数理解析研究所講究録
1187 巻 2001 年 27-36

OPTIMAL CONTROL PROBLEM FOR THE NONLINEAR HYPERBOLIC SYSTEMS

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ABSTRACT. In this paper we study parameter optimal control monitored by nonlinear hyperbolic systems. We show that for every value of the parameter, the optimal control problem has a solution. Moreover we obtain the necessary optimality condition on the control system.

1. INTRODUCTION

The optimal control problems have been extensively studied by many authors [1,3,5,7,10,13 and reference there in] and also identification problem for damping parameters in the second order hyperbolic systems have been dealt with by many authors [6,8,12 and there reference in]. In this paper, we consider the following control systems

$$\left\{\begin{array}{l} y'' + A_2(t,q)y' + A_1(t,q)y + N^*g(Ny) = Bu + f(t,q) \\ y(q,u)(0) = y_0 \in V, y'(q,u)(0) = y_1 \in H \end{array}\right. \tag{1.1}$$

and the cost functional given by the quadratic form

$$J(q,u) = \frac{1}{2}||Cy(q,u) - z_d||_M^2. \tag{1.2}$$

Here $A_1(t,q)$, and $A_2(t,q)$ are differential operators containing unknown parameter $q \in Q$ and there are given by some bilinear forms on Hilbert spaces, $N^*g(Ny)$ is a nonlinear term, B is a controller, $u \in U$ is a control, f is a forcing term and C is an observation operator defined on an observation space M, z_d is a desired value. The optimal control problem subject to (1.1) with (1.2) is to find an element $(\bar{q},\bar{u}) \in Q \times U$ such that $\inf_{(q,u)\in Q\times U} J(q,u) = J(\bar{q},\bar{u})$. In this paper we will study the optimal control to the system (1.1) with (1.2) and the existence of weak solution for (1.1). It is not easy to find the optimal control pairs $(\bar{q},\bar{u})$ belonging to a general admissible set $Q \times U$ of parameters and controls subject to (1.1) with (1.2). Hence we will show the existence of such $(\bar{q},\bar{u})$ when

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1991 *Mathematics Subject Classification.* 93C20, 49J20, 49K20.
Key words and phrases. Optimal control, nonlinear hyperbolic systems, quadratic cost function.
This work was supported by Brain Korea 21, 1999.

$Q \times U$ is a compact subset of a topological space. Recently, inspired by the optimal control theoretical studies of Euler-Bernoulli Beam Equations with Kelvin-Voigt Damping, and Love-Kirchoff Plate Equations with various damping terms, the appeared numerous paper studying optimal control theory and identification problems. In Banks et al.[4], Banks and Kunisch [5], they treated the existence of the optimal control (or minimizing parameters) by using the methods of approximations, but they didn't deal with the necessary conditions (or characterizations) on them. When $A_1(t,q) \equiv \gamma A_2(t,q), \gamma > 0$ and $N^*g(Ny) = 0$ in (1.1), the identification problem estimating q via output least-square identification problem is studied by Ahmed [1,2] based on the transposition methods. In the nonlinear parabolic type case, Papageorgiou [11] treated with the optimal control problems contained parameter and control. But we deal with the second order nonlinear hyperbolic systems.

In specially, in this paper we study the optimal control (or minimizing parameters) problems to (1.1) with (1.2) on the Gelfand five fold and the necessary conditions.

