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鉛直に加振した粒状体薄層における
準結晶的 パターン

Pattern formation of thin granular layer
due to vertical vibration

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Abstract

Experimental study of the pattern formation on the thin horizontal granular layer, which oscillated vertically with frequency f under atmospheric pressure, was made. Patterns were observed by a high-speed video camera and classified by means of two-dimensional Fourier transform, as well as a direct measurement of the images. In addition to spots, square cells, polygons and stripes with frequency $f/2$, we observed squares, polygons and quasi-crystal patterns with frequency $f/4$. The dependence of oscillation amplitude and frequency on the pattern size was analyzed.

1　INTRODUCTION

Pattern formation of thin granular layer due to vertical vibration has been extensively studied since the pioneering work by M. Faraday (1831). Recently many experimental(Douady et al. 1989, Melo et al. 1994, 1995, Goldshtein et al. 1995, Umbanhower et al. 1996, Metcalf et al. 1997, Sano & Suzuki 1998, Sano et al. 1999) and numerical(Aoki et al. 1996, Clement et al. 1996, Lan & Rosato 1997, Bizon et al. 1998) investigations have revealed wavy motions on this layer. In spite of industrial importance and academic interest, however, the basic understanding of the physical mechanisms underlying the collective behaviors of this material is still inadequate (Jaeger & Nagel 1992, 1996, Lubkin 1995, Jaeger et al. 1996, etc.). In this paper we shall elucidate the detailed pattern diagram and dispersion relation to understand the fundamental processes of pattern formation.

2 EXPERIMENTAL APPARATUS

An experimental study of the pattern formation on thin granular materials in a vessel, which is vibrated vertically under atmospheric pressure, was made. A circular cylindrical vessel of a diameter 106mm and a height 31mm was mounted with its principal axis parallel to gravity on an electro-mechanical vibration generator. The block diagram of our experimental apparatus is shown in Figure 1.
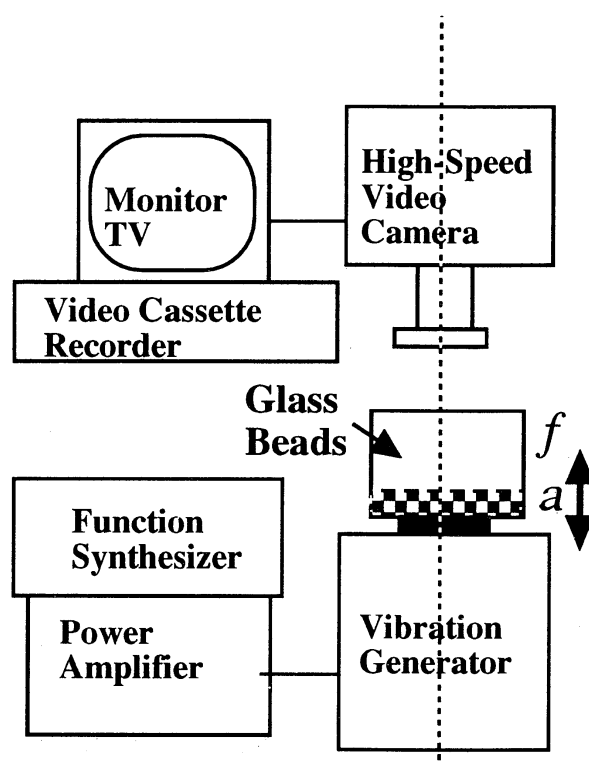

Figure 1. Experimental Apparatus.

The vertical oscillation of the container was given by $z = a\sin(2\pi ft)$, where the frequency f and amplitude a were given by a function synthesizer and an amplifier. No deformation of the bottom of the container was recognized. The observation was made by means of a high-speed video camera to which close-up lens is mounted. These images were later reproduced and the two-dimensional Fourier image, in addition to direct observation of their planar form, were analyzed.

