数理解析研究所講究録
1184巻 2001年 152-160

Theoretical analysis of stress distribution in sand piles: the result of an exactly solvable model and its controversial points

Hisao Hayakawa (早川尚男)

Graduate School of Human and Environmental Studies, Kyoto University, Sakyo-ku, Kyoto, 606-8501, Japan (京都大学人間・環境学研究科)

The current understanding on the sandpile problem is reviewed. In particular, the current development of exactly solvable continuous models is summarized. A try to get stress distribution based on an oriented stress linearity (OSL) model is explained, but its result except for the fixed principal axis ansatz does not work well.

1 Introduction

Physics of dry, cohesionless granular matter is one of challenging subjects in condensed matter physics[1, 2, 3]. In particular, many papers on static stresses of granular matter from different approaches have been published. Nevertheless, there is few consensus on what is the basic mechanism involved in it. One of characteristic phenomena for static granular matter is the dip formation of sand piles[4, 5]: Namely, the pressure measured at the bottom of sand piles has the minimum at the central part, and it has the maximum far from the center. The dip is reproducible even in two dimensional situations from simulations.[6, 7, 8, 9, 10]. There is an experimental paper to show the dip formation in a quasi two-dimensional situation[5].

To construct a statistical mechanics of discrete elements is a challenging approach, but this approach will encounter the difficulty how to obtain a closure form of the correlation functions at present[11]. Thus, the most of theoretical arguments are based on the continuum mechanics. Amongst many papers Edwards and Oakeshott[12] gave an explanation of the dip formation with the introduction of a model in which the force is propagated with a fixed angle from the slope. Later, Wittmer et al.[13] have revised Edwards' model as a continuum model for the stress tensor with the aid of the Fixed principal axis (FPA) ansatz where the major principal axis (MAPA) is inclined with a fixed angle. They succeeded to explain the dip formation and have published series of papers[14] based on the idea of FPA or its generalized model, the oriented stress linearity (OSL) model in spite of the strong criticism by Savage[6]. The most controversial part of FPA is that the symmetric axis becomes a singular line which is the source of MAPA, though the singularity causes the arching of the stress field.

On the other hand, the classical theory of granular continuum mechanics assumes a constitutive relation of the Incipient Failure Everywhere (IFE) where the material is everywhere just on the point of failure[15]. Although the IFE model is regarded as a standard one in the engineering literature, its validity is not clear. In fact, the static (active) mode of the IFE model cannot describe the dip formation, and a recent simulation by Inagaki[16] clearly demonstrates that the material does not satisfy the Mohr-Coulomb yield criterion everywhere. It is, however,

worthwhile to indicate that MAPA and the minor principal axis (MIPA) which is perpendicular to MAPA are not straight lines but curved lines due to the effect of boundaries in the IFE model[15, 17].

Numerical simulations of the sand piles are helpful to understand the behavior of the stress field. For example, Luding[8] analyzed sand piles based on the distinct element method (DEM). The initial configurations of his simulation are almost in regular lattice structure. His results are summarized as follows: (i) For monodisperse particles, he found that the distance between two center particles in the lower-most row is an important parameter. If the distance is a little smaller than the regular value, he observed distinct dip in the vertical stress. (ii) The arching can be produced by a defect (absent hole of a particle) of the lattice structure. (iii) For polydisperse particles, he may find the dip after the average of 100 runs, though the fluctuation is very large.

Oron and Herrmann[9] performed the exact calculation of force networks in regular piles. They obtained a wide variety of results. Arching can occur by pushing corner particles, where the clear dip can be observed. However, in many cases without external constraints, arching is difficult to be observed but small dips can be observed. They also confirm the sensitivity of results to the boundary condition at the base of piles.

Inagaki[16] performed DEM to construct sand piles under the realistic situation: Particles (circular) are poured from the top of piles. Her result suggests that the dip formation seems to be unstable for circular particles. In fact, Matuttis *et al.*[10] stressed that circular particles cannot produce stable dips, but to adopt polygonal particles is a reasonable way to reproduce dips and to simulate realistic sand piles. Note that real sand particles have random oriented flat surfaces. The contact between flat surfaces is stable and sticky to support piles. Thus, metastable structure of sand piles with polygonal particles is more stable than that with circular particles. At least, it is easy to imagine that arching is easily achieved by the polygonal particles.