2. PRELIMINARIES

Let X be a real Hilbert spaces. $(\cdot,\cdot)_X$ and $||\cdot||_X$ denote the inner product and the induced norm on X. X^* the dual space of X and $\langle\cdot,\cdot\rangle_{X^*,X}$ denotes the dual pairing between X^* and X. Let us introduce underlying Hilbert spaces to describe the nonlinear hyperbolic systems. Let H be a real pivot Hilbert space, its norm $||\cdot||_H$ is denoted simply by $|\cdot|_H$. Throughout this paper we assume there is a sequence of real separable Hilbert spaces V_1, V_2, V_1^*, V_2^* forming a Gelfand quintuple satisfying $V_1 \hookrightarrow V_2 \hookrightarrow H \equiv H^* \hookrightarrow V_2^* \hookrightarrow V_1^*$. And also we assume that the embedding $V_1 \hookrightarrow V_2$ is dense and continuous with $||\phi||_{V_2} \leq c||\phi||_{V_1}$ for $\phi \in V_1$ and $V_2 \hookrightarrow H$ is a densely compact embedding. From now on, we write $V_1 = V$ for convenient of notation. We assume that the equalities $\langle\phi,\varphi\rangle_{V^*,V} = \langle\phi,\varphi\rangle_{V_2^*,V_2}$ for $\phi \in V_2^*, \varphi \in V$ and $\langle\phi,\varphi\rangle_{V^*,V} = (\phi,\varphi)_H$ for $\phi \in H, \varphi \in V$. We shall give an exact description of the nonlinear hyperbolic systems. We suppose that Q is algebraically contained in a linear topological vector space with topology τ and $Q_\tau = (Q,\tau)$ is compact. And also we suppose that U is compact subspace of Hilbert space Y. Let $I = [0,T], T \geq 0$ be fixed and $t \in [0,T]$. Let $q \in Q_\tau$.

We will need the following hypotheses on the data.

H(A) : $A_i : I \times Q \longrightarrow \mathcal{L}(V_i, V_i)$ is an operator $(i = 1, 2)$.

(1) $a_i(t, q; \phi, \varphi) = a_i(t, q; \varphi, \phi)$, where $a_i(t, q; \phi, \varphi) = \langle A_i(t, q)\phi, \varphi\rangle_{V_i^*, V_i}, \forall \phi, \varphi \in V_i$.

(2) There exists $c_{i1} > 0$ such that $|a_i(t, q; \phi, \varphi)| \leq c_{i1}||\phi||_{V_i}||\varphi||_{V_i}, \forall \phi, \varphi \in V_i$.

(3) There exists $\alpha_i > 0$ and $\lambda_i \in R$ such that $a_i(t, q; \phi, \varphi) + \lambda_i|\phi|_H^2 \geq \alpha_i||\phi||_{V_i}^2, \forall \phi \in V_i$.

(4) The function $t \mapsto a_i(t, q; \phi, \varphi)$ is continuously differentiable in $[0, T]$.

(5) There exists $c_{i2} > 0$ such that $|a_i'(t, q; \phi, \varphi)| \leq c_{i2}||\phi||_{V_i}||\varphi||_{V_i}, \forall \phi, \varphi \in V_i$, where $' = \frac{d}{dt}$ and $a_i'(t, q; \phi, \varphi) = \langle A_i'(t, q)\phi, \varphi\rangle_{V_i^*, V_i}$.

H(f) : $f : I \times Q \longrightarrow V_2^*$ is the forcing term such that $f(t, q) \in L^2(0, T; V_2^*)$.

H(B) : $B : Y \longrightarrow V_2^*$ is a bounded linear operator such that $B \in L^\infty(0, T; \mathcal{L}(Y, V_2^*))$.

H(N) : $N : V_i \longrightarrow H$ is a linear operator such that $N \in \mathcal{L}(V_i, H)$ with $||N\varphi|| \leq \sqrt{k_i}||\varphi||_{V_2}$, k_i is constant and the range of N on V_i is dense in H.

H(g) : $g : H \longrightarrow H$ is a continuous nonlinear mapping of real gradient(or potential)type such that

(1) $||g(\varphi)|| \leq c_1||\varphi|| + c_2, \ \varphi \in H$ and for some constant c_1, c_2,

(2) $||g(\varphi) - g(\phi)|| \leq c_3||\varphi - \phi||, \ \varphi, \phi \in H$ and for some constant c_3.

