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ON THE CONSTRUCTION OF CONFORMAL MEASURES FOR PIECEWISE C^0-INVERTIBLE SYSTEMS

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ABSTRACT. We present a new method for the construction of conformal measures ν for infinite to one piecewice C^0-invertible Markov systems. We direct our attention to potentials ϕ which may fail both summable variations and bounded distortion but satisfy the *weak bounded variation*. Our results apply to higher-dimensional maps which are not necessarily conformal and admit certain nonhyperbolic periodic orbits.

§0 Introduction

Let $(T, X, Q = \{X_i\}_{i\in I})$ be a piecewise C^0-invertible system i.e., X is a compact metric space with metric d, $T : X \to X$ is a noninvertible map which is not necessarily continuous, and $Q = \{X_i\}_{i\in I}$ is a countable disjoint partition $Q = \{X_i\}_{i\in I}$ of X such that $\bigcup_{i\in I} intX_i$ is dense in X and satisfy the following properties.

(01) For each $i \in I$ with $intX_i \neq \emptyset, T|_{intX_i} : intX_i \to T(intX_i)$ is a homeomorphism and $(T|_{intX_i})^{-1}$ extends to a homeomorphism v_i on $cl(T(intX_i))$.

(02) $T(\bigcup_{intX_i=\emptyset} X_i) \subset \bigcup_{intX_i=\emptyset} X_i$.

(03) $\{X_i\}_{i\in I}$ generates $\mathcal{F}$, the sigma algebra of Borel subsets of X.

Let $\underline{i} = (i_1 \dots i_n) \in I^n$ satisfy $int(X_{i_1} \cap T^{-1}X_{i_2} \cap \dots T^{-(n-1)}X_{i_n}) \neq \emptyset$. Then we define $X_{\underline{i}} := X_{i_1} \cap T^{-1}X_{i_2} \cap \dots T^{-(n-1)}X_{i_n}$ which is called a cylinder of rank n and write $|\underline{i}| = n$. By (01), $T^n|_{intX_{i_1\dots i_n}} : intX_{i_1\dots i_n} \to T^n(int(X_{i_1\dots i_n}))$ is a homeomorphism and $(T^n|_{intX_{i_1\dots i_n}})^{-1}$ extends to a homeomorphism $v_{i_1} \circ v_{i_2} \circ \dots \circ v_{i_n} = v_{i_1\dots i_n} : cl(T^n(intX_{\underline{i}})) \to cl(intX_{\underline{i}})$.

We impose on (T, X, Q) the next condition which gives a nice countable states symbolic dynamics similar to sofic shifts (cf. [5]):

(Finite Range Structure). $\mathcal{U} = \{int(T^n X_{i_1\dots i_n}) : \forall X_{i_1\dots i_n}, \forall n > 0\}$ consists of finitely many open subsets $U_1 \dots U_N$ of X.

In particular, we say that (T, X, Q) satisfies Bernoulli property if $cl(T(intX_i)) = X (\forall i \in I)$ so that $\mathcal{U} = \{intX\}$ and that (T, X, Q) satisfies Markov property if $int(cl(intX_i) \cap cl(intTX_j)) \neq \emptyset$ implies $cl(intTX_j) \supset cl(intX_i)$. (T, X, Q) satisfying Bernoulli (Markov) property is called a piecewise C^0-invertible Bernoulli (Markov) system respectively.

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For given a subset A of X, let $R_A : A \to \mathbb{N} \cup \infty$ be the first return function over A and we define $D_n^A = \{x \in A : R_A(x) > n\}$. If we have previously a reasonable measurable dynamics e.g. $(T, X, Q, \mathcal{F}, \nu)$, where $\mathcal{F}$ denotes the σ- algebra of Borel subsets of X and ν is a nonsingular ($\nu T^{-1} \sim \nu$) probability measure with $\nu(A) > 0$ satisfying

$$(1) : \lim_{n \to \infty} \nu(D_n^A) = \nu(\bigcap_{n \geq 0} D_n^A) = 0,$$

then the induced map T_A over A can be defined almost everywhere on A and all iterations $\{T_A^n\}_{n \geq 1}$, too. Furthermore, if we can construct a T_A-invariant ergodic probability measure μ_A absolutely continuous with respect to ν , then the integrability of the first return function with respect to ν (which is equivalent to (2) : $\sum_{n \geq 0} \nu(D_n^A) < \infty$ so that (1) is automatically satisfied) is sufficient for the existence of T-invariant ergodic probability measure μ absolutely continuous with respect to ν which is given by the well-known Kac formula as follows : for all $f \in L^1(\nu)$, $\frac{1}{\mu(A)} \int_X f d\mu = \int_A f_A d\mu_A$, where $f_A(x) = \sum_{i=0}^{R_A(x)-1} fT^i(x)$.

Those results and a generalized Thermodynamic Formalism for potential ϕ of weak bounded variation were established in [6] for piecewise C^0-invertible Bernoulli systems by assuming some regulalr condition on T_A and on the associated potential ϕ_A. In particular, this approach works satisfactory to establish Thermodynamic formalism for piecewise C^1- invertible maps with the Bernoulli property admitting certain nonhyperbolic periodic orbits (e.g.,indifferent periodic points) and for the natural potential $\phi = -\log |detDT|$. In fact, A can be taken as a hyperbolic region which is away from the nonhyperbolic periodic orbits (see [6] for details) and the absolutely continuous invariant measure μ with respect to the normalized Lebesgue measure ν attains the measure theoretical pressure for $\phi = -\log |detDT|$. On the other hand, when (T, X, Q) does not satisfy the Bernoulli property we have no evidence of the existence of nonsingular reference measure ν even if the Markov property is satisfied. If we restrict our attention to (countable) Markov shifts then we can find some answer to this problem (e.g.,[4]). However, if the systems are not symbolic dynamics the existence problem is still remain open (cf.[1]). In this talk, for infinite to one piecewise C^0-invertible transitive Markov systems we shall give a partial answer to this problem. For this purpose, we first clarify properties of topological pressure for ϕ and for the associated potential ϕ_A defined on a single cylinder $A \in Q$. Then we introduce Schweiger's jump transformations T^* over cylinders which are mapped onto X under T. We shall see a good relation between the topological pressure for ϕ_A and the topological pressure for ϕ^* associated to T^*. This observation allows one to establish the existence of an eigenvalue 1 of the Perron-Frobenius operator associated to ϕ_A by using a formula of zeta function for ϕ in terms of zeta function for ϕ^* obtained in [5]. Finally we can construct a conformal measure ν by using a result in [1]. We also establish the existence of conformal measures by using jump transformation defined. Again the existence of an eigenvalue 1 of the Perron-Frobenius operator associated to ϕ^* plays an important role for the construction of ν.

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