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OPTIMAL CONTROL FOR SEMILINEAR ABSTRACT EQUATIONS OF PARABOLIC TYPE

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1. Introduction

In the preceding paper [8], the authors studied the optimal control problems for the Keller-Segel equations. In that paper we showed the existence of optimal control and the first order necessary condition by formulating the Keller-Segel equations as a semilinear abstract equation. Many papers have already been published to study the control problems for nonlinear parabolic equations. In the books Ahmed [1] and Barbu [2], some general frameworks are given for handling the semilinear parabolic equations with monotone perturbations. In [1] the nonlinear terms are monotone functions with linear growth, and in [2] they are generalized to the multivalued maximal monotone operators determined by lower semicontinuous convex functions. Papageorgiou [7] and Casas et al. [3] have studied some quasilinear parabolic equations of monotone type. This note is the generalization of [8] as a semilinear abstract equation of non-monotone type.

Notations. $\mathbb{R}$ denotes the sets of real numbers. Let I be an interval in $\mathbb{R}$. $L^p(I;\mathcal{H})$, $1 \le p \le \infty$, denotes the L^p space of measurable functions in I with values in a Hilber space $\mathcal{H}$. $\mathcal{C}(I;\mathcal{H})$ denotes the space of continuous functions in I with values in $\mathcal{H}$. Let $\mathcal{D}(I)$ denote the space of $\mathcal{C}^\infty$-functions with compact support on I and $\mathcal{D}'(I)$ denote the space of distributions on I. For simplicity, we shall use a universal constant C to denote various constants which are determined in each occurrence in a specific way by δ, M, and so forth. In a case when C depends also on some parameter, say θ, it will be denoted by C_θ.

2. The formulation of problem

Let $\mathcal{V}$ and $\mathcal{H}$ be two separable real Hilbert spaces with dense and compact embedding $\mathcal{V} \hookrightarrow \mathcal{H}$. Identifying $\mathcal{H}$ and its dual $\mathcal{H}'$ and denoting the dual space of $\mathcal{V}$ by $\mathcal{V}'$, we have $\mathcal{V} \hookrightarrow \mathcal{H} \hookrightarrow \mathcal{V}'$. We denote the scalar product of $\mathcal{H}$ by $(\cdot,\cdot)$ and the norm by $|\cdot|$. The duality product between $\mathcal{V}'$ and $\mathcal{V}$ which coincides with the scalar product of $\mathcal{H}$ on $\mathcal{H}\times\mathcal{H}$ is denoted by $\langle\cdot,\cdot\rangle$, and the norms of $\mathcal{V}$ and $\mathcal{V}'$ by $\|\cdot\|$ and $\|\cdot\|_*$, respectively. $\mathcal{U} = L^2(0,T;\mathcal{V}')$ and $\mathcal{U}_{ad}$ is closed, bounded and convex subset of $\mathcal{U}$.

We consider the following Cauchy problem

$$\text{(E)} \qquad \begin{cases} \dfrac{dY}{dt} + AY = F(Y) + U(t), \quad 0 < t \le T, \\ Y(0) = Y_0 \end{cases}$$

in the space $\mathcal{V}'$. Here, A is the positive definite self-adjoint operator of $\mathcal{H}$ defined by a symmetric sesquilinear form $a(Y,\widetilde{Y})$ on $\mathcal{V}$, $\langle AY,\widetilde{Y}\rangle = a(Y,\widetilde{Y})$, which satisfies

$$\text{(a.i)} \qquad |a(Y,\widetilde{Y})| \le M\|Y\|\|\widetilde{Y}\|, \quad Y,\widetilde{Y}\in\mathcal{V},$$
$$\text{(a.ii)} \qquad a(Y,Y) \ge \delta\|Y\|^2, \quad Y\in\mathcal{V}$$

with some δ and $M > 0$. A is also a bounded operator from $\mathcal{V}$ to $\mathcal{V}'$. $F(\cdot)$ is a given continuous function from $\mathcal{V}$ to $\mathcal{V}'$ satisfying

(f.i) For each $\eta > 0$, there exists an increasing continuous function $\phi_\eta : [0,\infty) \to [0,\infty)$ such that

$$\|F(Y)\|_* \le \eta\|Y\| + \phi_\eta(|Y|), \quad Y\in\mathcal{V};$$

(f.ii) For each $\eta > 0$, there exists an increasing continuous function $\psi_\eta : [0,\infty) \to [0,\infty)$ such that

$$\|F(\widetilde{Y}) - F(Y)\|_* \le \eta\|\widetilde{Y} - Y\| + (\|\widetilde{Y}\| + \|Y\| + 1)\psi_\eta(|\widetilde{Y}| + |Y|)|\widetilde{Y} - Y|, \quad \widetilde{Y},Y\in\mathcal{V}.$$

$U(\cdot) \in L^2(0,T;\mathcal{V}')$ is a given function and $Y_0 \in \mathcal{H}$ is an initial value.