We consider the following problem for the nonlinear second order evolution equations of the form :

$$y'' + A_2(t, q)y' + A_1(t, q)y + N^*g(Ny) = f(t, q) \tag{2.1}$$

$$y(q)(0) = y_0 \in V, y'(q)(0) = y_1 \in H, \tag{2.2}$$

where $y' = \dfrac{dy}{dt}, y'' = \dfrac{d^2y}{dt^2}$. We define a Hilbert space, which will be a space of solutions, as

$$W(0, T) = \{y | y \in L^2(0, T; V), y' \in L^2(0, T; V_2), y'' \in L^2(0, T; V^*)\}$$

with an inner product

$$(y_1, y_2)_{W(0,T)} = \int_0^T \{(y_1(t), y_2(t))_V + (y_1'(t), y_2'(t))_{V_2} + (y_1''(t), y_2''(t))_{V^*}\}dt$$

and the induced norm

$$||y||_{W(0,T)} = (||y||^2_{L^2(0,T);V)} + ||y'||^2_{L^2(0,T;V_2)} + ||y''||^2_{L^2(0,T;V^*)})^{\frac{1}{2}}.$$

We denote by $\mathcal{D}(0, T)$ the space of distributions on $(0, T)$.

Definition 2.1. A function y is said to be a weak solution of (2.1)-(2.2) if $y \in W(0,T)$ and y satisfies

$$(2.3)\quad \langle y''(\cdot),\phi\rangle_{V^*,V} + a_2(\cdot,q;y'(\cdot),\phi) + a_1(\cdot,q;y(\cdot),\phi) + \langle g(Ny(\cdot)),N\phi\rangle_H = \langle f(\cdot,q),\phi\rangle_{V_2^*,V_2}$$

$$\text{for all } \phi \in V \text{ in the sense of } \mathcal{D}(0,T),$$

$$(2.4)\quad y(q)(0) = y_0 \in V,\ \frac{dy}{dt}(q)(0) = y_1 \in H.$$

By Definition2.1 it is verified that a weak solution y of (2.1) satisfies

$$(2.5)\qquad \int_0^T \langle y''(t) + A_2(t,q)y'(t) + A_1(t,q)y(t) + N^*g(Ny(t)),\phi(t)\rangle_{V_2^*,V_2}dt$$
$$= \int_0^T \langle f(t,q),\phi(t)\rangle_{V_2^*,V_2}dt,\ \forall\phi \in L^2(0,T;V_2).$$

We state the existence and uniqueness results of a weak solution of (2.1)-(2.2).

Theorem 2.1. *If* $H(A)$,$H(f)$,$H(B)$,$H(N)$ *and* $H(g)$ *hold and* $L(t)$ *satisfy* $L(\cdot) \in L^\infty(0,T;\mathcal{L}(V_2,V_2^*))$. *Then the equation*

$$(2.6)\qquad \begin{cases} y'' + A_2(t,q)y' + A_1(t,q)y + N^*g(Ny) = L(t)y + f(t,q) \ \ in\ (0,T), \\ y(q)(0) = y_0 \in V, y'(q)(0) = y_1 \in H, \end{cases}$$

has a unique weak solution $y \in W(0,T) \cap C(0,T;V) \cap C^1(0,T;H)$. *Here the concept of a weak solution for (2.6) is defined as*

$$\langle y''(\cdot),\phi\rangle_{V^*,V} + a_2(\cdot,q;y'(\cdot),\phi) + a_1(\cdot,q;y(\cdot),\phi) + \langle g(Ny(\cdot)),N\phi\rangle_H$$
$$= \langle L(\cdot)y(\cdot) + f(\cdot,q),\phi\rangle_{V_2^*,V_2}, \forall\phi \in V \ \ in\ the\ sense\ of\ \mathcal{D}'(0,T)$$

with the initial conditions $y(q)(0) = y_0 \in V, y'(q)(0) = y_1 \in H$.

PROOF. We can prove by using the method Lions [9] and Ha [8].

3. EXISTENCE OF BOTH PARAMETERS AND CONTROLS FOR OPTIMALITY

In this section we consider the optimal control problem for the following system:

$$(3.1)\qquad \begin{cases} y'' + A_2(t,q)y' + A_1(t,q)y + N^*g(Ny) = Bu + f(t,q) \ \text{ in } (0,T) \\ y(q,u)(0) = y_0 \in V, y'(q,u)(0) = y_1 \in H, q \in Q_T, u \in U. \end{cases}$$

Note that since there is a unique solution y to (3.1) for given $(q,u) \in Q_T \times U$, we have a well-defined mapping $y = y(q,u)$ of $Q_T \times U$ into $W(0,T)$.

