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MCKAY CORRESPONDENCE AND T-DUALITY

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CONTENTS

1. INTRODUCTION

From 1985 to 1995, there was a development on the exsitence of a crepant resolution for quotient singularities which were given by finite subgroups of $SL(3,\mathbb{C})$. This problem was based on the conjecture by Hirzebruch and Höfer ([7]) which came from the result in the superstring theory and some people called it Vafa's formula, ([4]).

The author proved the conjecture for nonabelian monomial groups around 1994 ([8], [9]) and recently there are some physical papers on D-branes which were related with these singularities ([6], [14]). The topological property can be explained by McKay correspondece in mathematics and also by T-duality in physics. They look different but it seems that they are deeply related.

In this section, we will introduce the McKay correspondence in dimension three and the relation with T-duality.

The McKay correspondence is originally a correspondence between the topology of the minimal resolution of the 2-dimensional rational double points (ADE singularities), which are quotient singularities by

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finite subgroups G of $SL(2,\mathbb{C})$, and the represenation theory (irreducible representations or conjugacy classes) of the group G. We can see the correspondence with Dynkin diagrams:

Let G be a finite subgroup of $SL(2,\mathbb{C})$, then the quotient space $X := \mathbb{C}^2/G$ has a rational double point at origin. As there exists the minimal resolution $\widetilde{X}$ of the singularity, we have the exceptional divisors E_i. The dual graph of the configuration of the exceptional divisors is just the Dynkin diagram of type A_n, D_n, E_6, E_7 or E_8.

On the other hand, we have the set of the non-trivial irreducible representations ρ_i of the group G up to isomorphism and let ρ be a regular representation in $SL(2,\mathbb{C})$. The tensor product of these representations

$$\rho_i \otimes \rho = \Sigma a_{ij}\rho_j$$

gives the set of integers a_{ij} and it determines the Cartan matrix which defines the Dynkin diagram.

Then there is a one-to-one numerical correspondence between $\{\rho_i\}$ and $\{E_i\}$, that is, the intersention matrix of the exceptional divisors can be written as $(-1)\times$ Cartan matrix.

In dimension three, we have several "McKay correspondences" but they are just bijection between two sets: Let X be the quotient singularity $\mathbb{C}^3/G$ where G is a finite subgroup of $SL(3,\mathbb{C})$. The X has a Gorenstein canonical singularity of index 1 but not a terminal singularity. It is known that there exist crepant resolutions $\widetilde{X}$ of this singularity. The crepant resolution is a minimal resolution and preserve the triviality of the canonical bundle in this case.

As the McKay correspondence, following bijections are known:

(1) (Ito-Reid [10]) cohomology group $H^{2i}(\widetilde{X},\mathbb{C}) \leftrightarrow$ {the conjugacy classes of "age" i in G}.
(2) (Ito-Nakajima [11])Grothendieck group $K(\widetilde{X}) \leftrightarrow$ {the irrducible representations of G}, where G is a finite abelian group.
(3) (Bridgeland-King-Reid [2]) Derived category $D(\widetilde{X}) \leftrightarrow$ {the irreducible representations of G} for any finite subgroups.

Remark 1.1. In (1), the **age** of $g \in G$ is defined as follows: After diagonalization, if $g^r = 1$, we obtain $g' = \mathrm{diag}(\epsilon^a, \epsilon^b, \epsilon^c)$ where ϵ is a primitive r-th root of unity. Then $\mathrm{age}(g) := (a+b+c)/r$. For identity element *id*, we define age(*id*)$= 0$ and all of ages are integer.

The correspondence (2) can be included in (3), but we can explain the 2-dimensional numerical McKay correspondence very clearly as a corollary of the result in [11].

In the first McKay correspondence (1), we have precise correspondence for each i-th cohomology with conjugacy classes, but in (2) we don't have such difference among representations like *age*. Then we would like to introduce T-duality which gives some difference between irreducible representations using resolution graphs.

T-duality for 3-dimensional quotient singularities can be explained as a correspondence between brane configuration of the D-branes of the quotient space and the dual graph of the exceptional locus of certain crepant resolution. The brane cofigurations are given by the quiver and they are obtained with the irreducible representations of the acting groups. This kind of explanation was of course given by phyicists in the theory of superstring and related with recent mathematical results, so called higher dimensional McKay correspondences. We will see this physical explanation and the relationship with crepant resolutions in the following chapters.

The author would like to thank T. Muto for valuable discussions.