3 RESULTS

3.1 *Planar form*

We filled the glass beads of a diameter $d = 0.13 \pm 0.05$mm in a vessel to a depth $h = 0.7$mm$\sim$1.2mm, and observed the planar form of the granular layer under vertical vibrations. Examples of the patterns are shown in Figure 2. The typical sequence of pattern changes, which we found under frequency forcing of fixed amplitude a of medium values is as follows: initially flat surface disrupted into spot-like patterns (Fig. 2a) for frequencies $f \approx 20$Hz, which diffused into sheet around spots for $f \approx 25$Hz, developed to square patterns (Fig. 2b) for $f = 25 \sim 35$Hz (sometimes hexagons/triangles (Fig. 2c) were observed), and stripe patterns (Fig. 2d) with some dislocations for $f = 40 \sim 45$Hz, which sometimes evolved into spirals (Fig. 2e).These patterns repeated with $2T$ (Fig. 3), where $T(= 1/f)$ is the period of external forcing. When we increased the frequency, patterns disappeared (Fig. 2f) for $f \approx 50$Hz, and cellular patterns of approximately hexagonal shape reappeared for $f \approx 60$Hz(Fig.2g), which repeated with period $4T$. Between the patterns (e) and (g), we observed almost flat surface, although constituent particles of the latter were not at rest. These findings agree with the previous works. We also observed quasi-crystal pattern(Penrose 1974, Mackay 1981, Shechtman et al. 1984, Levine & Steinhardt 1984, Christiansen et al. 1992, Edwards & Fauve 1994) with 8-fold symmetry (Fig. 2h). Typical Fourier patterns corresponding to Figures 2b, 2c and 2h are shown in Figure 4. Figure 5 is the pattern diagram in normalized frequency $f^{*}(= f\sqrt{h/g})$ against normalized acceleration amplitude Γ $(=4\pi^2 f^2 a/g)$, where g is the acceleration of gravity. In contrast to the previous works which were made under evacuated circumstance, we have wider region of convection regime as well as the region of quasi-crystal patterns in the phase diagram.
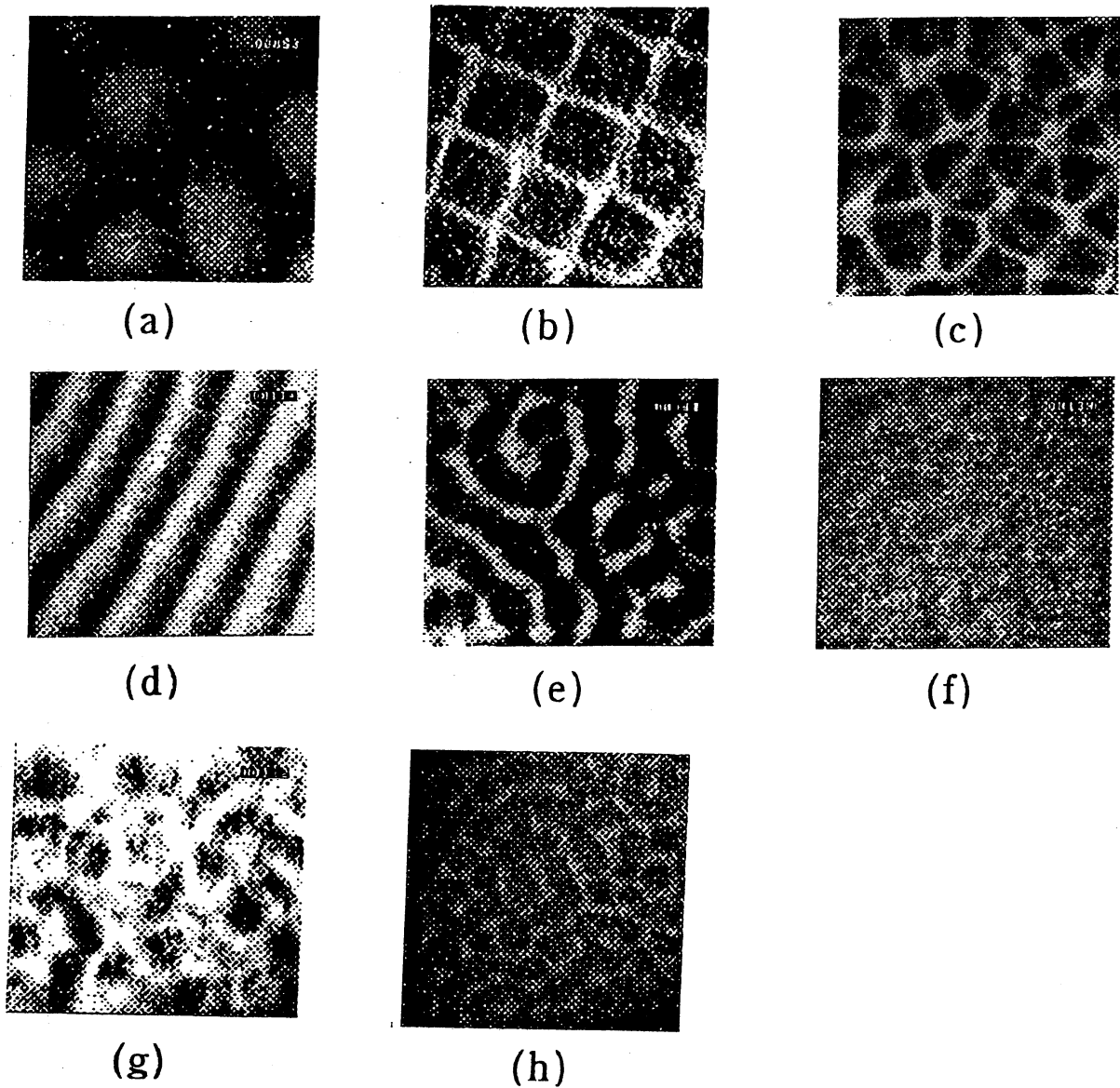
Figure 2. Typical patterns observed on vertically vibrated thin granular layer:

