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Can We Explain Traffic Congestion?

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1 Introduction

Recently physical understanding of traffic flow phenomena becomes active research area. Phase transition from freely moving state to jam state can be understood with simple models. More complex behavior, for example, spontaneous emergence of traffic jam and congestion in two-lane systems, have attracted attentions[1, 2, 3].

The methods for modeling traffic flow phenomena is divided into two approaches: microscopic and macroscopic ones. In the traffic flow in expressways, the motion of cars and their interaction are the microscopic processes. In cellular automaton models, the motions of cars are treated as particle hopping processes in one-dimensional lattice systems. The interaction between cars is described as exclusion effects. The simplest choice is Wolfram's rule-184 cellular automaton.

In car-following models, the motions of cars are described with sets of differential equations. Cars are controlled under the stimuli from the preceding car through the headway or the relative velocity.

The microscopic models are based on hydrodynamic approaches. They needs the macroscopic relation between the density and the velocity of the flow, which the microscopic models try to describe.

2 The Optimal Velocity Model

The Optimal Velocity (OV) model is one of car-following models[4, 5]. The essential point of the OV model is the introduction of an Optimal Velocity function. In the OV model, each car controls its acceleration to tune its velocity to the optimal (safety) velocity V_{optimal} which depends only on the headway Δx to the preceding car:

$$\frac{d^2x}{dt^2} = \alpha \left[V_{\text{optimal}}\left(\Delta x\right) - \frac{dx}{dt} \right], \tag{1}$$

where a constant α is the sensitivity corresponding to the inverse of the time scale for tuning to the optimal velocity.

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Because the OV model has a simple form, we can obtain some analytical results[5]. By linear stability analysis, the phase transition point ρ_c is obtained independently of the detail form of V_{optimal}:

$$\frac{\alpha}{2} = V'_{\mathrm{optimal}}(\rho_c), \tag{2}$$

where $'$ denotes the first derivative. In the high density state $\rho > \rho_c$, the system contains jam clusters (Fig. 1).

The form of the OV function can be chosen arbitrarily under some conditions. Here we employ a sigmoidal one after Ref.[4]:

$$V_{\mathrm{optimal}}(\Delta x) = \frac{v_{\max}}{2}\left[\tanh\left(2\frac{\Delta x - d}{w}\right) + c\right]. \tag{3}$$

The parameters can be determined from observed data[6].

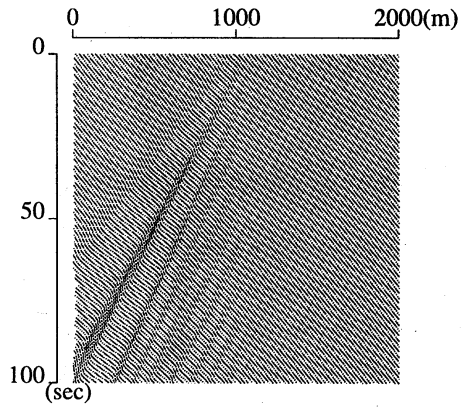

Fig. 1: The uniform flow at the initial time with $\rho > \rho_c$ decays to the jam state. The jam clusters propagate upstream. The horizontal axis is the position of cars and the vertical axis denotes the time.

The density can be a control parameter only in periodic systems. The phase transition in equilibrium sense also can be described with eq. 2 only in periodic systems. In realistic traffic flow systems, the density is a state variable induced by dynamics. We need to construct a model applicable in open boundary systems.

3 The Coupled Map Optimal Velocity Model

There are some difficulties to apply the OV model to realistic situations. The main reason is that the model is described with a set of differential equation and each equation depends on the position of the preceding car. To write down the set of equations, the sequence of cars must be fixed previously. Therefore the model can not be applied to open boundary systems. Moreover there are no room to introduce lane-changing rules into the model.

To overcome the difficulties, we constructed the Coupled Map Optimal Velocity (CMOV) model by simple discretization in time[7]. The position x and the velocity v of a car is updated as follows:

$$x(t+1) = x(t) + v(t)\Delta t, \tag{4}$$

$$v(t+1) = v(t) + \alpha\left[V_{\mathrm{optimal}}(\Delta x(t)) - v(t)\right]\Delta t. \tag{5}$$

The time step Δt is a constant and in an order of 0.1 second, which corresponds to the time scale of human response.

The merits of the CMOV model is based on flexible sequences of cars. The sequences of cars can be changed at any time steps. We can apply the model to open boundary systems, in which cars are appended to the tail of the sequences of cars at the entrance and are removed from the head of the sequences of cars at the exit. Two-lane expressway systems can be constructed with the CMOV model by introducing appropriate lane-changing rules. Moreover various types of noises can be introduced.

4 Noise Induced Congested Flow

The system used for simulating the flow on one-lane expressways is shown in Fig. 2. The system has no car at the initial state. From the left side, a car is injected every second if the distance to the preceding car is larger than the minimum value Δx_{min}. The initial speed of injected cars is zero. The car arriving the right end of the system is removed. Without noise, injected cars just catch up the preceding one. No congestion happens.