We often call (3.1) the state equation and $y(q,u)$ the state with respect to (3.1). Let us consider a quadratic cost functional attached to (2.6) as

$$J(q,u) = \frac{1}{2}||Cy(q,u) - z_d||_M^2, (q,u) \in Q_T \times U \tag{3.2}$$

where M is a Hilbert space of observations, $C \in \mathcal{L}(W(0,T), M)$ is an observer and z_d is a desired value belonging to M. Our main aim is to find $(\bar{q}, \bar{u}) \in Q_T \times U$ satisfying

$$J(\bar{q},\bar{u}) = \min_{(q,u)\in Q_T \times U} J(q,u) \tag{3.3}$$

and to give a characterization of such $(\bar{q},\bar{u})$. We call $(\bar{q},\bar{u})$ the optimal control to the system (3.1) and (3.2). Furthermore, we will give an assumption to $a_i(t,q;\phi,\varphi), i = 1,2$ and f:

$H(A)_1$: $q \longrightarrow a_i(t,q;\phi,\varphi) : Q_T \longrightarrow R$ is continuous for all $t \in [0,T], \phi, \varphi \in V_i$.

Note that for each $q \in Q_T, \phi, \varphi \in V_i$ the following equalities hold :

$$\sup_{||\varphi||_{V_i}=1} |a_i(t,q;\phi,\varphi)| = \sup_{||\varphi||_{V_i}=1} |\langle A_i(t,q)\phi, ,\varphi\rangle_{V_i^*,V_i}| = ||A_i(t,q)\phi||_{V_i^*},$$

whence the assumption $H(A)_1$ and the above equality imply that $||A_i(t,q)\phi||_{V_i^*}$ is continuous on q.

$H(f)_1$: $q \longrightarrow f(\cdot,q) : Q_T \longrightarrow V_2^*$ is continuous.

Lemma 3.1. *If H(A),H(f),H(B),H(N),$H(A)_1$ and $H(f)_1$ hold and also $L(t)$ satisfy $L(\cdot) \in L^\infty(0,T;\mathcal{L}(V_2,V_2^*))$. Then $y(q,u)$ is strongly continuous on (q,u), i.e., $y(q,u) \in C(Q_T \times U, W(0,T))$.*

PROOF. It can be proved by using the method of Ahemd[2] and Ha[8].

Theorem 3.1. *If H(A),H(f),H(B),H(N),$H(A)_1$ and $H(f)_1$ hold and also $L(t)$ satisfy $L(\cdot) \in L^\infty(0,T;\mathcal{L}(V_2,V_2^*))$. Then there is at least one optimal control $(\bar{q},\bar{u})$ if $Q_T \times U$ is compact.*

PROOF. It is clear from Lemma 3.1 and continuity of norm. □

4. NECESSARY CONDITION OF OPTIMALITY FOR BOTH PARAMETERS AND CONTROLS

Here we present the necessary condition (the minimizing condition) for the optimal controls $(\bar{q},\bar{u}) \in Q_T \times U$ to the system (3.1) with the cost functional $J(p,u)$ given by

(3.2). If $J(p,u)$ is Gâteaux differentiable at $(\bar{q},\bar{u})$ in the direction $(q-\bar{q},u-\bar{u})$, the necessary condition on $(\bar{q},\bar{u})$ is characterized by the following inequality

$$DJ(\bar{q},\bar{u};q-\bar{q},u-\bar{u})\geq 0,\ \ \forall(q,u)\in Q_T\times U, \tag{4.1}$$

where $DJ(\bar{q},\bar{u};q-\bar{q},u-\bar{u})$ denotes the Gâteaux derivative at $(\bar{q},\bar{u})$ in the direction $(q-\bar{q},u-\bar{u})$.

Note that since $J(q,u)$ composed of the term $y(q,u)$, the Gâteaux differentiability of $J(q,u)$ follows from that of $y(q,u)$. Hence to obtain that of $y(q,u)$ we will need the following condition:

$H(A)_2$: $q\longrightarrow A_i(\cdot,q)$ is Gâteaux differentiable for all t and $DA_i(t,q)(p)\equiv DA_i(t,q;p)\in L^2(0,T;\mathcal{L}(V_i,V_i^*))$ for all $q\in Q_T$, where $DA_i(t,q;p)$ denotes the Gâteaux derivative at q in the direction of p.