(a) spots ($f = 20$Hz, $a =0.80$mm),

(b) squares ($f/2$) ($f = 25$Hz, $a =1.20$mm),

(c) polygons ($f/2$) ($f = 30$Hz, $a =2.18$mm),

(d) stripes ($f/2$) ($f = 45$Hz, $a =1.00$mm),

(e) spirals ($f/2$) ($f = 45$Hz, $a =1.20$mm),

(f) flat ($f = 45$Hz, $a =1.60$mm),

(g) hexagons ($f/4$) ($f = 60$Hz, $a =0.60$mm),

(h) quasi-crystal pattern ($f/4$) ($f = 75$Hz, $a =1.00$mm).

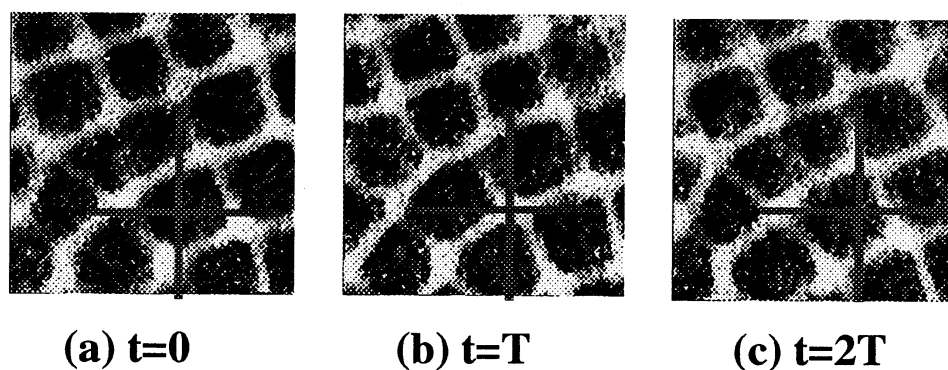(a) t=0   (b) t=T   (c) t=2T

Figure 3. Time sequence of the square pattern; (b) and (c),respectively, are the patterns at a time T and $2T$ after (a), where T is the period of external vibration. Crosses indicate the same position.