2. T-DUALITY AND BRANE CONFIGURATION

Let us see the T-duality and brane configuration. In this paper, we don't give precise definition of branes, and we just regard p-brane as a real $(p+1)$-dimensional object.

Here we consider real 10-dimensional superstring theory of type IIA and IIB. They may have fundamental strings, NS5-branes and Dp-branes. For IIA (resp. IIB) string theory, p is even (resp. odd).

Later, we will consider real 11-dimensional theory, so called M-theory, for generalized T-duality which is related with complex 3-dimensional crepant resolutions.

compactification is not same as that in mathematics. For example, S^1-compactification means replacing $\mathbb{R}$ by S^1.

2(i). **T-duality.** Let us consider IIA and IIB string theories and both of them have real 10 dimensional space-time ($\mathbb{R}^{1,9}$) and we fix the coodinate $x_0, x_1, \cdots, x_9$.

There are 3 types of T-duality:

(1) Flat case: S^1-compactification of IIA and IIB string theories.

$$\begin{array}{c} \text{IIA theory over } \mathbb{R}^{1,8}(x_0,\cdots,x_8)\times S^1(x_9) \text{ (radius } R_9) \\ \big\downarrow \text{T-duality}(*) \\ \text{IIB theory over } \mathbb{R}^{1,8}(x_0,\cdots,x_8)\times S^1(x_9') \text{ (radius } R_9') \end{array}$$

where $R_9' = 1/R_9$ and this relations is called a basic T-duality.

2) With D-brane: There are two types of T-duality.

Type I

$$\begin{array}{c} \text{IIA over D6-brane } (x_0, \cdots, x_6) \times \mathbb{R}^3(x_7, \cdots, x_9) \\ \Big\downarrow \text{T-duality}(**) \\ \text{IIB over D5-brane } (x_0, \cdots, x_5) \times \mathbb{R}^4(x_6', x_7, \cdots, x_9) \end{array}$$

This T-duality occars at the coodinates x_6 and x_6'.

Type II This case was construdted by Ooguri and Vafa ([17]) and they called this T-duality as CFT equivalence.

$$\begin{array}{c} \text{IIA over } \mathbb{R}^{1,5}(x_0, \cdots, x_5) \times \text{multi Taub NUT space } (x_6, \cdots, x_9) \\ \Big\downarrow \text{T-duality}(***) \\ \text{IIB over } k \text{ NS5-brane } (x_0, \cdots, x_5) \times \mathbb{R}^4(x_6, \cdots, x_8, x_9') \end{array}$$

Let $(a_i^6, a_i^7, a_i^8, a_i^9)$ be a coordinate of i-th NS5-brane. The compactification with the coodinate x_9 is T-duality of the compactification with coodinate x_9'. And multi Taub NUT space has S^1-fibration with a coordinate (x_9) over $\mathbb{R}^3$ (x_6, x_7, x_8) and sigularities at (a_i^6, a_i^7, a_i^8). That is, the space is the same as the minimal resolution of A_k-singularities (see figure 2.1).

