ is a minimal injective hull of $Hom_A(Im\delta, U_i)$. Now the result follows by Step 1.

Step 4. *$\Omega^{-1}(Hom_A({}_BN_A, U_i)) \cong Hom_A({}_BN_A, W_i)^\circ$ as B-modules.*

Proof. Apply $Hom_A({}_BN_A, -)$ for the exact sequence : $0 \to U_i \to P_i \to W_i \to 0$. Then we obtain the exact sequence of B-modules ;

$$0 \to Hom_A({}_BN_A, U_i) \to Hom_A({}_BN_A, P_i) \to Hom_A({}_BN_A, W_i) \to 0$$

As $Hom_A({}_BN_A, P_i)$ is an injective (projective) B-module, the result follows.

Set $A^{(1)} = End_{K^b(P_A)}(P^*(N, I_0))$.

$A^{(1)}$ is Morita equivalent to $C = End_{K^b(P_A)}(P^*\{I_0\})$,the algebra discussed in section 1. Let $S_i^{(1)}$ be a simple $A^{(1)}$-module corresponding to an indecomposable direct summand P_i^* of $P^*(N, I_0)$, $i \in I$.

As each term in $P^*(N, I_0)$ is a (B, A)-bimodule, there exists an algebra map $B \rightarrow A^{(1)}$ induced by left multiplication by elements in B. And $A^{(1)}$-modules are considered as B-modules via this map.

A proof of the above theorem shows the following.

Corollary 1. *The followings hold.*

(1) *For* $i \notin I_0, S_{iB}^{(1)} \cong Hom_A({}_BN_A, S_i)$ *as B-modules*
(2) *For* $i \in I_0, S_{iB}^{(1)} \cong Hom_A({}_BN_A, W_i)^\circ$ *as B- modules*

The above fact says that ${}_{A^{(1)}}A_B^{(1)}$ is indecomposable as an $(A^{(1)}, B)$-bimodule.

Proposition 2. *The followings hold.*

(1) ${}_BA_B^{(1)} \cong B\oplus$ *projective* (B, B)*-module*
(2) $A^{(1)} \otimes_B Hom_k(A^{(1)}, k) = A^{(1)}\oplus$ *projective* $(A^{(1)}, A^{(1)})$*-module modules*

Proof. $A^{(1)}$ is connected and symmetric as so is A. And B is also connected and symmetric by our assumption. So the asertion (2) follows from (1).

From the complex $P^*(N, I_0) : {}_BP_A \xrightarrow{\delta} {}_BN_A$ we obtain the following sequences of (B, B)-bimodules;

$$Hom_A(N, P) \xrightarrow{\tau} Hom_A(N, N) \oplus Hom_A(P, P) \xrightarrow{\sigma} Hom_A(P, N)$$

where τ is defined by $f \mapsto (\delta \circ f, f \circ \delta)$ for $f \in Hom_A(N, P)$ and σ is defined by $(h, g) \mapsto h \circ \delta - \delta \circ g$. As $P^*(N, I_0)$ is a tilting complex for A , τ is injective , σ is surjective and $Ker\sigma/Im\tau \cong A^{(1)}$.

$Hom_A({}_BN_A, {}_BP_A), Hom_A({}_BP_A, {}_BN_A), Hom_A({}_BP_A, {}_BP_A)$ are all projective as (B, B)-modules and $Hom_A({}_BN_A, {}_BN_A) \cong B\oplus$ projective (B, B)-module. Now the assertion (1) follows a standard argument.

The following corollary shall be shown by the above discussion and the result of Linckelmann.

Corollary 2. *Suppose that* $Hom_A({}_BN_A, S_i), i \notin I_0$ *and* $Hom_A({}_BN_A, W_i)^\circ, i \in I_0$ *are all simple B-modules. Then* $B = A^{(1)}$ *and* $P^*(N, I_0)$ *is a split-endmorphism twosided tilting complex for* (B, A).

For the definition of split-endmorphism twosided tilting complexes, see [12]

3. Splendid Tilting Complexes

We shall use the same notaions as for sections 1 and 2.

Let ${}_{A^{(1)}}L_A^{(1)}$ be a non projective $(A^{(1)}, A)$-summand of $A^{(1)} \otimes_B N$.

Set $\mathcal{P}(A^{(1)}, I_0) = add(A^{(1)} \otimes_k \oplus \sum_{i \in I_0} P_i)$ (this is a subcategory of the category of $(A^{(1)}, A)$-modules). And let $P^*(L^{(1)}, I_0)$ be a complex of $(A^{(1)}, A)$ -bimodules ;

$P^*(L^{(1)}, I_0) : \cdots \longrightarrow 0 \longrightarrow {}_{A^{(1)}}P_A^{(1)} \xrightarrow{\delta^{(1)}} {}_{A^{(1)}}L_A^{(1)} \longrightarrow 0 \longrightarrow \dots ,$

where $\delta^{(1)} : {}_{A^{(1)}}P_A^{(1)} \to {}_{A^{(1)}}L_A^{(1)}$ is a minimal right $\mathcal{P}(A^{(1)}, I_0)$-approximation of ${}_{A^{(1)}}L_A^{(1)}$.

Theorem 3.
$P^*(L^{(1)}, I_0)$ is a split-endmorphism twosided tilting complex for $(A^{(1)}, A)$. And as complexes of (B, A)-modules , ${}_BP^*(L^{(1)}, I_0)_A \cong P^*(N, I_0)$.

