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New Kinetic Laws of Cluster Formation in N-body Hamiltonian Systems

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Abstract

Cluster formation is a generic phenomenon in many-body systems, and have been studied in various context(see Aizawa, Sato and Ito, 2000; Nakato and Aizawa, 2000). A cluster is not the closed system but the open system with finite lifetime. It survives by exchanging the member particles with the external environment, and self-regulates the inherent inner structure through the interactions among member particles. The peripheral region of a cluster, so-called "interface" plays a significant role in the response to the change of environment, i.e., evaporation or absorption of particles is sensitively controlled in the interface which surrounds the core-part of the cluster. Recent results of our simulations, carried out with many-body systems under short-ranged attractive forces, will be briefly discussed in the following.

1. Two Kinetic Phases embedded in the Cluster

An equilibrium cluster is described by the ergodic measure $\mu(\boldsymbol{x})$ defined in the one-particle phase space $\boldsymbol{x}$,

$$\mu(\boldsymbol{x}) = \langle r\rangle\, \mu_c(\boldsymbol{x}) + (1-\langle r\rangle)\, \mu_g(\boldsymbol{x}) \tag{1}$$

where $\langle r\rangle$ stands for the mean fraction of the phase space corresponding to the clustering motions; $\mu_c(\boldsymbol{x})$ and $\mu_g(\boldsymbol{x})$ are the normalized characteristic measure to describe the cluster phase and the gaseous phase, respectively. The probability density $P_g(T_g)$ for the residence time T_g in gaseous phase is usually approximated by the poissonian, $P_g(T_g) = \langle T_g\rangle^{-1} \exp[-T_g/\langle T_g\rangle]$, but on the other hand the probability density $P_c(T_c)$ for the residence time T_c in cluster phase is quite different from the poissonian;

$$P_c(T_c) = p \cdot \frac{dQ_W}{dT_c} + (1-p)\cdot\frac{dQ_L}{dT_c} = \frac{dQ_c}{dT_c} \tag{2}$$

where $Q_W = \exp[-AT_c^{-\alpha}]$ (negative Weibull distribution), $Q_L = \exp[-B(\log T_c)^{-\beta}]$ (Log-Weibull distribution), and p is the fraction of the negative Weibull component (Aizawa, Sato and Ito, 2000; Aizawa, 2000). Figure 1 demonstrates the accumulated probability $Q_c(x)(x = T_c)$, where the scaling regimes corresponding to two components Q_W (or P_W) and Q_L (or P_L) are clearly observed, and particularly the intrinsic long time tails of $Q_L(x)$ are systematically prolonged when the cluster size becomes large; when the total energy E decreases the size of the cluster increases. Two kinetic phases generally coexist in big clusters.

The shape of a cluster depends on the strength of interaction between the cluster and the environment. When the member particles are violently exchanged in the case of a small cluster, the shape is quite irregular, but in a large cluster the shape is almost globular and the variation of the shape is very slow and majestic. This is the reason why the $1/f$ fluctuation is often observed in clustering motions. Figure 2 is a typical example of cluster formation in N-body hamiltonian systems which we have reported in the previous paper

ato and Ito, 2000), where the stability of member particles is demon
cale which represents the Gauss-Riemannian curvature; the inside o
e curvature and the outside has negative one. The Riemannian geom
used in the analysis of the Mixmaster universe model (Aizawa, K
1997), is successfully applied for the rigorous definition of the cluste