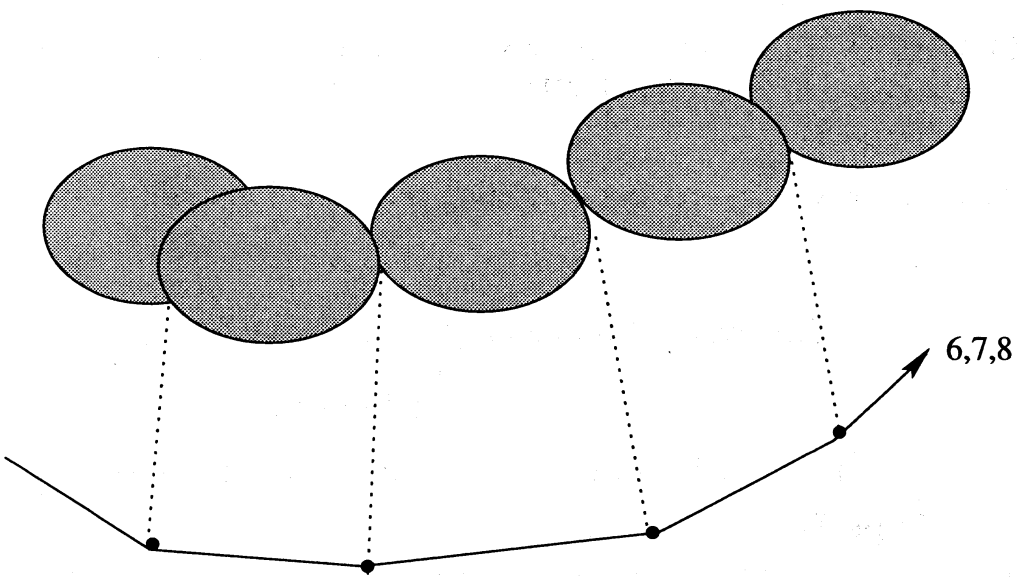

FIGURE 2.1. Multi Taub NUT space

Let us consider the T-duality with D-branes over a quotient space $/\mathbb{Z}_k$ using these T-dualities. For a compactification of type IIA ing, we consider the space D4-brane $\times\mathbb{R} \times \mathbb{C}^2/\mathbb{Z}_k$. Then replace quotient space by the mult Taub NUT space, i.e., the minimal resolution. Finally, we obtain k NS5-branes and k D5-b the brane configuration of these 5-branes, if you draw NS5-brane and an edge for D5-brane, then the diagram extended Dynkin diagram of type $\widetilde{A}_{k-1}$.

2(ii). **Generalized T-duality.** Now we condiser a gen T-duality via M-theory. This is due to the result by J. H. This T-duality is in general a correspondence between T^2- of the M-theory and S^1-compactification of type IIB stri

In usual T-duality, we consider the exchanging the ra and this generalized case also has same way. Note tha ways for it because there are 2 S^1s in T^2. This is the di the previous T-dualities.

(1) Flat case: It is a generalization of T-duality $(*)$.

$$\begin{array}{c}
\text{M theory over } \mathbb{R}^{1,8}(x_0,\cdots,x_8)\times T^2(x_9,x_{10}) \\
\Big\downarrow{\scriptstyle \text{radius } R_{10}\longrightarrow 0} \\
\text{IIA theory over } \mathbb{R}^{1,8}(x_0,\cdots,x_8)\times S^1(x_9) \text{ (radius} \\
\Big\downarrow{\scriptstyle \text{T-duality}(*)} \\
\text{IIB theory over } \mathbb{R}^{1,8}(x_0,\cdots,x_8)\times S^1(x_9')\text{(radius}
\end{array}$$

(2) With D-brane: There are 2 ways because there a the space of multi Taub NUT space $\times S^1$.

Type I

$$\begin{array}{c}
\text{M theory over } \mathbb{R}^{1,5}(x_0,\cdots,x_5)\times \text{multi Taub NUT space} \\
\Big\downarrow{\scriptstyle \text{radius } R_9\longrightarrow 0} \\
\text{IIA over D6-brane } (x_0,\cdots,x_6)\times \mathbb{R}^3(x_7, \\
\Big\downarrow{\scriptstyle \text{T-duality}(**)} \\
\text{IIB over D5-brane } (x_0.\cdots,x_5)\times \mathbb{R}^4(x_6',x_7
\end{array}$$

Type II

$$\begin{array}{c}
\text{M theory over } \mathbb{R}^{1,5}\times \text{multi Taub NUT space } (x_6\cdots x_9 \\
\Big\downarrow{\scriptstyle \text{radius } R_{10}\longrightarrow 0} \\
\text{IIA over } \mathbb{R}^{1,5}(x_0,\cdots,x_5)\times \text{multi Taub NUT space } (x \\
\Big\downarrow{\scriptstyle \text{T-duality}(***)} \\
\text{IIB over } k \text{ NS5-brane } (x_0,\cdots,x_5)\times \mathbb{R}^4(x_6',\cdots
\end{array}$$

More generally, we can obtain both of D5-brane and NS5-branes. The D5-brane will appear if a $(0,1)$ cycle of the torus T^2 shrinks. On the other hand, if a $(1,0)$ cycle of T^2 shrinks, then we have NS5-brane. In general, if (p,q) cycle shrink, then we have p NS5-brane and q D5-brane. Let us call this (p,q)5-brane. The brane configuration can show how the (p,q) cycles shrink. And Leung and Vafa ([12]) explained the brane configuration for the T-duality of M-theory over a quotient space $\mathbb{R}^3/G$, where G is a finite abelian subgroup of $SL(3,\mathbb{C})$, in terms of toric geometry. By their result, we can see the T-duality as follows:

$$\begin{array}{c}\text{M theory over } \mathbb{R}^{1,4}(x_0,\cdots,x_4)\times\mathbb{C}^3/G(x_5,\cdots,x_{10})\\ \big\downarrow{\scriptstyle \text{T-duality}}\\ \text{IIB over } p \text{ NS5-brane } (x_0,\cdots,x_5) \text{ and } q \text{ D5-brane}\end{array}$$

The brane configuration of (p,q)5-brane can be draw as a toric diagram which is obtained from toric resolution (cf. Fugure 3.1). And the slope of the lines in the dual graph of the diagram correspond to the shrinking (p,q) cycles.