Proof. For any A-module X , $Hom_A(A^{(1)} \otimes_B N_A, X)$ is isomorphic to a direct sum of $Hom_A(L^{(1)}, X)$ and a projective $A^{(1)}$-module as $A^{(1)}$-modules. And we have the following isomorphisms ; $Hom_A(A^{(1)} \otimes_B N_A, X) \cong Hom_B(A_B^{(1)}, Hom_A({}_BN_A, X))$ $\cong Hom_A({}_BN_A, X) \otimes_B A^{(1)}$. Thus for $i \notin I_0, Hom_A(A^{(1)} \otimes_B N_A, S_i) \cong$ $Hom_A({}_BN_A, S_i) \otimes_B A^{(1)} \cong S_i^{(1)} \otimes_B A^{(1)} = S_i^{(1)} \oplus$ projective $A^{(1)}$-module (the second isomorphism follows from Corollary 1 and the third follows from Proposition 2).

For $i \in I_0, Hom_A(A^{(1)} \otimes_B N_A, W_i) \cong Hom_A({}_BN_A, W_i) \otimes_B A^{(1)} \cong S_i^{(1)} \otimes_B A^{(1)} =$ $S_i^{(1)} \oplus$ projective $A^{(1)}$-module by the same reason as above.

Now applying Corollary 2 for $A^{(1)}$ and A , we can conclude that $P^*(N^{(1)}, I_0)$ is a split-endmorphism twosided tilting complex for $(A^{(1)}, A)$.

${}_BL_A^{(1)} \cong N \oplus$ projective (B, A)-module by Proposition 2. As a complex of (B, A)-modules , ${}_BP^*(L^{(1)}, I_0)_A$ is a right $\mathcal{P}(B, I_0)$-approximation of ${}_BL_A^{(1)}$. So ${}_BP^*(L^{(1)}, I_0)_A$ is a direct sum of $P^*(N, I_0)$ and some complex Q^* of projective (B, A)-bimodules. Because endmorphism algebras over $K^b(P_A)$ are $A^{(1)}$ both for ${}_BP^*(N^{(1)}, I_0)_A$ and $P^*(N, I_0)$, we have that $Hom_{K^b(P_A)}(P^*(N, I_0), Q^*) = 0$. Thus Q^* is 0.

Let L_1 be an (A, C)-bimodule corresponding to an $(A, A^{(1)})$-module $Hom_k({}_{A^{(1)}}N_A^{(1)}, k)$ under the Morita equivalence between C and $A^{(1)}$ described in the above. Then by a proof of Theorem 3 , we may choose L_1 as an (A, C)-bimodule L discussed in the end of Section 1 (See the properties of L).

Now we shall summarize our discussions in the above.

We are given

(0-1) two symmetric algebras B and A which are stably equivalent of Morita type and the sets of their simple modules are indexed by the same set I.

(0-2) a (B, A)-bimodule ${}_BN_A$ which gives a stable equivalence of Morita type between B and A.

Then we took a subset $I_0 \subset I$

(0-3) construct a complex $P^*(N, I_0)$ of (B, A)-bimodules and set $A^{(1)} = End_{K^b(P_A)}(P^*(N, I_0))$.

Then ${}_BA_{A^{(1)}}^{(1)}$ gives a stable equivalence of Morita type between B and $A^{(1)}$.

(0-4) costruct a complex $P^*(L^{(1)}, I_0)$ of $(A^{(1)}, A)$-bimodules where ${}_{A^{(1)}}L^{(1)}_A$ is a non projective $(A^{(1)}, A)$-summand of ${}_{A^{(1)}}A^{(1)} \otimes_B N_A$ and ${}_BL^{(1)}_A \cong N \oplus$ projective (B, A)-module.

$P^*(L^{(1)}, I_0)$ is a split-endmorphism twosided tilting complex for $(A^{(1)}, A)$. It has $L^{(1)}$ in degree 0 term and a projective $(A^{(1)}, A)$-module in degree (-1).

Set ${}_BN^{(1)}_{A^{(1)}} = {}_BA^{(1)}_{A^{(1)}}$. Then B , $A^{(1)}$ and ${}_BN^{(1)}_{A^{(1)}}$ satisfy the situations (0-1) and (0-2).

Suppose that repeating the above procedures we could find a sequece of "nice" subsets $I_0, I_1, \ldots, I_{s-1}$ of I for $A^{(0)} = A, A^{(1)}, \ldots, A^{(s-1)}$ respectively,where $A^{(t)} = End_{K^b(P_{A^{(t-1)}})}(P^*(N^{(t)}, I_{t-1}))$, $t = 1, \ldots, s$ such that in the final stage, the assumptions in Corollay 2 are satisfied (This is the situation discussed in [7]).

Then $A^{(s)} \cong B$ and we can conclude that B and A are derived equivalent.

Actually the tensor product $P^*(N; I_0, \ldots, I_{s-1}) :=$
$P^*(L^{(s)}, I_{s-1}) \otimes_{A^{(s-1)}} P^*(L^{(s-1)}, I_{s-2}) \otimes \cdots \otimes_{A^{(1)}} P^*(L^{(1)}, I_0)$ is a split-endmorphism twosided tilting complex for $(A^{(s)}, A)$. We can easily see that the module in degree 0 of $P^*(N; I_0, \ldots, I_{s-1})$ is isomorphic to a direct sum of N and a projctive (B, A)-module and the modules in other degree are projective (B, A)- modules.

In the all examples we discussed in [7], we could choose the first (B, A)-module N from a bimodule direct summand of group algebras. Therefore the resulting complex $P^*(N; I_0, \ldots, I_{s-1})$ will be a splendid tilting complex.

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