We then obtain the following result (For the proof, see Ryu and Yagi [8]).

Theorem 2.1. *Let* (a.i), (a.ii), (f.i), *and* (f.ii) *be satisfied. Then, for any* $U \in L^2(0,T;\mathcal{V}')$ *and* $Y_0 \in \mathcal{H}$, *there exists a unique weak solution*

$$Y \in H^1(0,T(Y_0,U);\mathcal{V}') \cap \mathcal{C}([0,T(Y_0,U)];\mathcal{H}) \cap L^2(0,T(Y_0,U);\mathcal{V})$$

to (E), *the number* $T(Y_0,U) > 0$ *is determined by the norms* $\|U\|_{L^2(0,T;\mathcal{V}')}$ *and* $|Y_0|$.

In this section we are concerned with the following problem

$$\text{(P)} \qquad \text{Minimize } J(U),$$

where the cost functional $J(U)$ is of the form

$$J(U) = \int_0^S \|DY(U) - Y_d\|^2 dt + \gamma\int_0^S \|U\|_*^2 dt, \quad U\in\mathcal{U}_{ad}.$$

Here, $Y(U)$, $U \in \mathcal{U}_{ad}$, is the weak solution of (E) and is assumed to exist on a fixed interval $[0,S]$. D is a bounded operator from $\mathcal{V}$ into $\mathcal{V}$ and Y_d is a fixed element of $L^2(0,S;\mathcal{V})$. γ is a nonnegative constant.

Remark. Let $Y_0 \in \mathcal{H}$ be fixed. By Theorem 2.1, for $U \in \mathcal{U}_{ad}$, $Y(U)$ exists on the interval $[0,T(U)]$ with $T(U) > 0$ depending on $\|U\|_{L^2(0,T;\mathcal{V}')}$. Hence, $0 < S \le \inf\{T(U); U \in \mathcal{U}_{ad}\}$.

We prove the following theorem.

Theorem 2.2. *There exists an optimal control* $\overline{U} \in \mathcal{U}_{ad}$ *for* (P) *such that*

$$J(\overline{U}) = \min_{U \in \mathcal{U}_{ad}} J(U).$$

Proof. The proof can be carried out in the same way as that of Theorem 2.1 (see [8, Theorem 2.1]). As it is standard (cf. [2, Chap. 5, Proposition 1.1] and [6, Chap. III, Theorem 15.1]), we will only sketch.

Let $\{U_n\} \subset \mathcal{U}_{ad}$ be a minimizing sequence such that $\lim_{n\to\infty} J(U_n) = \min_{U \in \mathcal{U}_{ad}} J(U)$. Since $\{U_n\}$ is bounded, we can assume that $U_n \to \overline{U}$ weakly in $L^2(0,S;\mathcal{V}')$. For simplicity, we will write Y_n instead of the solution $Y(U_n)$ of (E) corresponding to U_n,

$$\begin{cases} \dfrac{dY_n}{dt} + AY_n = F(Y_n) + U_n(t), \quad 0 < t \le S, \\ Y_n(0) = Y_0. \end{cases}$$

Taking the scaler product of the equation and Y_n, we obtain that

$$\frac{1}{2}\frac{d}{dt}|Y_n(t)|^2 + \langle AY_n(t), Y_n(t)\rangle = \langle F(Y_n(t), Y_n(t)\rangle + \langle U_n(t), Y_n(t)\rangle.$$

Then, from (a.ii) and (f.i),

$$\frac{1}{2}\frac{d}{dt}|Y_n(t)|^2 + \delta\|Y_n(t)\|^2 \le \eta\|Y_n(t)\|^2 + \{\phi_\eta(|Y_n(t)|) + \|U_n(t)\|_*\}\|Y_n(t)\|.$$