For nonabelian case in dimension three, Muto ([15]) showed that the brane configuration can be explained by the quiver and we will see this result and relation with crepant resolution with an example in the following section. You may feel the T-duality looks like the McKay correspondence.

3. Example: Triheadral case

Trihedral singularity is a quotient singularity by trihedral group defined as follows: It is a nonabelian group generated by abelian subgroup of $SL(3,\mathbb{C})$ and a non-diagonal matrix

$$T=\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}.$$

The name *trihedral* is an analogy of *dihedral* and named by the author. Moreover trihedral singularities can be resolved similarly as D_n singularities (cf. [8]).

3(i). Crepant resolution of trihedral singularity. [8]

Let H be the normal abelian subgroup of G then we have the following diagram:

$$\begin{tikzcd} & & \widetilde{X} \arrow[d, "\tau"] \\ & \widetilde{Y} \arrow[r, "\mu'"] \arrow[d, "\pi"] & \widetilde{Y}/\mathfrak{A}_3 \arrow[d, "\pi'"] \\ \mathbb{C}^3 \arrow[r] & \mathbb{C}^3/H = Y \arrow[r, "\mu"] & \mathbb{C}^3/G = X \end{tikzcd}$$

where π is a resolution of the singularity of Y, and π' is the induced morphism, τ is a resolution of the singularity by $\mathfrak{A}_3$, and $\tau \circ \pi'$ is a crepant resolution of the singularity of X.

Sketch of the proof As a resolution π of Y, we take a toric resolution, which is also crepant. Then we lift the $\mathfrak{A}_3$-action on Y to its crepant resolution $\widetilde{Y}$ and form the quotient $\widetilde{Y}/\mathfrak{A}_3$. This quotient gives in a natural way a partial resolution of the singularities of X. The crepant resolution $\widetilde{X}$ of the singularities of $\widetilde{Y}/\mathfrak{A}_3$ induces a complete resolution of X.

Under the action of $\mathfrak{A}_3$, the singularities of $\widetilde{Y}/\mathfrak{A}_3$ lie in the union of the image of the exceptional divisor of $\widetilde{Y}$ under $\widetilde{Y} \longrightarrow \widetilde{Y}/\mathfrak{A}_3$ and the image of the locus $C : (x = y = z)$.

In the resolution $\widetilde{Y}$ of Y, the group $\mathfrak{A}_3$ permutes exceptional divisors. So the fixed points on the exceptional divisors consist of one point or three points. $\square$

To get $\widetilde{Y}$, we consider the special toric resolution. For the abelian quotient singularity $\mathbb{C}^3/H$, we have the following toric resolution:

$Y = \mathbb{C}^3/H$ is a toric variety. Then we can use toric geometry for the resolution, and there is the useful fact that the toric resolution is a crepant resolution. This was proved by Markushevich in 1987 [13], and by Roan in 1989 [19] independently. Both of them used toric geometry for the construction.

Let $\mathbb{R}^3$ be the 3-dimensional real vector space, $\{e^i | i = 1, 2, 3\}$ its standard base, L the lattice generated by e^1, e^2 and e^3, $N := L + \sum \mathbb{Z}v$, where the summation runs over all the elements $v = 1/r(a, b, c) \in H$, and

$$\sigma := \left\{ \sum_{i=1}^{3} x_i e^i \in \mathbb{R}^3, \quad x_i \geq 0, \forall i, 1 \leq i \leq 3 \right\}$$

the naturally defined rational convex polyhedral cone in $N_{\mathbb{R}} = N \otimes_{\mathbb{Z}} \mathbb{R}$. The corresponding affine torus embedding Y_σ is defined as $\operatorname{Spec}(\mathbb{C}[\check{\sigma} \cap M])$, where M is the dual lattice of N and $\check{\sigma}$ is the dual cone of σ in $M_{\mathbb{R}}$ defined as $\check{\sigma} := \{\xi \in M_{\mathbb{R}} | \xi(x) \geq 0, \forall x \in \sigma\}$.