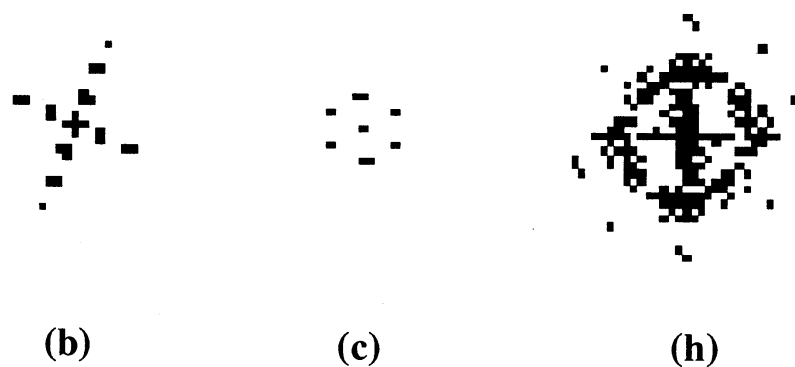(b)   (c)   (h)

Figure 4. Fourier patterns of Figures 2b, 2c and 2h, showing 4-fold, 6-fold and 8-fold symmetries, respectively.
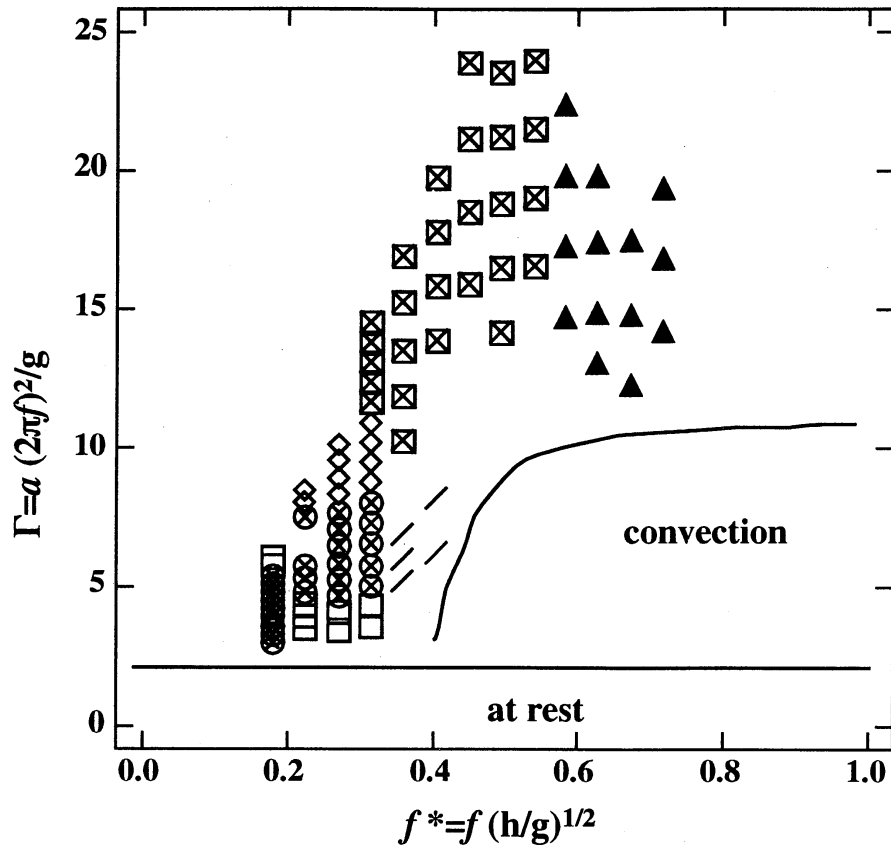
Figure 5. Phase diagram.