In this paper we try to discuss whether arching is needed for dip formation by the introduction of a simple continuum model. The model (at present) is similar to FPA, where the MAPA and MIPA are straight lines outside the central region, while the symmetric axis is not the singular source of MAPA and MIPA. In the next section, we will survey the outline of our idea. In section 3, we will propose a new OSL model

2 The outline of our idea

For simplicity we restrict ourselves to two dimensional cases: the governing equation is given by

$$\nabla \cdot \sigma = \rho \mathbf{g}, \quad \sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xz} \\ \sigma_{zx} & \sigma_{zz} \end{pmatrix}, \tag{1}$$

where $\sigma_{xz} = \sigma_{zx}$, x and z represent the horizontal and the vertical coordinate, and ${}^T\mathbf{g} = (0, g)$ with the gravitational acceleration g. Note that the origin is assumed to be the top of sand pile, and the positive direction of z coordinate is downward. We assume that the density ρ is a constant, and adopt the unit of $\rho g = 1$ and the height of sand pile is 1. We cannot solve (1) without introducing a constitutive relation. The conventional elastic theory assumes that

the strain is small and the resulting equations become elliptic, but the most of granular models adopt the constitutive relation which lead to a set of hyperbolic equations.

As already introduced the classical constitutive relation is the Mohr-Coulomb yield condition. This condition may be reasonable, because sand piles are easily mobilized by adding very small forces. The Mohr-Coulomb condition satisfies that the ratio of the shear stress to the normal stress on crack planes is $\tan\phi_0$ where ϕ_0 is the maximal internal friction angle or the angle of maximum static friction constant[15]. Another simple constitutive one is Janssen's one (or BCC model) which satisfies $\sigma_{xx} = K\sigma_{zz}$ where K is the coefficient of earth pressure[18, 21]. When we adopt Janssen's condition we obtain the one-dimensional wave equation for the stress variables σ_{xx}, σ_{zz} and σ_{xz}, where z can be regarded as the time variable. Janssen's condition is widely accepted in powder technology , civil engineering and soil mechanics, and is sometimes regarded as equivalent one to the Mohr-Coulomb condition[15]. However, this statement is only valid when the effects of boundaries can be neglected. Similarly, the FPA model assumes a mixed constitutive equation $\sigma_{xx} = \sigma_{xz} - 2\tan\phi|\sigma_{xz}|$. FPA is also reduced to the wave equation for combination of stress variables[13].

The studies introduced above may underestimate the effects of boundaries, because the most of MAPA (or MIPA) may be a curve line[17]. For such the general situation, at first, let the stress tensor be diagonalized as

$$\sigma = R(\theta)\begin{pmatrix}\sigma_1 & 0\\ 0 & \sigma_2\end{pmatrix}R(-\theta): \quad R(\theta) = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}, \tag{2}$$

where σ_1, σ_2 and θ are respectively the major principal stress, the minor principal stress ($\sigma_1 > \sigma_2$) and the angle between MAPA and x axis. In general the angle θ and ϕ (the internal friction angle) are the functions of position. It becomes a constant when the boundary effects are negligible. Thus, the independent variables of this problem is the mean principal stress $\sigma_m \equiv (\sigma_1 + \sigma_2)/2$ and θ or ϕ supplemented by a suitable constitutive relation. Using σ_m, θ and ϕ, eq.(1) can be rewritten as

$$\partial_x\{\sigma_m(1 + \cos\theta\sin\phi)\} + \partial_z(\sigma_m\sin\phi\sin 2\theta) = 0 \tag{3}$$

and

$$\partial_x(\sigma_m\sin\phi\sin 2\theta) + \partial_z\{\sigma_m(1 - \cos 2\theta\sin\phi)\} = 1 \tag{4}$$

Equations (3) and (4) supplemented by a suitable constitutive equation is the basic equation of this problem.