We define: $\Delta :=$ the simplex in $N_{\mathbb{R}}$

$$= \left\{ \sum_{i=1}^{3} x_i e^i \quad ; x_i \geq 0, \quad \sum_{i=1}^{3} x_i = 1 \right\},$$

$$t : N_{\mathbb{R}} \longrightarrow \mathbb{R} \qquad \sum_{i=1}^{3} x_i e^i \longmapsto \sum_{i=1}^{3} x_i$$

and

$$\Phi := \left\{ \frac{1}{r}(a, b, c) \in H \mid a + b + c = r \right\}.$$

Then $X = \mathbb{C}^3/H$ corresponds to the toric variety which is induced by the cone σ within the lattice $N = L + \sum_{v \in \Phi} \mathbb{Z}v$.

Fact 1 we can construct a simplicial decomposition S of the triangle determined by e^1, e^2, e^3 with $\Phi \cup \bigcup_{i=1}^{3} \{e^i\}$ as the set of its vertices.

Fact 2 If $\widetilde{Y} := Y_S$ is the corresponding torus embedding, then Y_S is non-singular. Then we obtain a crepant resolution $\pi = \pi_S : \widetilde{Y} = Y_S \longrightarrow \mathbb{C}^3/G = Y$, because Y_S is non-singular and Gorenstein. Moreover, each lattice point in above triangle corresponds to one exceptional divisor.

Let us see a concrete example of a trihedral singularity and compare the crepant resolution and the T-duality. We will see the case the group $G \cong \mathbb{Z}/4\mathbb{Z} \oplus \mathbb{Z}/4\mathbb{Z} \rtimes \mathbb{Z}/3\mathbb{Z}$ and let H be the abelian subgroup $\mathbb{Z}/4\mathbb{Z} \oplus \mathbb{Z}/4\mathbb{Z}$. The dual graph of the exceptional locus in $\widetilde{Y}$ become as Figure 3.1.

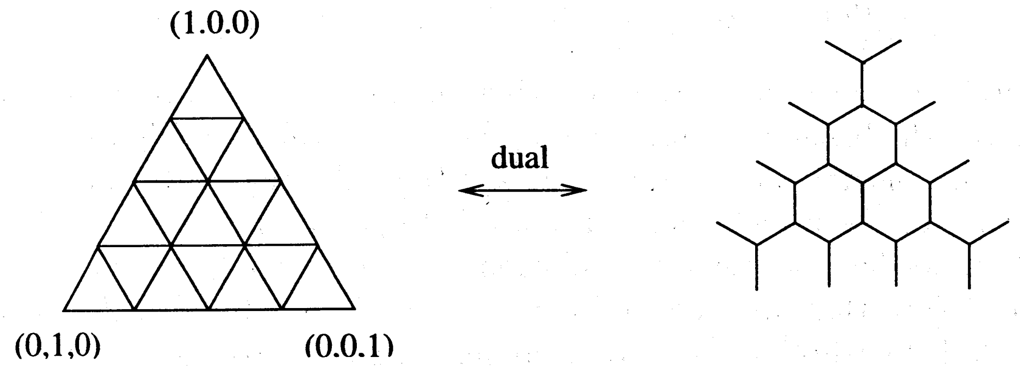

FIGURE 3.1. Toric resolution and the exceptional locus

After the action of $\mathfrak{A}_3$, we can identify some of the exceptional divisors and there exists a singularity of type $\frac{1}{3}(0,1,2)$ (see figure 3.2).

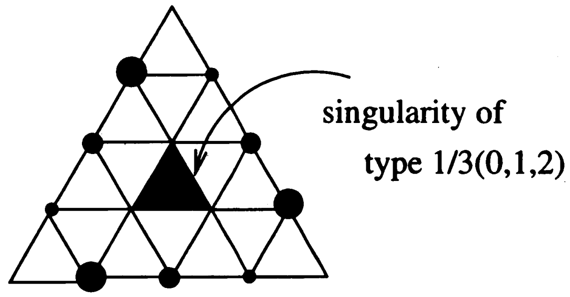

FIGURE 3.2. Singularity

Finally, we obtain 3 exceptional divisors which came from previous toric resolution and new 3 exceptional divisors.

3(ii). **Brane configuration.** [14], [15]

Let us see the brane configuration for this case. Brane configuration is described as a dual graph of the quiver (it is called *quiver diagram* in [15]) and the quiver is determined by the irreducible representations of the group G.