With some increasing, locally Lipschitz continuous function $\phi\colon [0,\infty) \to [0,\infty)$, it follows that

$$\text{(2.1)} \qquad \begin{cases} \dfrac{d}{dt}|Y_n(t)|^2 + \delta\|Y_n(t)\|^2 \le \phi(|Y_n(t)|^2) + \dfrac{8}{\delta}\|U_n(t)\|_*^2, \quad 0 < t \le S, \\ |Y_n(0)|^2 = |Y_0|^2. \end{cases}$$

Let $z_n(t) = |Y_n(t)|^2 - \frac{8}{\delta}\int_0^t \|U_n(s)\|_*^2 ds$, $0 \le t \le S$. Since $\int_0^S \|U_n(s)\|_*^2 ds \le C$, it follows that

$$\frac{dz_n}{dt} \le \phi(z_n + 8C\delta^{-1}).$$

On the other hand, let $z(t)$ be a solution to the ordinary differential equation

$$\begin{cases} \dfrac{dz}{dt} = \phi(z + 8C\delta^{-1}), \quad 0 \le t \le S, \\ z(0) = |Y_0|^2. \end{cases}$$

Then, by the theorem of comparison, $z_n(t) \le z(t)$ for all $0 \le t \le S$. Hence, $|Y_n(t)|^2 \le \|z\|_{C([0,S])} + 8C\delta^{-1}$.

The sequence $\{Y_n\}$ is thus bounded in $L^\infty(0,S;\mathcal{H})$. As a consequence, it follows from (2.1) that $\{Y_n\}$ is bounded in $L^2(0,S;\mathcal{V})$ also. Moreover, from (f.i), $\{dY_n/dt\}$ is bounded in $L^2(0,S;\mathcal{V}')$. Therefore, choosing a subsequence if necessary, we can assume that

$$Y_n \to \overline{Y} \quad \text{weakly in} \quad L^2(0,S;\mathcal{V}),$$
$$\frac{dY_n}{dt} \to \frac{d\overline{Y}}{dt} \quad \text{weakly in} \quad L^2(0,S;\mathcal{V}').$$

Since $\mathcal{V}$ is compactly embedded in $\mathcal{H}$, it is shown by [5, Chap. 1, Theorem 5.1] that

$$Y_n \to \overline{Y} \text{ strongly in } L^2(0,S;\mathcal{H}). \tag{2.2}$$

Let us verify that $\overline{Y}$ is a solution to (E) with the control $\overline{U}$. Let $\xi \in \mathcal{D}(0,S)$ and $V \in \mathcal{V}$, and put $\Phi(t) = \xi(t)V$. Then,

$$\int_0^S \langle Y_n'(t), \Phi(t)\rangle dt + \int_0^S \langle AY_n(t), \Phi(t)\rangle dt$$
$$= \int_0^S \langle F(Y_n(t), \Phi(t)\rangle dt + \int_0^S \langle U_n(t), \Phi(t)\rangle dt.$$

Let here n tend to infinity. It is then observed from (f.ii) that

$$\int_0^S |\langle F(Y_n(t) - F(\overline{Y}(t), \Phi(t)\rangle|\, dt \le \eta \int_0^S \|Y_n(t) - \overline{Y}(t)\| \|\Phi(t)\| dt$$
$$+ \int_0^S (\|Y_n(t)\| + \|\overline{Y}(t)\| + 1)\psi_\eta(|Y_n(t)| + |\overline{Y}(t)|)|Y_n(t) - \overline{Y}(t)| \|\Phi(t)\| dt,$$

where $\eta > 0$ is arbitrary. From (2.2) it is seen that $\int_0^S \langle F(Y_n), \Phi(t)\rangle dt$ converges to $\int_0^S \langle F(\overline{Y}(t)), \Phi(t)\rangle dt$ as $n \to \infty$. Therefore, we obtain that

$$\int_0^S \langle \overline{Y}'(t), \Phi(t)\rangle dt + \int_0^S \langle A\overline{Y}(t), \Phi(t)\rangle dt$$
$$= \int_0^S \langle F(\overline{Y}(t), \Phi(t)\rangle dt + \int_0^S \langle \overline{U}(t), \Phi(t)\rangle dt.$$

This then shows that $\overline{Y}(t)$ satisfies the equation of (E) for almost all $t \in (0,S)$. In a similar way it is also shown that $\overline{Y}(0) = Y_0$, note from [4, Chap. XVIII, Theorem 1] that $\overline{Y} \in \mathcal{C}([0,S];\mathcal{H})$. Hence, $\overline{Y}$ is the unique solution to (E) with the control $\overline{U}$; that is, $\overline{Y} = Y(\overline{U})$.