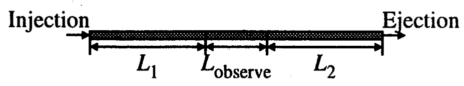

Fig. 2: The system setup for simulation. The left L_1 and right L_2 segments are used for relaxation. The middle L_{observe} segment is used for measuring the density and the velocity.

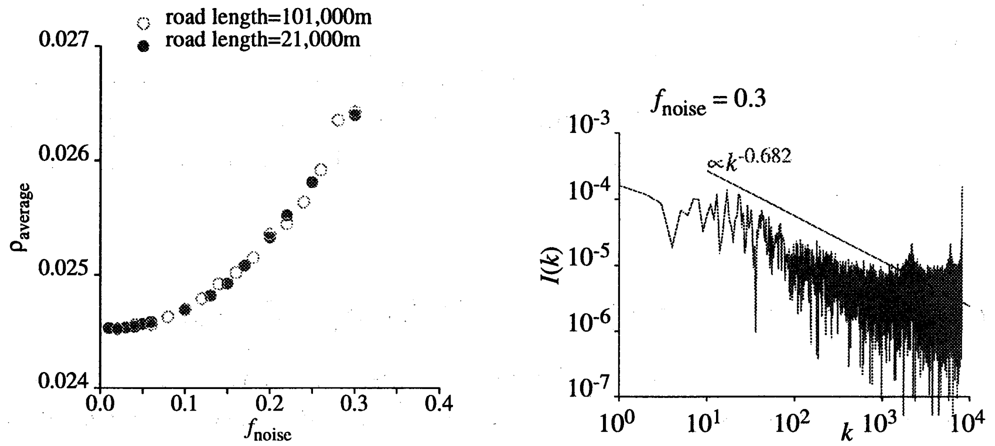

Fig. 3: The density increases in proportion to the square of the noise level (left). Above a critical noise level, the temporal fluctuations of the density shows $1/f$ spectra (right).

To investigate the effect of noise, we introduce noise in the velocity update rule in multiplicative form by replacing eq. (5) to

$$v(t+1) = \left(v(t) + \alpha\left[V_{\text{optimal}}\left(\Delta x(t)\right) - v(t)\right]\Delta t\right)\left(1 + f_{\text{noise}}\xi\right), \tag{6}$$

where $\xi \in [-0.5, 0.5]$ is an uniform random noise and the parameter f_{noise} controls the level of the noise. With increase of the noise level f_{noise}, the density ρ increases in proportion to the square of the noise level. If the noise level exceeds the critical value, the temporal fluctuation of the density shows $1/f$ spectra (Fig. 3). The congestion of this type is called *Noise Induced Congested Flow*[8].

5 Modeling Two-Lane Expressways

To extend the CMOV model to two-lane expressways, a set of lane-changing rules should be introduced. Currently observational data on lane-changing behavior are not available. Therefore we install a simple intuitive set of lane-changing rules as a test.

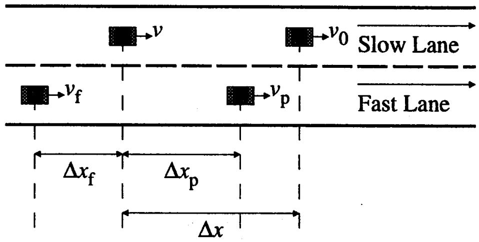

Fig. 4: The lane-changing rules for moving from the slow lane to the fast one.

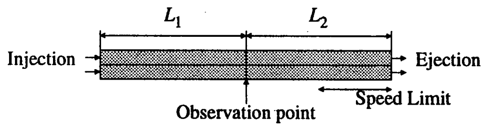

Fig. 5: The system setup for two-lane expressway. In the right half of the L_2 segments, the desired speed $v_{\max}$ reduces to $\beta v_{\max}$ ($\beta < 1$). At the observational point, the flow and the average velocity are recorded.

If a car can not run at the desired speed, the car hopes to change the lane. A lane-changing is allowed when the car can enter the target lane safely and the lane-changing is effective to run faster. If the two conditions mentioned above are satisfied, the car changes the lane probabilistically. The lane-changing rules are applied for all cars parallelly.

Figure 4 shows a set of lane-changing rules for moving from the slow lane to the fast one. If the headway Δx is shorter than a critical value Δx_{safe}, the car hopes to change lane. The lane-changing is allowed when the car can run faster in the fast lane ($\Delta x < \Delta x_p$) and enter the fast lane safely ($\Delta x > V^{-1}_{\mathrm{optimal}}(v)$). If these conditions are satisfied, the car moves to the fast lane with a probability P_{up}.

The lane-changing rules for moving from the fast lane to the slow one is almost the same as those above. Because we construct a model for expressways, an extra rule is added to require cars to run on the slow lane if overtaking is not needed.