Let ρ_i be a irreducible representation of G and ρ be a regular representation in $SL(3,\mathbb{C})$. Then one can take the tensor product

$$\rho_i \otimes \rho = a_{ij}\rho_j.$$

If $a_{ij} = k$, then you must draw k arrows from a vertex i to a vertex j, and you can obtain the quiver. In our example, we have 16 1-

dimensional irreducible representations in the group H. Let us denote them as $R_1^{(l_1,l_2)}$ where $(l_1,l_2) \in \mathbb{Z}_4 \times \mathbb{Z}_4$. As a regular representation ρ, if we take $R_3 = R_1^{(1,1)} \oplus R_1^{(-1,0)} \oplus R_1^{(0,-1)}$, then we obtain the tensor product

$$R_1^{(l_1,L_2)} \otimes R_3 = R_1^{(l_1+1,l_2+1)} \oplus R_1^{(l_1-1,l_2)} \oplus R_1^{(l_1,l_2-1)}.$$

Therefore, we can draw the following diagram as the quiver for H and also see the brane configuration as the dual graph (see Figure 3.3)

If we consider the $\mathfrak{A}_3$ in G, we have the following quiver and finally we will see the brane configuration (cf. Figure 3.4).

Now we have a crepant resolution and brane configuration for one case. Note that we take unique toric resolution for $\mathbb{C}^3/H$ which is the H-Hilb $\mathrm{Hilb}^{|H|}(\mathbb{C}^3)$. It is almost same as H-fixed part of the Hilbert scheme of $|H|$-points on $\mathbb{C}^3$. You can find the construction and properties in papers [16], [18] or [11].