One of the most important characteristics of eqs.(3) and (4) is its hyperbolictity. Actually, introducing a new variable

$$\Psi(x, z) = \sigma_m\sin\phi, \tag{5}$$

by eliminating σ_m from (3) and (4) we obtain

$$2\partial_x\partial_z(\Psi\cos 2\theta) + (\partial_z^2 - \partial_x^2)(\Psi\sin 2\theta) = 0. \tag{6}$$

Regarding this equation as a partial differential equation of Ψ, it is easy to obtain its characteristic equation is

$$\lambda^2 - 2\csc 2\theta\lambda - 1 = 0. \tag{7}$$

The roots of this equation $\lambda = \lambda_{\pm}$ are obtained as

$$\lambda_+ = -\cot\theta, \quad \lambda_- = \tan\theta. \tag{8}$$

Thus, if we know the constitutive equation for θ, eq.(6) is a standard linear partial differential equation (PDE). Even if we adopt a constitutive equation for ϕ, θ becomes a function of ϕ and σ_m. In this case, as long as θ is real, its hyperbolicity is invariant.

The most of conventional or standard PDE models of granular continuum mechanics belong to hyperbolic equations. For example, Janssen's one and FPA[13] are reduced to wave equations. The hyperbolicity itself has a serious weak point to describe sand piles, because any hyperbolic equation cannot contain the boundary condition in 'future', *i.e.* the model does not include the effect of the plate to support piles. It is obvious that the boundary effect is important, in particular, in granular materials. Actually, Janssen's law is caused by the boundary effects[18]. If we adopt Mohr-Coulomb condition it is not difficult to show the bent of MAPA and MIPA as a result of the boundary.

In addition, there is a gap between granular mechanics based on a set of hyperbolic equation and conventional theory of elasticity which can be described by a set of elliptic equations. Thus, e.g. Cantelaube *et al.*[19] assume that granular matter is at critical stress state which satisfies Mohr-Coulomb condition in the outer region but obeys the conventional theory of elasticity in the inner region. Quite recently, with the aid of the master equation of the probability of finding an oriented force chain of intensity in a direction around a point, Bouchaud *et al.*[20] shows that hyperbolic feature is unstable for the chain splitting mechanism, and pseudo-elastic behavior is recovered.

Thus, we do not have to insist on the hyperbolicity of granular continuum mechanics. Here, however, we keep to assume the hyperbolicity because (i) the most of conventional approaches assume it, (ii) we can draw MAPA and MIPA[8, 9], which means that the diagonalization of the stress tensor in eq.(2) is possible, and (iii) to summarize what we can conclude from the conventional view point is important.

3 a new OSL model

3.1 General Framework

As introduced in the previous section, there are many objections to the oriented stress linearity (OSL) ansatz which is generalization of FPA model. Nevertheless, the simplest choice is the OSL ansatz, because the problem is reduced to a linear PDE. In this section, through our new OSL model we summarize the current understanding on the OSL closure.

In an OSL model, the functional form of θ between MAPA and the plate is assumed to be given. As a result, eqs.(3) and (4) become a closed set of equation for σ_m and $\Psi = \sigma_m \sin\phi$ as

$$\begin{aligned} &\partial_x\sigma_m + \partial_x(\Psi\cos 2\theta) + \partial_z(\Psi\sin 2\theta) = 0, \\ &\partial_x(\Psi\sin 2\theta) + \partial_z\sigma_m - \partial_z(\Psi\cos 2\theta) = 1. \end{aligned} \tag{9}$$

Here we note that θ is not always a constant as in FPA model even in a generalized OSL model but can be a function of position. Thus, it is possible to create a lot of variations from FPA.

Here, we introduce a simple OSL model which does not have any singularity inside the sandpile. Of course, if there is an arch structure, it is easy to get the dip and there remains the singularity along z axis. While the arch structure is not stable for spherical (or circular) particles, where the distinct dip may disappear. Through the construction of the solution of this problem we may be able to clarify what the point is.

Let us consider a two-dimensional sand pile whose surface is flat and its top is located at the origin. To describe such the pile it is convenient to introduce the polar coordinate (r, φ), where r is the distance from the top (the origin) and φ is the angle variable. Note that φ satisfies

$$\varphi = \frac{\pi}{2} - \theta, \tag{10}$$

where θ is again the angle between MAPA and x axis. Now, we assume that the surface of the sand pile satisfies the Mohr-Coulomb condition. In this case, the slope is equivalent to the maximal friction angle ϕ_0 or the angle of repose. As a result, it is easy to show that the angle between the surface and MAPA should be $\varphi_0 \equiv \pi/4 - \phi_0/2$[15]. If we adopt an OSL model, thus, MAPA is assumed to keep the angle $\theta_0 \equiv \pi/4 + \phi_0/2$ with the horizontal axis. Such the situation is the same as that in FPA model. The variables Ψ and σ_m are thus governed by

$$\begin{aligned} &\partial_x \sigma_m - \sin\phi_0 \partial_x \Psi + \cos\phi_0 \partial_z \Psi = 0, \\ &\cos\phi_0 \partial_x \Psi + \partial_z \sigma_m + \sin\phi_0 \partial_z \Psi = 0. \end{aligned} \tag{11}$$