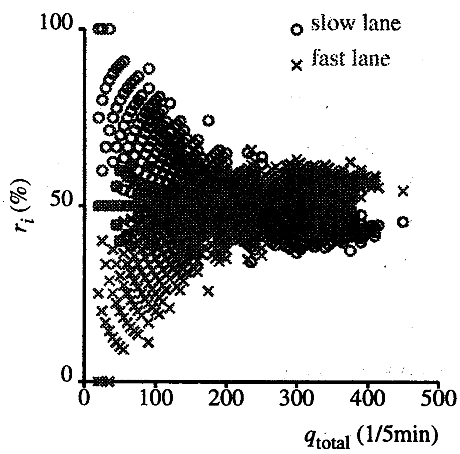

Fig. 6: The lane usage characteristics.

As in simulations for one-lane expressways, no congestion will happen without noise nor any bottleneck structures. To observe the formation of congestion, the maximum speed is restricted in the right half of the second segment (Fig. 5). The method for injecting cars is the same as in the case for simulations of one-lane expressway. At the observational point, the flow q and the average v velocity are recorded. Moreover, the desired speed $v_{\max}$ for each car is distributed in $[0.8v_{\max}, 1.2v_{\max}]$.

The lane usage characteristics is the ratio $r_i = q_i/q_{\mathrm{total}}$ ($i \in \{\mathrm{slow}, \mathrm{fast}\}$) of the flow q_i to the total one $q_{\mathrm{total}} = q_{\mathrm{slow}} + q_{\mathrm{fast}}$. In the low flow region, most of cars run on the slow lane. With increase of the total flow, the flow on the fast lane exceeds one on the slow lane. We call this phenomenon *reverse lane usage.*

6 Comparison with Observations

To validate the results of our simulation, the observational data are shown. The observational data was obtained with a set of induction loop detectors installed by the Japan Highway Corporation. The observational points locates at 170.64km point in Tomei Expressway bound for Tokyo. There is Nihonzaka tunnel 2km downstream. The observation was done on August 2. 1996 (Friday).

Figure 7 shows the lane usage characteristics. As in the results of simulations, we can observe the reverse lane usage ($q_{\text{flow}} > q_{\text{slow}}$) in the high flow state. The macroscopic features, including fundamental diagrams, observed in Tomei expressway can be reproduced by simulations.

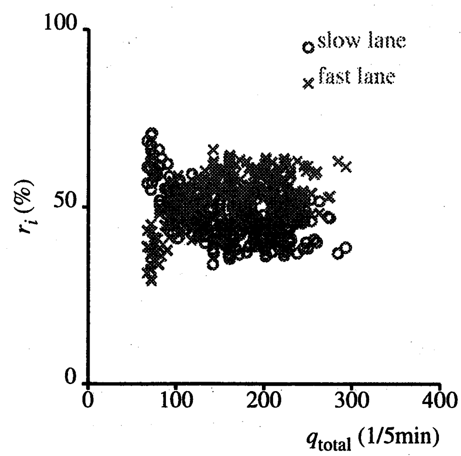

Fig. 7: The lane usage characteristics.

We can understand the origins of the reverse lane usage from the temporal sequences of the data. In congested states, the difference of the velocity between lanes remains small. The density on the fast lane, however, exceeds one on the slow lane. In freely moving states, on the contrary, the difference of the density between lanes remains small. The velocity on the fast lane exceeds one on the slow lane.

7 Summary

We introduced the Coupled Map Optimal Velocity model of traffic flow by discretizing the Optimal Velocity model. The model can be applied to open boundary systems and two-lane expressway.

In simulations of one-lane expressways, we observed the noise induced congested flow. Above the critical value of the noise level, the headway shortage created by noise propagates upstream. As a result, noisy hysteresis loops appear in the headway-velocity plane. The average density increases in proportion to the square of the noise level. The temporal behavior of the density shows $1/f$ fluctuations.

We introduced a simple intuitive set of lane-changing rules for simulating two-lane expressways. The simulation results can reproduce the macroscopic observed properties qualitatively. We observed the reverse lane usage ($q_{\text{fast}} > q_{\text{slow}}$) both in simulations and observations. The origins were discussed with the time sequences of observed data.

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References

[1] D. Helbing, H. J. Herrmann, M. Schreckenberg and D. E. Wolf ed., *Traffic and Granular Flow '99* (Springer, Berlin, 2000).

[2] S. Tadaki, M. Kikuchi, Y. Sugiyama and S. Yukawa, Butsuri **55** (2000) 166 (in Japanese).

[3] D. Chowdhury, L. Santen and A. Schadschneider, Phys. Rep. **329** (2000) 199.

[4] M. Bando, K. Hasebe, A. Nakayama, A. Shibata and Y. Sugiyama, Jpn. J. Ind. Appl. Math. **11** (1994) 203.

[5] M. Bando, K. Hasebe, A. Nakayama, A. Shibata and Y. Sugiyama, Phys. Rev. E**51** (1995) 1035.

[6] M. Bando, K. Hasebe, K. Nakanishi, A. Nakayama, A. Shibata and Y. Sugiyama, J. Phys. I (France) **5** (1995) 1389.

[7] S. Tadaki, M. Kikuchi, Y. Sugiyama and S. Yukawa, J. Phys. Soc. Japan **67** (1998) 2270.

[8] S. Tadaki, M. Kikuchi, Y. Sugiyama and S. Yukawa, J. Phys. Soc. Japan **68** (1999) 3110.