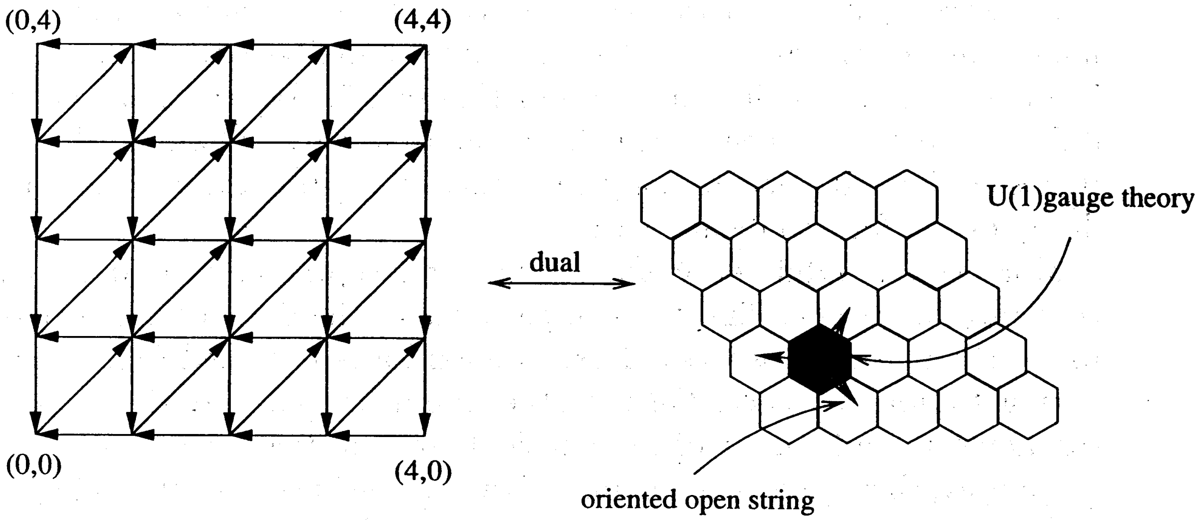

FIGURE 3.3. Quiver and brane configuration for H

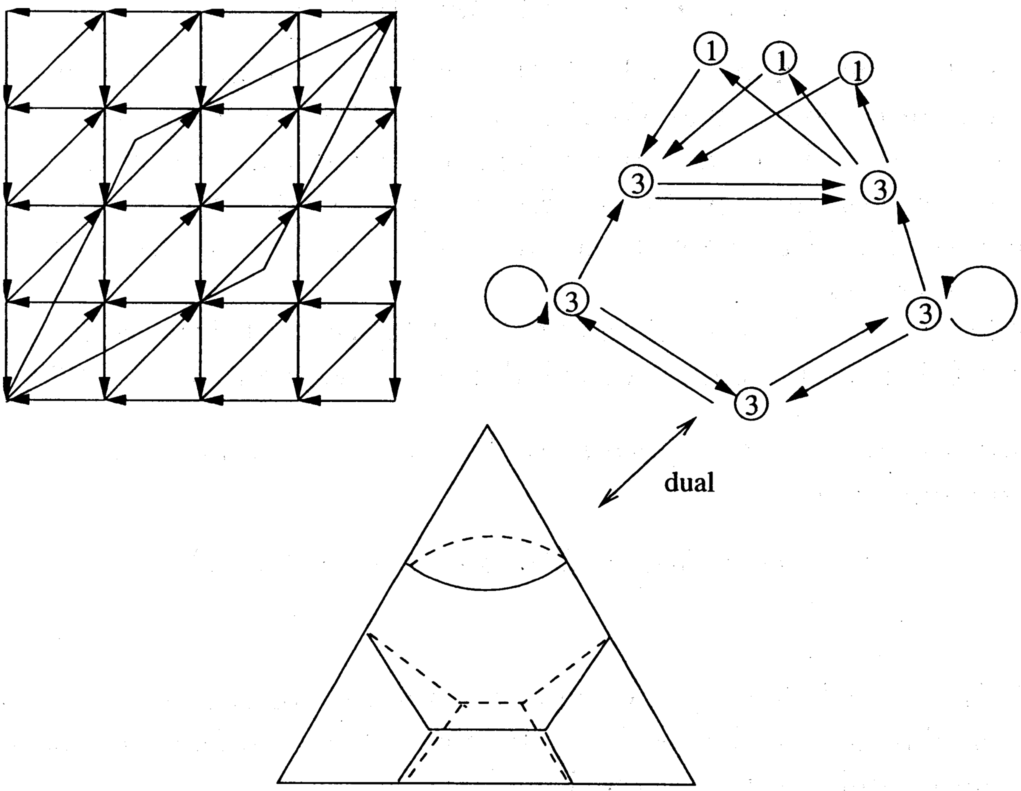

FIGURE 3.4. Quotient of quiver and brane configuration for G

Let us compare the pictures for the crepant resolution and brane configurations for the group H. They looks very similar but the daigrams in Figure 3.1 is just half of those of Figure 3.3. We can imagine the difference between them comes from the difference among irreducible representaions like *age* for conjugacy classes in [10]. And we have similar question for nonabelian cases.

Moreover, as a difference of diagrams for exceptional locus and brane configuration we can find the compactness of the diagrams. It is easy to see the non-compactness of exceptional locus obtained from a non-compact quotient space. On the other hand, we may say the physical situaion for the D-brane of the quotient singularities are compact.

Anyway, these phenomenon may give us some ideas for cohomological McKay correspondence with representations like [10].

4. McKay correspondence

After the symposium at RIMS, the author participated a summer school on toric geometry at Fourier Institute, Grenoble in France and Craw gave a talk there on a cohomological McKay correspondence and they discussed on these problems. So we would like to recall the known 3-dimensional McKay correspondence and announce the very recent result by Craw [3].

We denote $X := \mathbb{C}^3/G$ where $G \in SL(3,\mathbb{C})$ through this section.

Type I. Ito-Reid's comohological McKay correspondence, (1996) [10]

Theorem 4.1. *For any crepant resolution $\widetilde{X}$ of X, there exists the following bijection:*

$$\{\textit{conjugacy class in } G \textit{ of age } i\} \leftrightarrow h^{2i}(\widetilde{X},\mathbb{Q})$$

This bijection holds for any crepant resolution of canonical Gorenstein singularities by the result by Batyrev [1] today, but we restrict our situation only in dimension three here.

Then we have conjugacy classes of age 0, 1 and 2. Obviously the age 0 element is the identity element. The age 1 elements are expressed as the lattice points in a triangle like Figure 3.1 and they correspond to the exceptional divisors. The age 2 elements themselves don't appear in the crepant resolution but the inverse elements of them are elements of age 1 and the corresponding lattice points are inner points of the triangle, that is, they are corresponding to the exceptional divisors in the fiber $f^{-1}(0)$ which form the basis of H^2_c. Therefore we have the bijection between H^4 and the age 2 element as Poincaré duality.

By this correspondence, we can see the difference among conjugacy classes by age grading.

Type II. Ito-Nakajima's McKay correspondence, (1998) [11] This is a McKay correspondence for a unique crepant resolution, so called G-Hilb.