□: squares $(f/2)$, ⊗: hexagons and triangles $(f/2)$, ∕: stripes $(f/2)$, ◇: squares $(f/4)$, ⊠: hexagons and triangles $(f/4)$, and ▲: octagons $(f/4)$.

3.2 Size of the cell

The size of the cell depends on f, a and h. We plot the wavelength of the pattern λ^* normalized by the depth (i.e. $\lambda^* = \lambda/h$) against normalized frequency f^* in Figure 6. Data estimated from previous works (Melo et al. 1994; Metcalf et al. 1997) are also plotted. By this scaling, each family of data belonging to $f/2$ and $f/4$ is well fitted by $\lambda^* \propto f^{*\alpha}$ with $\alpha \approx -1$. Their intercepts in log-log plot differ by about 0.3. Note that in the case of capillary/gravity waves of the water, the dispersion relation leads to $\alpha = -2 \sim -1$ for lower frequency, while $\alpha = -2/3 \sim -1/2$ for higher frequency (Miles & Henderson 1990,

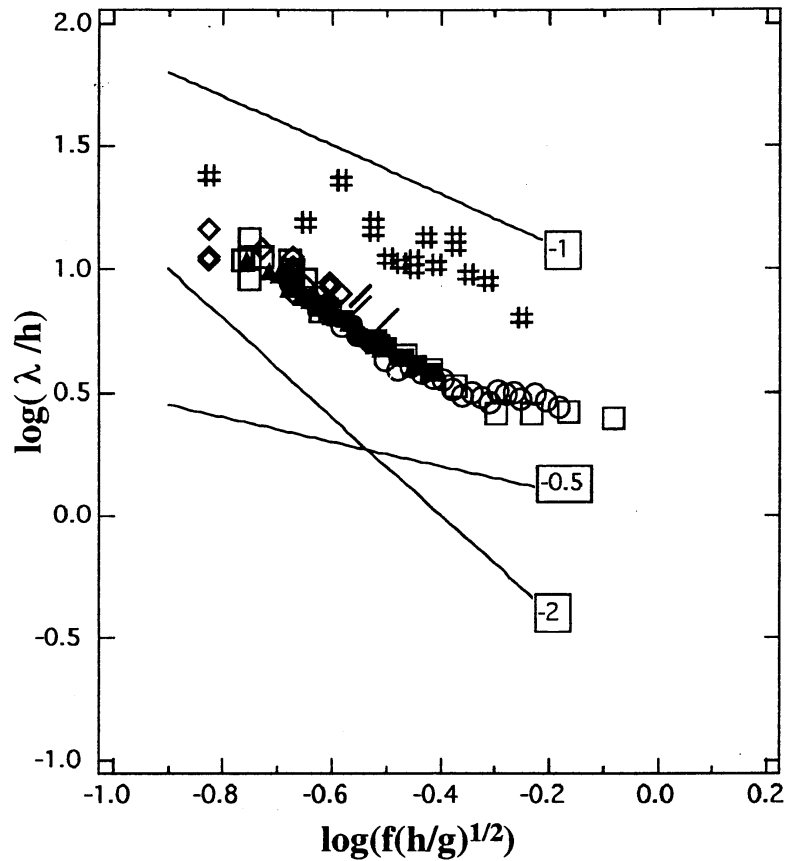Tufillaro et al. 1989, Milner 1991, Müller 1993, Edwards & Fauve 1994).

Figure 6. Dispersion relation.

present result

 d =0.13mm, $h/d = 4 \sim 13$, P =1atm,

 □: squares, ◇: triangles and hexagons,

 ╱: stripes, ♯: $f/4$ patterns.

Melo. et al. (1994) ($\Gamma = 3.5$),

 h/d =7, P =1atm,

 ▲: d =0.2mm, ●: d =0.3mm,

 ■: d =0.4mm.