$H(g)_1$: For any $\varphi\in H$ the Fréchet derivative of g exists and satisfies $g_\varphi(\varphi)\in\mathcal{L}(H,H)$ with $||g_\varphi(\varphi)||_{\mathcal{L}(H,H)}\leq c_4$, where $g_\varphi(\varphi)$ is the Fréchet derivative of g at φ and c_4 is constant.

$H(f)_2$: $q\longrightarrow f(t,q)$ is Gâteaux differentiable for all t and $f_q(t,q)p\equiv f_q(t,q;p)\in L^2(0,T,V_2^*)$, where $f_q(t,q;p)$ is Gâteaux derivative at q in the direction of p.

Lemma 4.1. *Assume that the conditions in Theorem 2.1, $H(A)_1$, $H(A)_2$, $H(f)_1$, $H(f)_2$ and $H(g)_1$ are satisfied. Then $y(q,u)$ is weakly Gâteaux differentiable at (q,u) in the direction $(q-\bar{q},u-\bar{u})$, denote the Gâteaux derivative of $y(q,u)$ by $z=Dy(\bar{q},\bar{u};q-\bar{q},u-\bar{u})$, which satisfies the following Cauchy problem:*

$$\left\{\begin{array}{l} z''+A_2(t,\bar{q})z'+A_1(t,\bar{q})z+N^*g_y(Ny(\bar{q},\bar{u}))Nz \\ =-DA_2(t,\bar{q};q-\bar{q})y'(\bar{q},\bar{u})-DA_1(t,\bar{q};q-\bar{q})y(\bar{q},\bar{u}) \\ \qquad +B(u-\bar{u})+f_q(t,\bar{q};q-\bar{q})\quad in\ (0,T) \\ z(0)=z'(0)=0. \end{array}\right. \tag{4.2}$$

Proof. We can prove by using the method of Ahemd [2] and Park et al. [12]. ☐

By Lemma 4.1, the cost functional $J(q,u)$ is Gâteaux differentiable at $(\bar{q},\bar{u})$ in the direction $(q-\bar{q},u-\bar{u})$, and so, the condition (4.1) is rewritten by

$$\begin{aligned} DJ(\bar{q},\bar{u};q-\bar{q},u-\bar{u}) &= \langle C^*\Lambda_M(Cy(\bar{q},\bar{u})-z_d),z\rangle_{W^*(0,T),W(0,T)} \\ &+\langle C^*\Lambda_M(Cy(\bar{q},\bar{u})-z_d),y_u(\bar{q},\bar{u};u-\bar{u})\rangle_{W^*(0,T),W(0,T)}\geq 0,\ \ \forall(q,u)\in Q_T\times U, \end{aligned} \tag{4.3}$$

where z is a unique weak solution to (4.2), $C^* \in \mathcal{L}(M^*, W^*(0,T))$ is the adjoint operator of C and Λ_M is the canonical isomorphism of M onto M^* in the sense that

(i) $\langle \Lambda_M \phi, \phi \rangle_{M^*,M} = ||\phi||_M^2$,

(ii) $||\Lambda_M \phi||_{M^*} = ||\phi||_M$ for all $\phi \in M$. In order to avoid the complexity of setting up observation spaces, we consider the following two types of distributive and terminal value observations in time sense. that is, the following cases :

(i) we take $C_1 \in \mathcal{L}(L^2(0,T;V_2), M)$ and observer $z(q,u) = C_1 y(q,u)$;

(ii) we take $C_2 \in \mathcal{L}(H, M)$ and observer $z(q,u) = C_2 y(q,u)(T)$.