Theorem 4.2. *If G is a finite abelian subgroup of $SL(3,\mathbb{C})$, there exists a bijection between the set of irreducible representations rho_i of G and $K(Y)$, Grothendieck group of algebraic vector bundles over G-Hilb Y.*

We can see this with tautological vector bundles $\mathcal{R}_i$ which corresponds to each irreducible representation ρ_i. And as a corollary of this theorem we have the following:

Corollary 4.3. *If you consider Chern character homomorphism*

$$ch \ : \ K(Y) \longrightarrow H^*(Y,\mathbb{Q}),$$

then $\{ch(\mathcal{R}_i)\}_{i=0}^r\}$ form a basis of $H^*(Y,\mathbb{Q})$.

Type III. Craw's McKay correspondence, (2000) [3] Now we have the following result for G-Hilb Y where G is a finite abelian subgroup of $SL(3,\mathbb{C})$:

Theorem 4.4. *There are three types of tautological vector bundles:*

(0) tirivial bundle $\mathcal{O}_Y = \mathcal{R}_0$ which forms a basis of H^0.

(1) line bundles whose first Chern classes $c_1(\mathcal{R}_i)$ intersect with exceptional rational curves transversally.

(2) line bundles which determine naturally a dual basis of H^4.

Interesting point of these result is the following: Of course, it coinside with the McKay correspondece [11] and moreover the line bundles of type (2) gives a dual basis of H^4 clearly. On the other hand, in the correspondeince in [10] gives the basis of H^2 more naturally.

Anyway by the result of Craw, we can find some difference among irreducible representations but the cohomological McKay correspondece for any finite subgroup of $SL(3,\mathbb{C})$ is still open and mysterious.

References

[1] V. Batyrev, *Non-Archimedean integrals and stringy Euler numbers of log-terminal pairs*, J. Eur. Math. Soc. **1** (1999), 5–33.

[2] T. Bridgeland, A. King and M. Reid, *Mukai implies McKay: the McKay correspondence as an equivalence of derived categories*, alg-geom/9908027

[3] A. Craw, talk at Fourier institute and University of Warwick, 2000 July and August.

[4] L. Dixion, J. Havey, C. Vafa and E. Witten, *Strings on orbifolds (I), (II)*, Nucl. Phys. **B261** (1985), 678–686 and **N274** (1986), 285–314.

[5] G. Gonzales-Sprinberg and J.L. Verdier, *Construction géometrique de la correspondence de McKay*, Ann. Sci. École Norm. Sup. **16** (1983), 409–449.

[6] B. Greene, C. Lazaroiu and M. Raugas, *D-branes on nonabelian threefold quotient singularities*, hep-th/9811201.

[7] F. Hirzebruch and T. Höfer, *On the Euler number of an orbifold*, Math. Ann. **286** (1990), 255–260.

[8] Y. Ito, *Crepant resolution of trihedral singularities and the orbifold Euler characteristic*, Intern. Jour. of Math. **6** (1995), 33-43.

[9] Y. Ito, *Gorenstein quotient singularities of monomial type in dimension three*, J. Math. Sci. Univ. Tokyo **2** (1995), 419–440.

[10] Y. Ito and M. Reid, *The McKay correspondence for finite subgroups of* $SL(3,\mathbb{C})$, in Higher Dimensional Complex Varieties, Proc. Internat. Conf., Trento 1994 (1996), 221–240.

[11] Y. Ito and H. Nakajima, *McKay correspondence and Hilbert schemes in dimension three*, Topology, **39** (2000), 1155–1191.

[12] N.C. Leung and C. Vafa, *Branes and toric geometry*, Adv. Theor. Math. Phys. **2** (1998), 91–118.

[13] D. G. Markushevich, M. A. Olshanetsky and A. M. Perelomov, *Description of a class of superstring compactifications related to semi-simple Lie algebras*, Comm. Math. Phys. **111** (1987), 247–274.

[14] T. Muto, *D-branes on three-dimensional nonabelian orbifolds*, J. High Energy Phys. **2** (1999), 8–.

[15] T. Muto, *Brane configurations for three-dimensional nonabelian orbifolds*, hep-th/9905230.

[16] I. Nakamura, *Hilbert scheme of G-orbits for abelian G*, to appear in Jour. of Algebraic Geometry.

[17] H. Ooguri and C. Vafa, *Two-dimensional black hole and singularities of CY manifolds*, Nucl. Phys. B463 (1996), 55-72.

[18] M. Reid, *McKay correspondence*, Proceedings of Kinosaki symposium, alg-geom/9702016.

[19] S.-S. Roan *On the generalization of Kummer surfaces*, J. Diff. Geometry **30** (1989), 523–537.

[20] J. H. Schwarz, *M Theory Extensions of T Duality*, hep-th/9601077.

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