However, inside the cone $\varphi < \varphi_0 = \pi/4 - \phi_0/2$, the line with the angle θ_0 cannot hit the surface. So if the angle θ is assumed to be a constant, z axis must be the singular source of MAPA. This model is nothing but FPA model.

To remove the singularity along the axis of symmetry (or z axis in our set up) we adopt the following model. We assume that the MAPA is emitted from the origin in a radial manner if $|\varphi|$ is less than φ_0. Thus, eqs.(3) and (4) are reduced to

$$\begin{aligned} &\partial_x \sigma_m - \partial_x(\Psi \cos 2\varphi) + \partial_z(\Psi \sin 2\varphi) = 0 \\ &\partial_x(\Psi \sin 2\varphi) + \partial_z \sigma_m + \partial_z(\Psi \cos 2\varphi) = 1 \end{aligned} \tag{12}$$

where φ satisfies

$$\tan\varphi = \frac{x}{z}. \tag{13}$$

Note that this choice is not unique but the simplest if we do not have any singularities in the sand pile.

3.2 Inner solution

In this subsection we construct the inner solution for $|\varphi| < \varphi_0 = \pi/4 - \phi_0/2$. We adopt the polar coordinate to describe the system, because r and φ satisfy the equation for the characteristic coordinate

$$\partial_x \varphi + \lambda_- \partial_z \varphi = 0, \quad \partial_x r + \lambda_+ \partial_z r = 0. \tag{14}$$

Then, eliminating σ_m from (12) we obtain a regularized equation for $\Psi(r,\varphi)$ as

$$\partial_\varphi(\partial_r\Psi + \frac{1}{r}\partial_\varphi\Psi) = 0. \tag{15}$$

It is easy to obtain the general solution of eq.(15) as

$$\Psi(r,\varphi) = f(r) + \frac{g(\varphi)}{r}, \tag{16}$$

where f and g are arbitrary functions at present.

Substituting (16) into (12) and with the simple modification, we obtain

$$\frac{1}{r}\partial_\varphi\sigma_m - \frac{g'(\varphi)}{r^2} = -\sin\varphi \tag{17}$$

and

$$\partial_r\sigma_m + f'(r) + \frac{2}{r}f(r) + \frac{g(\varphi)}{r^2} = \cos\varphi. \tag{18}$$

Thus, we obtain the solution of (17) and(18) as

$$\sigma_m(r,\varphi) = h(r) + r\cos\varphi + \frac{g(\varphi)}{r}, \tag{19}$$

where h is an undetermined function, which satisfies

$$h'(r) = -f'(r) - \frac{2}{r}f(r). \tag{20}$$

We note the followings about the inner solution. (i) $\Psi \sim 1/r$ is the most natural r dependence of Ψ but $\Psi = f(r)$ can be the solution. (ii) There are no equations to determine $g(\varphi)$.

3.3 Outer solution

Now, let us discuss the outer solution for $|\varphi| > \varphi_0$. The characteristic coordinates should satisfy

$$\partial_x\alpha + \mu_-\partial_z\alpha = 0, \quad \partial_x\beta + \mu_+\partial_z\beta = 0, \tag{21}$$

where

$$\mu_\pm = \pm\tan(\frac{\pi}{4} \pm \frac{\phi_0}{2}). \tag{22}$$

On the other hand, α and β are given by the rotational transform of x and z, where the rotational angle is $\theta_0 = \pi/4 + \phi_0/2$. Thus, we obtain

$$\begin{aligned} \alpha &= \sin\theta_0(\cot\theta_0 x + z), \\ \beta &= \cos\theta_0(-\tan\theta_0 x + z). \end{aligned} \tag{23}$$

With the help of the characteristic coordinates we obtain the standard form of PDE as

$$\partial_\alpha\partial_\beta\Psi = 0. \tag{24}$$

It is easy to obtain the general solution of (24) as

$$\Psi(\alpha,\beta) = \tilde{f}(\alpha) + \tilde{g}(\beta), \tag{25}$$

where $\tilde{f}$ and $\tilde{g}$ are arbitrary functions.