Since $Y_n - Y_d$ is weakly convergent to $\overline{Y} - Y_d$ in $L^2(0,S;\mathcal{V})$, we have:

$$\min_{U \in \mathcal{U}_{ad}} J(U) \le J(\overline{U}) \le \underline{\lim}_{n \to \infty} J(U_n) = \min_{U \in \mathcal{U}_{ad}} J(U).$$

Hence, $J(\overline{U}) = \min_{U \in \mathcal{U}_{ad}} J(U)$. □

3. First order necessary condition

In this section, we show the first order necessary condition for the Problem (P). We denote the scalar products in $\mathcal{V}$ and $\mathcal{V}'$ by $\langle\cdot,\cdot\rangle_{\mathcal{V}}$ and $\langle\cdot,\cdot\rangle_{\mathcal{V}'}$, respectively. In order to the necessary conditions of optimality, we need some additional assumptions:

(f.iii) The mapping $F(\cdot):\mathcal{V}\to\mathcal{V}'$ is Fréchet differentiable and for each $\eta>0$, there exists an increasing continuous functions $\mu_\eta,\nu:[0,\infty)\to[0,\infty)$ such that

$$|\langle F'(Y)Z,P\rangle|\le\begin{cases}\eta\|Z\|\|P\|+(\|Y\|+1)\mu_\eta(|Y|)|Z|\|P\|, & Y,Z,P\in\mathcal{V},\\ \eta\|Z\|\|P\|+(\|Y\|+1)\mu_\eta(|Y|)\|Z\||P|, & Y,Z,P\in\mathcal{V},\\ \nu(|Y|)\|Z\|\|P\|, \quad Y,Z,P\in\mathcal{V}.\end{cases}$$

(f.iv) $F'(\cdot)$ is continuous from $\mathcal{H}$ into $\mathcal{L}(\mathcal{V},\mathcal{V}')$.

Proposition 3.1. *Let* (a.i), (a.ii), (f.i), (f.ii), (f.iii), *and* (f.iv) *be satisfied. The mapping* $Y:\mathcal{U}_{ad}\to H^1(0,S;\mathcal{V}')\cap\mathcal{C}([0,S];\mathcal{H})\cap L^2(0,S;\mathcal{V})$ *is Gâteaux differentiable with respect to* U. *For* $V\in\mathcal{U}_{ad}$, $Y'(U)V=Z$ *is the unique solution in* $H^1(0,S;\mathcal{V}')\cap\mathcal{C}([0,S];\mathcal{H})\cap L^2(0,S;\mathcal{V})$ *of the problem*

$$\text{(3.1)}\qquad\begin{cases}\dfrac{dZ}{dt}+AZ-F'(Y)Z=V(t), \quad 0<t\le S,\\ Z(0)=0.\end{cases}$$

Proof. Let $U,V\in\mathcal{U}_{ad}$ and $0\le h\le 1$. Let Y_h and Y be the solutions of (E) corresponding to $U+hV$ and U, respectively.

Step 1. $Y_h\to Y$ *strongly in* $\mathcal{C}([0,S];\mathcal{H})$ *as* $h\to 0$. Let $W=Y_h-Y$. Obviously, W satisfies

$$\text{(3.2)}\qquad\begin{cases}\dfrac{dW}{dt}+AW-(F(Y_h(t))-F(Y(t)))=hV(t), \quad 0<t\le S,\\ W(0)=0.\end{cases}$$

Taking the scalar product of the equation (3.2) with W, we obtain that

$$\frac{1}{2}\frac{d}{dt}|W(t)|^2+\langle AW(t),W(t)\rangle=\langle F(Y_h(t))-F(Y(t)),W(t)\rangle+\langle hV(t),W(t)\rangle.$$

Using (a.ii) and (f.ii), we have

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}|W(t)|^2+\delta\|W(t)\|^2\\ &\quad\le\frac{\delta}{2}\|W(t)\|^2+\left(\|Y_h(t)\|^2+\|Y(t)\|^2+1\right)\psi_{\frac{\delta}{4}}\left(|Y_h(t)|+|Y(t)|\right)^2|W(t)|^2\\ &\qquad\qquad+4h^2\delta^{-1}\|V(t)\|_*^2.\end{aligned}$$

