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Lie groups and complex geometry

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By a pre-Cartan structure (modeled after a homogeous space G/H), we mean a mathematical structure on a manifold M by which, at each point $p \in M$, we can identify M infintesimally (of order possibly ≥ 1) with G/H. Such structure comes with its frame bundle, Among possible Cartan connections on the fame bundle, we choose one. We call such a structure a Cartan geometry. We thus associate the curvature to the Cartan geometry. In complex analysis CR geometry of a strongly pseudoconvex CR structure can be regarded as an example of Cartan geometry. In this case the model is the boundary of the unit ball B in the complex space, i.e. G/H, where G is the group of holomorphic automorphisms of the unit ball and H is the isotropy group of a point in the boundary. In this case, G with the standard projection $\pi : G \rightarrow G/H$ is the frame bundle. We also have a projection $\rho : G \rightarrow B = G/H_1$ where H_1 is the isotropy group of an interior point in B, In the strongly pseudoconvex case we can also construct locally (at least in real analytic cases with dimension ≥ 5) a projection ρ_M of an open subset of its frame bundle to its inside tubular neighborhood. This provide a new proof of embedding theorem of strongly pseudoconvex CR structures.

Since the above construction is based on the properties of the Lie group G, there is a strong possiblity that the above construction can be also carried out in the case of the automorphism groups of bounded symmetric domains.. We may thus use the notion of Cartan geometry as a guideline to develop the function theory of CR structure of codimension ≥ 2 which looks infinitesimally like the Silov boundary of bounded symmetric domains.