With the help of (25) eq.(12) can be rewritten as

$$\partial_\alpha \sigma_m + \tilde{f}'(\alpha) = \frac{1}{2}\cos\phi_0 \sec\theta_0 \tag{26}$$

and

$$\partial_\beta \sigma_m - \tilde{g}'(\beta) = \frac{1}{2}\cos\phi_0 \csc\theta_0. \tag{27}$$

Thus, we can solve σ_m along MAPA and MIPA separately.

In the outer region, MAPA and MIPA hit the surface. The boundary condition at the surface is the force free condition as

$$\sigma_m(\alpha, \beta) = 0. \tag{28}$$

Note that the surface is represented by the line

$$\beta = -\cot\theta_0 \alpha, \quad \theta_0 = \frac{\pi}{4} + \frac{\phi_0}{2}. \tag{29}$$

Thus, $\tilde{f}$ and $\tilde{g}$ can be determined as

$$\tilde{f}(\alpha) = \frac{\alpha}{2}\sin\phi_0, \quad \tilde{g}(\beta) = \frac{\beta}{2}\sin\phi_0 \sec\theta_0. \tag{30}$$

As a result, σ_m and Ψ are respectively given by

$$\begin{aligned} \sigma_m(\alpha, \beta) &= \frac{\alpha + \tan\theta_0 \beta}{2}\csc\theta_0, \\ \Psi(\alpha, \beta) &= \frac{\alpha + \tan\theta_0 \beta}{2}\csc\theta_0 \sin\phi_0, \end{aligned} \tag{31}$$

which means that the outer region satisfies the Mohr-Coulomb condition everywhere.

3.4 Matching

The outer solution and the inner solution should be matched at the boundary. The boundary is represented by $\beta = 0$ for the outer region and by $\varphi = \pm\varphi_0 = \pm\pi/4 - \phi_0/2$. Let the suffix $(+/-)$ put the outer/inner solution. Thus, the outer solution should have

$$\sigma_m^+(\alpha, 0) = \frac{\alpha}{2}\csc\theta_0, \quad \Psi^+(\alpha, 0) = \frac{\alpha}{2}\csc\theta_0 \sin\phi_0. \tag{32}$$

On the other hand, the inner solution is given by

$$\sigma_m^-(r, \varphi_0) = cr + r\cos\varphi_0, \quad \Psi^-(r, \varphi_0) = \tilde{c}r, \tag{33}$$

where c and $\tilde{c}$ are constants to be determined. Thus, to match the outer solution with the inner solution, we should assume that $f(r)$ and $h(r)$ are linear functions of r.

From (32) and (33) with $r = \alpha$ on the boundary line c and $\tilde{c}$ must satisfy

$$c = -\frac{\sin\phi_0}{2\sin\theta_0}, \quad \tilde{c} = -c. \tag{34}$$

However, this problem is over-complete. In fact, from eq.(20) we must have

$$\tilde{c} = -\frac{1}{3}c. \tag{35}$$

It is obvious that (34) is contradicted with (35).

We may guess that this mismatch is intrinsic, because the inner solution prefers $1/r$ behavior but the outer solution should be proportional to the distance from the top. This contradiction cannot be absorbed if we assume that the singular top is out of the sand pile. It is possible to construct an OSL model when we assume that the boundary near the top is represented a parabolic curve. In this case, however, analytic exact solutions cannot be constructed.

In any case, it is not difficult to show the following. If we adopt (35) we obtain

$$c = -\frac{\sin\phi_0}{6\sin\theta_0}. \tag{36}$$

Both (34) and (36) lead to the negative c. With the aid of $\sigma_{zz} = \sigma_m + \Psi\cos 2\varphi$ it is easy to show $d\sigma_{zz}/dx$ is proportional to c and the positive definite function of x. In addition, $d\sigma_{zz}/dz = 0$ at $x = 0$ (the symmetric axis). Thus, this model without defects does not have any dip.