4.1. The case where $C_1 \in \mathcal{L}(L^2(0,T;V_2), M)$

In this case the cost functional is given by

$$J(q,u) = \frac{1}{2}||C_1 y(q,u) - z_d||_M^2, \forall q \in Q_T \times U,$$

and then the necessary condition (4.3) is equivalent to

$$\begin{aligned}(4.4)\qquad & \int_0^T \langle C_1^* \Lambda_M (C_1 y(\bar q, \bar u)(t) - z_d), z(t) \rangle_{V_2^*, V_2} dt \\ & + \int_0^T \langle C_1^* \Lambda_M (C y(\bar q, \bar u)(t) - z_d), y_u(\bar q, \bar u; u - \bar u) \rangle_{V_2^*, V_2} dt \geq 0, \quad \forall (q,u) \in Q_T \times U,\end{aligned}$$

Let us introduce an adjoint state $\eta(\bar q, \bar u)$ satisfying

$$\begin{aligned}(4.5)\qquad & \eta''(\bar q,\bar u) - A_2(t,\bar q)\eta'(\bar q,\bar u) + [(A_1(t,\bar q) - A_2'(t,\bar q)) + (N^* g_y(Ny(\bar q,\bar u)N)^*]\eta(\bar q,\bar u) \\ & \qquad = C_1^* \Lambda_M (C_1 y(\bar q,\bar u) - z_d), \\ & \eta(\bar q,\bar u)(T) = 0, \quad \eta'(\bar q,\bar u)(T) = 0.\end{aligned}$$

Since $C_1^*\Lambda_M(C_1 y(\bar q,\bar u) - z_d) \in L^2(0,T;V_2^*)$ and $A_2'(t,\bar q) \in L^\infty(0,T;\mathcal{L}(V_2,V_2^*))$, the equation (4.5) is well-posed and permits a unique weak solution $\eta(\bar q,\bar u) \in W(0,T)$ if we consider the change of the time variable as $t \to T - t$. Multiplying (4.5) by z, which is the weak solution to (4.2), integrating it by parts after integrating it on $[0,T]$, we obtain

$$\begin{aligned}& \int_0^T \langle \eta(\bar q,\bar u)(t), z''(t) + A_2(t,\bar q)z'(t) + [A_1(t,\bar q) + N^* g_y(Ny(\bar q,\bar u)(t))N]z(t) \rangle_{V^*,V} dt \\ (4.6)\qquad & = \int_0^T \langle \eta(\bar q,\bar u)(t), -DA_2(t,\bar q; q - \bar q)y'(\bar q,\bar u)(t) - DA_1(t,\bar q; q-\bar q)y(\bar q,\bar u)(t) \rangle_{V^*,V} dt \\ & + \int_0^T \langle \eta(\bar q,\bar u)(t), B(u - \bar u)(t) + f_q(t,\bar q; q - \bar q) \rangle_{V^*,V} dt \geq 0, \forall (q,u) \in Q_T \times U.\end{aligned}$$

From (4.3) and (4.4), we obtain the inequality

$$\begin{aligned}
&\int_0^T \langle \eta(\bar q,\bar u)(t), z''(t) + A_2(t,\bar q)z'(t) + [A_1(t,\bar q) + N^* g_y(Ny(\bar q,\bar u)(t))N]z(t)\rangle_{V^*,V}dt \\
&\quad + \int_0^T \langle C_1 y_u(\bar q,\bar u; q-\bar q)(t), C_1 y(\bar q,\bar u)(t) - z_d\rangle dt \\
&= \int_0^T \langle \eta(\bar q,\bar u)(t), -DA_2(t,\bar q; q-\bar q)y'(\bar q,\bar u)(t) - DA_1(t,\bar q; q-\bar q)y(\bar q,\bar u)(t)\rangle_{V^*,V}dt \\
&\quad + \int_0^T \langle \eta(\bar q,\bar u)(t), B(u-\bar u)(t) + f_q(t,\bar q; q-\bar q)\rangle_{V^*,V}dt \\
&\quad + \int_0^T \langle C_1 y_u(\bar q,\bar u; q-\bar q)(t), C_1 y(\bar q,\bar u)(t) - z_d\rangle_{V^*,V}dt \geq 0, \forall (q,u) \in Q_T \times U.
\end{aligned}$$

Here we used the inequality (4.4). Summarizing these we have the following theorem.