Therefore,

$$(3.3)\quad \frac{1}{2}|W(t)|^2 + \frac{\delta}{2}\int_0^t \|W(s)\|^2 ds$$
$$\leq \int_0^t \left(\|Y_h(s)\|^2 + \|Y(s)\|^2 + 1\right)\psi_{\frac{\delta}{4}}\left(|Y_h(s)| + |Y(s)|\right)^2 |W(s)|^2 ds$$
$$+ 4h^2\delta^{-1}\int_0^S \|V(s)\|_*^2 ds.$$

Using Gronwall's lemma, we obtain that

$$|W(t)|^2 \leq Ch^2\|V\|^2_{L^2(0,S;\mathcal{V}')} e^{\int_0^S (\|Y_h(s)\|^2 + \|Y(s)\|^2 + 1)\psi_{\frac{\delta}{4}}(|Y_h(s)| + |Y(s)|)^2 ds}$$

for all $t \in [0, S]$. Hence, $Y_h \to Y$ strongly in $\mathcal{C}([0, S]; \mathcal{H})$ as $h \to 0$.

Step 2. $\frac{Y_h - Y}{h} \to Z$ *strongly in* $H^1(0, S; \mathcal{V}') \cap \mathcal{C}([0, S]; \mathcal{H}) \cap L^2(0, S; \mathcal{V})$ *as* $h \to 0$. We rewrite the problem (3.2) in the form

$$(3.4)\quad \begin{cases} \dfrac{d}{dt}\dfrac{Y_h - Y}{h} + A\dfrac{Y_h - Y}{h} - \dfrac{F(Y_h) - F(Y)}{h} = V(t), \quad 0 < t \leq S, \\ \dfrac{Y_h - Y}{h}(0) = 0. \end{cases}$$

On the other hand, we consider the linear problem (3.1). From (a.i), (a.ii), (f.i), (f.ii), and (f.iii), we can easily verify that (3.1) possesses a unique weak solution $Z \in H^1(0, S; \mathcal{V}') \cap \mathcal{C}([0, S]; \mathcal{H}) \cap L^2(0, S; \mathcal{V})$ on $[0, S]$ (cf. [4, Chap. XVIII, Theorem 2]). Define $F'_h = \int_0^1 F'\big(Y + \theta(Y_h - Y)\big)d\theta$. Then $\widetilde{W} = \frac{Y_h - Y}{h} - Z$ satisfies

$$(3.5)\quad \begin{cases} \dfrac{d\widetilde{W}(t)}{dt} + A\widetilde{W}(t) - F'_h\widetilde{W}(t) = (F'_h - F'_0)Z(t), \quad 0 < t \leq S, \\ \widetilde{W}(0) = 0. \end{cases}$$

Taking the scalar product of the equation of (3.5) with $\widetilde{W}$, we obtain that

$$\frac{1}{2}\frac{d}{dt}|\widetilde{W}(t)|^2 + \langle A\widetilde{W}(t), \widetilde{W}(t)\rangle$$
$$= \langle F'_h\widetilde{W}(t), \widetilde{W}(t)\rangle + \langle (F'_h - F'_0)Z(t), \widetilde{W}(t)\rangle.$$
$$\leq \frac{\delta}{2}\|\widetilde{W}(t)\|^2 + (\|Y(t)\|^2 + \|Y_h(t) - Y(t)\|^2 + 1)\mu(|Y_h|^2 + |Y|^2)|\widetilde{W}(t)|^2$$
$$+ \frac{4}{\delta}\|(F'_h - F'_0)Z(t)\|_*^2,$$

where $\mu : [0, \infty) \to [0, \infty)$ is some increasing continuous function. Therefore,

$$(3.6)\quad |\widetilde{W}(t)|^2 + \delta\int_0^t \|\widetilde{W}(s)\|^2 ds$$
$$\leq \int_0^t (\|Y(s)\|^2 + \|Y_h(s)\|^2 + 1)\mu(|Y_h|^2 + |Y|^2)|\widetilde{W}(s)|^2 ds$$
$$+ \frac{8}{\delta}\|(F'_h - F'_0)Z(t)\|^2_{L^2(0,S;\mathcal{V}')}.$$