4 Discussion

In this paper, we introduce a try to construct a solvable model of sand piles. Unfortunately, this try is not successful, and we encounter the contradiction about the solution. However, this fault suggests that the dip may not be stable without defects of the stress field or the strong boundary effects.

Even when we can construct the exact solution of a toy model, the result may not be interesting. To discuss how to produce defects of the stress field as a function of history and the boundary is more important and exciting. In this sense, we may expect quite recent try by Sasa[22] to emphasize the historical structure of sand piles. For this purpose, we may need to construct a simplified model including effects of defects and history of the construction of the sand pile.

The author thank S. Inagaki, S. Sasa, K. Ichiki, and H-G. Matuttis for valuable discussion.

参考文献

[1] H.M. Jaeger, S. R. Nagel and R. B. Behringer Rev. Mod. Phys. **68**, 1259 (1996).

[2] H.J.Herrmann, J-P. Havi and S. Luding eds. Physics of Dry Granular Media (Kluwer Academic 1998).

[3] 早川尚男、那須野悟、現代物理最前線 1 (共立出版、2000): 物理学論文選集 XI: 粉粒体の物理学 (西森拓、早川尚男、田口善弘編、日本物理学会, 1999).

[4] D.Trollope and B.Burman : *Géotechnique* **30**, 137 (1980): J.Smid and J.Novosad : *Int. Chem. Eng. Symp.* **63** D3/V/1 (1981) R.Brockbank, J.M.Huntley and R.C.Ball : *J. Phys. II France* **7**, 1521 (1997).

[5] L. Vanel, D. Howell, D. Clark, R.P. Behringer and E. Clement, Phys. Rev. E **60**, 5040 (1999).

[6] S. B. Savage, in ref.[2].

[7] D. W. Taylor, Reviews of Pressure Distribution Theories, Earth Pressure Cell Investigation and Pressure Distribution Data (Progress Report, Waterway Experiment Station, Vickersburh, VA 1947).

[8] S. Luding, Phys. Rev. E **55**, 4720 (1997).

[9] G. Oron and H. J. Herrmann, Phys. Rev. E **58**, (1998) 2079.

[10] H-G. Matuttis, in this volume and private communication.

[11] S. F. Edwards and D. V. Grinev, Phys. Rev. Lett. **82**, 5397 (1999).

[12] S.F.Edwards and R. B. Oakeshott : *Physica D* **38**, 88 (1989).

[13] J.P.Wittmer, P.Claudin, M.E.Cates and J.-P.Bouchaud : *Nature* **382**, 336 (1996); J.P.Wittmer, M.E.Cates and P.Claudin : *J. Phys. I France* **7**, 39 (1997). J. -P. Bouchaud, P. Claudin, M. E. Cates and J. P. Wittmer, in ref.[2].

[14] see e.g. P. Claudin, J-P. Bouchaud, M. E. Cates & J. P. Wittmer, Phys. Rev. E **57**, 4441 (1998): L. Vanel, P. Claudin, J.-P. Bouchaud, M. E. Cates, E. Clement and J. P. Wittmer, Phys. Rev. Lett. **84**, 1439 (2000).

[15] e.g. 三輪茂雄、粉体工学通論 (日刊工業新聞社,1981).

Kogyo Sninbun, Tokyo, 1981) *written in Japanese* : R. M. Needderman, Statics and Kinematics of Granular Materials (Cambridge Univ. Press, Cambridge, 1992).

[16] S. Inagaki, to be published in the Proceedings of Traffic Flows and Granular Flows 99 (Springer, Berlin, 2000). see also her paper in this volume.

[17] D.M. Walker, Chem. Eng. Sci. **21**, 975 (1966): J. K. Walters, Chem. Eng. Sci. **28**, 13 (1973).

[18] H. A. Janssen, V.D.I. -Zeit, **39**, 1045 (1895).

[19] see e.g. F. Cantelaube, A. K. Didwania and J. D. Goddard., in ref.[2].

[20] J.-P. Bouchaud. P. Claudin, D. Levine and M. Otto, cod-mat/0011213.

[21] J.-P. Bouchaud, M. E. Cates and P. Claudin, J. Phys. I (France) **5**, 639 (1995).

[22] S. Sasa, in private communications.