Theorem 4.1. *Assume that H(A), H(f), H(B), H(N), H(g),* $H(A)_1$, $H(A)_2$, $H(f)_1$, $H(f)_2$, $H(g)_1$ *hold. Then the optimal control* $(\bar q,\bar u)$ *is characterized by state and adjoint equations and inequality:*

$$\begin{cases} y''(\bar q,\bar u) + A_2(t,\bar q)y'(\bar q,\bar u) + A_1(t,\bar q)y(\bar q,\bar u) + N^* g(Ny(\bar q,\bar u)) = B\bar u + f(t,\bar q) \quad \text{in } (0,T) \\ y(\bar q,\bar u)(0) = y_0 \in V, y'(\bar q,\bar u)(0) = y_1 \in H, \end{cases}$$

$$\begin{cases} \eta''(\bar q,\bar u) - A_2(t,\bar q)\eta'(\bar q,\bar u) + [(A_1(t,\bar q) - A_2'(t,\bar q)) + (N^* g_y(Ny(\bar q,\bar u)N)^*]\eta(\bar q,\bar u) \\ \qquad\qquad = C_1^*\Lambda_M(C_1 y(\bar q,\bar u) - z_d) \quad \text{in } (0,T), \\ \eta(T,\bar q) = 0, \eta'(T,\bar q) = 0 \end{cases}$$

$$\begin{cases} \int_0^T \langle \eta(\bar q,\bar u)(t), B(u-\bar u)(t) + f_q(t,\bar q; q-\bar q)\rangle_{V^*,V}dt \\ \qquad + \int_0^T \langle C_1 y_u(\bar q,\bar u; u-\bar u)(t), C_1 y(\bar q,\bar u)(t) - z_d\rangle_{V^*,V}dt \\ \quad \geq \int_0^T \langle \eta(\bar q,\bar u)(t), DA_2(t,\bar q; q-\bar q)y'(\bar q,\bar u)(t) + DA_1(t,\bar q; q-\bar q)y(\bar q,\bar u)(t)\rangle_{V^*,V}dt, \\ \qquad\qquad \forall (q,u) \in Q_T \times U. \end{cases}$$

$\square$

4.2. The case where $C_2 \in \mathcal{L}(H,M)$

In this case the cost functional is given by

$$J(q,u) = \frac{1}{2}||C_2 y(q,u)(T) - z_d||_M^2, (q,u) \in Q_T \times U$$

and then the necessary condition (4.3) is equivalent to

$$\begin{aligned} (4.7)\qquad &(C_2^*\Lambda_M(C_2 y(q,u)(T) - z_d), z(T))_H \\ &+(C_2^*\Lambda_M(C_2 y(q,u)(T) - z_d), y_u(\bar q,\bar u; u-\bar u)(T))_H \geq 0, \quad \forall (q,u) \in Q_T \times U. \end{aligned}$$

Let us introduce an adjoint state $\eta(\bar{q},\bar{u})$ satisfying

$$(4.8)\qquad \begin{cases} \eta''(\bar{q},\bar{u}) - A_2(t,\bar{q})\eta'(\bar{q},\bar{u}) + [(A_1(t,\bar{q}) - A_2'(t,\bar{q})) \\ \qquad\qquad\qquad\qquad +(N^*g_y(Ny(\bar{q},\bar{u})N)^*]\eta(\bar{q},\bar{u}) = 0 \\ \eta(\bar{q},\bar{u})(T) = 0, \\ \eta'(\bar{q},\bar{u})(T) = -C_2^*\Lambda_M(C_2y(\bar{q},\bar{u})(T) - z_d). \end{cases}$$

It follows by the same reason as the case 4.1 that there is a unique weak solution $\eta(\bar{q},\bar{u}) \in W(0,T)$, because $C_2^*\Lambda_M(C_2y(\bar{q},\bar{u})(T) - z_d) \in H$.