From (f.iii), we have $\|F_h'Z(t)\|_* \le C\|Z(t)\|$, $t \in [0,S]$. Since $Y_h \to Y$ strongly in $\mathcal{H}$, it follows from (f.iv) that

$$F_h'Z(t) \to F_0'Z(t) \quad \text{strongly in } \mathcal{V}' \text{ a.e..}$$

By the dominated convergence theorem, we have

$$\|(F_h' - F_0')Z(t)\|^2_{L^2(0,S;\mathcal{V}')} \to 0 \quad \text{as } h \to 0.$$

Using Gronwall's lemma, it follows from (3.6) that $\frac{Y_h - Y}{h}$ is strongly convergent to Z in $H^1(0,S;\mathcal{V}') \cap \mathcal{C}([0,S];\mathcal{H}) \cap L^2(0,S;\mathcal{V})$. □

With the aid of this proposition, we can easily show the first order necessary condition.

Theorem 3.2. *Let $\overline{U}$ be an optimal control of* (P) *and let $\overline{Y} \in L^2(0,S;\mathcal{V}) \cap \mathcal{C}([0,S];\mathcal{H}) \cap H^1(0,S;\mathcal{V}')$ be the optimal state, that is $\overline{Y}$ is the solution to* (E) *with the control $\overline{U}(t)$. Then, there exists a unique solution $P \in L^2(0,S;\mathcal{V}) \cap \mathcal{C}([0,S];\mathcal{H}) \cap H^1(0,S;\mathcal{V}')$ to the linear problem*

$$\text{(3.7)} \qquad \begin{cases} -\dfrac{dP}{dt} + AP - F'(\overline{Y})^*P = D^*\Lambda(D\overline{Y} - Y_d), \quad 0 \le t < S, \\ P(S) = 0 \end{cases}$$

in $\mathcal{V}'$, where $\Lambda : \mathcal{V} \to \mathcal{V}'$ is a canonical isomorphism; moreover,

$$\int_0^S \langle \Lambda P + \gamma\overline{U}, V - \overline{U}\rangle_{\mathcal{V}'} dt \ge 0 \qquad \textit{for all } V \in \mathcal{U}_{ad}.$$

Proof. Since J is Gâteaux differentiable at $\overline{U}$ and $\mathcal{U}_{ad}$ is convex, it is seen that

$$J'(\overline{U})(V - \overline{U}) \ge 0 \qquad \text{for all } V \in \mathcal{U}_{ad}.$$

On the other hand, we verify that

$$\text{(3.8)} \qquad J'(\overline{U})(V-\overline{U}) = \int_0^S \langle DY(\overline{U}) - Y_d, DZ\rangle_{\mathcal{V}} dt \;+\; \gamma\int_0^S \langle \overline{U}, V - \overline{U}\rangle_{\mathcal{V}'} dt$$

with $Z = Y'(\overline{U})(V - \overline{U})$. Let P be the unique solution of (3.7) in $H^1(0,S;\mathcal{V}') \cap \mathcal{C}([0,S];\mathcal{H}) \cap L^2(0,S;\mathcal{V})$. From (a.i), (a.ii), (f.i), (f.ii), and (f.iii), we can guarantee that such a solution P exists (cf. [4, Chap. XVIII, Theorem 2]). Thus, in view of Proposition 3.1 the first intergal in the right hand side of (3.8) is shown to be

$$\begin{aligned} \int_0^S \langle DY(\overline{U}) - Y_d, DZ\rangle_{\mathcal{V}} dt &= \int_0^S \langle D^*\Lambda(DY(\overline{U}) - Y_d), Z\rangle dt \\ &= \int_0^S \langle -\frac{dP}{dt} + AP - F'(\overline{Y})^*P, Z\rangle dt = \int_0^S \langle P, \frac{dZ}{dt} + AZ - F'(\overline{Y})Z\rangle dt \\ &= \int_0^S \langle \Lambda P, V - \overline{U}\rangle_{\mathcal{V}'} dt. \end{aligned}$$

Hence,

$$\int_0^S \langle \Lambda P + \gamma \overline{U}, V - \overline{U} \rangle_{\mathcal{V}'} dt \geq 0, \qquad \text{for all } V \in \mathcal{U}_{ad}. \quad \square$$

Remark. Note that our result covers that of [8, 9] when the sensitivity function $\chi(\rho)$ is linear function of ρ, $\chi(\rho) = b\rho$ (b being a positive constant). Furthermore, since all assumptions of our abstract result are satisfied when $\chi(\rho) = \frac{b\rho}{1+\rho}$, our result is also applied in this case.

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