Metcalf. et al. (1997) ($\Gamma = 3.0$),

 ○: d =0.5mm, h/d =6, P =10torr,

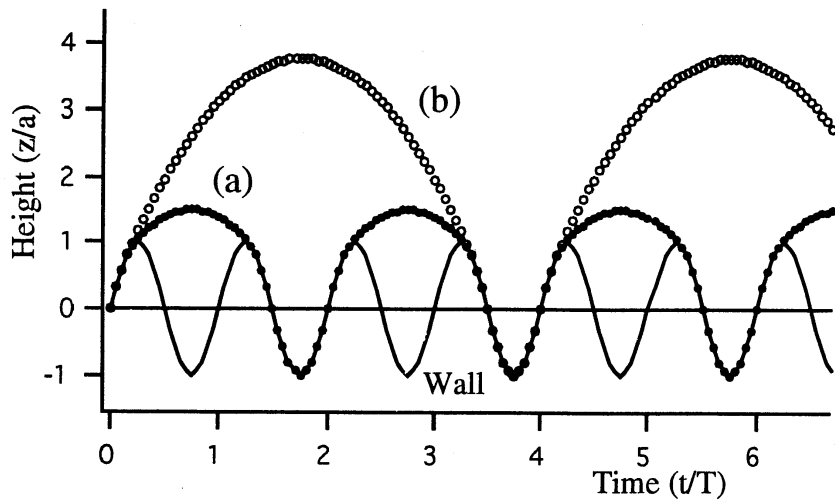 □: d =0.5mm, h/d =8, P = 0.04torr.

Figure 7. Schematic picture of the orbit of a granular particle: (a)$f/2$-pattern, and (b) $f/4$-pattern.

4 DISCUSSION

Our results are explained as follows: Granular particles pushed up by the upward motion of the vessel obtain upward momentum and make a free flight during the time interval in which the upward acceleration due to external forcing exceeds that of the gravity. When granular particles fall down and touch the bottom of the vessel, they are reflected and make successive free flights with given positions and velocities as new 'initial conditions'. These conditions, however, are in general random, so that no collective motion is observed. On the other hand, if particles make 'soft-landing' so that they have the same velocities as the wall velocity at the time of collision, they are convected by the downward motion of the vessel, which are duly released in the same phase of the next elevation as they did previously, which forms the periodic and correlated motion of the granular particles, i.e. 'patterns'.

The above-mentioned process is possible for a particular combination of f, a, h and g, whose minimum time interval is $2T$ (subharmonic oscillation). The container may oscillate even-number times during

the free flight, so that similar patterns reappear for a period $4T$, or $6T$, or $\cdots$ (Fig. 7), although the observation becomes increasingly difficult.

Finally we shall consider the experimentally observed dispersion relation. To this end we assume

$$\text{(assumption 1)} \qquad 2\lambda_1 = \lambda_2, \quad f_2 = f_1/2,$$

$$\text{(assumption 2)} \qquad \lambda = cf^{\alpha},$$

where c is a constant. From assumption 1, we have

$$\log\lambda_2 = \log 2 + \log\lambda_1 = 0.301 + \log\lambda_1,$$

which explains the difference 0.3 of the intercepts. From assumptions 1 and 2, we have

$$2cf_1^{\alpha} = cf_2^{\alpha}, \quad \text{or} \quad 2f_1^{\alpha} = (f_1/2)^{\alpha},$$

which explains the exponent $\alpha = -1$, i.e. $\lambda = cf^{-1}$.

5 CONCLUSIONS

Pattern formation on vertically oscillating thin granular layer under atmospheric pressure was investigated by means of a high-speed video camera.

1. Pattern diagram is obtained by means of direct and/or Fourier images, and the sequence of transitions are elucidated.

2. Pattern formation process, temporal periodicities, translational and/or rotational symmetries of the patterns are checked.

3. In addition to regular patterns(squares, triangles/hexagons, etc of subharmonic type), we found quasi-crystal pattern of 8-fold symmetry with frequency $f/4$, as the external forcing frequency f^* and/or Γ are increased.

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