Theorem 4.2. *We assume that H(A), H(f), H(B), H(N), H(g), $H(A)_1$, $H(A)_2$, $H(f)_1$, $H(f)_2$ and $H(g)_1$ hold. Then the optimal control $(\bar{q},\bar{u})$ is characterized by state and adjoint equations and inequality:*

$$\begin{cases} y''(\bar{q},\bar{u}) + A_2(t,\bar{q})y'(\bar{q},\bar{u}) + A_1(t,\bar{q})y(\bar{q},\bar{u}) + N^*g(Ny(\bar{q},\bar{u})) = B\bar{u} + f(t,q) \quad \text{in } (0,T) \\ y(\bar{q},\bar{u}) = y_0 \in V, y'(\bar{q},\bar{u}) = y_1 \in H, \end{cases}$$

$$\begin{cases} \eta''(\bar{q},\bar{u}) - A_2(t,\bar{q})\eta'(\bar{q},\bar{u}) + [(A_1(t,\bar{q}) - A_2'(t,\bar{q})) + (N^*g_y(Ny(\bar{q},\bar{u})N)^*]\eta(\bar{q},\bar{u})(T) = 0, \\ \eta(\bar{q},\bar{u})(T) = 0, \\ \eta'(\bar{q},\bar{u})(T) = -C_2^*\Lambda_M(C_2y(\bar{q},\bar{u})(T) - z_d) \end{cases}$$

$$\begin{cases} (C_2^*(C_2y(\bar{q},\bar{u})(T) - z_d)_H + \int_0^T \langle B(u-\bar{u})(t) + f_q(t,\bar{q};q-\bar{q}), \eta(\bar{q},\bar{u})(t)\rangle_{V^*,V}dt \\ \geq \int_0^T \langle DA_2(t,\bar{q};q-\bar{q})y'(\bar{q},\bar{u})(t) + DA_1(t,\bar{q};q-\bar{q})y(\bar{q},\bar{u})(t), \eta(\bar{q},\bar{u})(t)\rangle_{V^*,V}dt, \forall q \in Q_T. \end{cases}$$

PROOF. We prove the inequality condition of optimal control only. Multiplying (4.8) by z, which is a weak solution to (4.2), integrating it by parts after integrating it on $[0,t]$, we obtain

$$\begin{aligned} &\int_0^T \langle \eta(\bar{q},\bar{u})(t), z''(t) + A_2(t,\bar{u})z'(t) + [(A_1(t,\bar{q}) + N^*g_y(Ny(\bar{q},\bar{u})(t)N]z(t)\rangle_{V^*,V}dt \\ &\qquad\qquad +(z(T), \eta'(\bar{q},\bar{u})(T))_H \\ &= \int_0^T \langle \eta(\bar{q},\bar{u})(t), -DA_2(t,\bar{q};q-\bar{q})y'(\bar{q},\bar{u})(t) - DA_1(t,\bar{q};q-\bar{q})y(\bar{q},\bar{u})(t)\rangle_{V^*,V}dt \\ &\qquad\qquad + \int_0^T \langle \eta(\bar{q},\bar{u})(t), B(u-\bar{u})(t) + f_q(t,\bar{q};q-\bar{q})\rangle_{V^*,V}dt \\ &\quad +(z(T), -C_2^*\Lambda_M(C_2y(\bar{q},\bar{u})(T) - z_d))_H = 0. \end{aligned}$$

Hence from (4.7) and (4.8) we conclude that

$$
\begin{aligned}
&(z(T), C_2^*\Lambda_M(C_2y(\bar{q},\bar{u})(T)-z_d))_H + (y_u(\bar{q},\bar{u};u-\bar{u})(T), C_2^*\Lambda_M(C_2y(\bar{q},\bar{u})(T)-z_d)_H \\
&= \int_0^T \langle \eta(\bar{q},\bar{u})(t), -DA_2(t,\bar{q};q-\bar{q})y'(\bar{q},\bar{u})(t) - DA_1(t,\bar{q};q-\bar{q})y(\bar{q},\bar{u})(t)\rangle_{V^*,V}dt \\
&\qquad + \int_0^T \langle \eta(\bar{q},\bar{u})(t), B(u-\bar{u})(t) + f_q(t,\bar{q};q-\bar{q})\rangle_{V^*,V}dt \\
&\quad +(y_u(\bar{q},\bar{u};u-\bar{u})(T), C_2^*\Lambda_M(C_2y(\bar{q},\bar{u})(T)-z_d)_H \geq 0, \quad (q,u) \in Q_T \times U.
\end{aligned}
$$

□

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