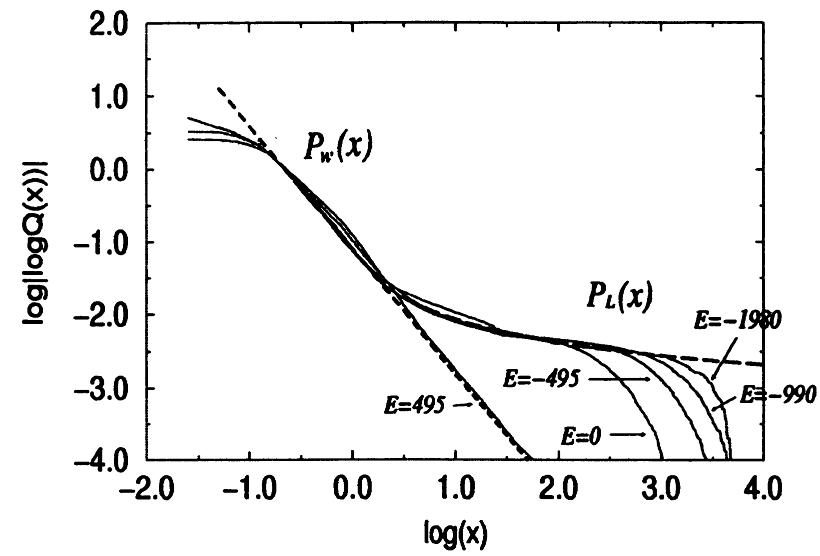

Fig.1 Distribution function $Q(x)$ for the trapping time x. $P_W(x)$ is the Negative-Weibull and $P_L(x)$ is the Log-Weibull. The parameter E is the total energy which controls the size of the cluster

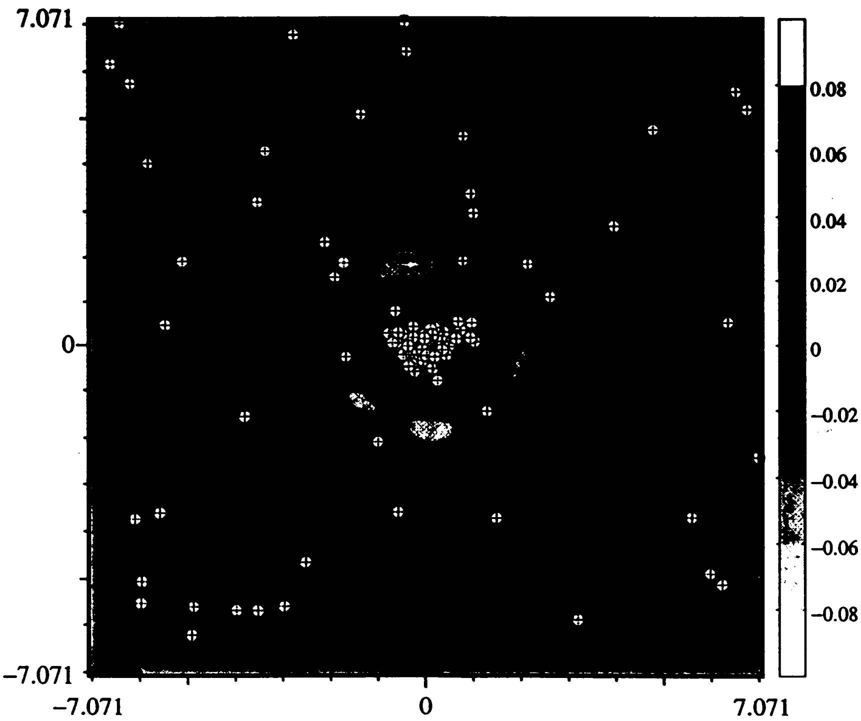

Fig.2 A snapshot of the cluster formation at $E = 0$, where the number of member particles is almost 50 percents of the total particles.

2. Universality of the Log-Weibull Distribution Characterizing Arnold Diffusion

The Log-Weibull component becomes larger and larger, when the cluster size increases and the cluster shape approaches to a globular one. This implies that the one-particle motion can be approximated by an integrable hamiltonian $H(p,q)$ if we consider the clustering motions in a large cluster;

$$H(p,q) = H_0(p) + \epsilon H_1(p,q,t) \tag{3}$$

where H_0 is the effective integrable hamiltonian and the $\epsilon H_1(p,q,t)$ is the perturbation due to the small derivation from the globular cluster. Equation (3) is the standard form of nearly integrable hamiltonian systems, where we can use the Nekhoroshev theorem (Nekhoroshev, 1977) for slowly drifting motions such as the Arnold diffusion in the Fermi-Pasta-Ulam models for quartz oscillators (Aizawa, 1995). The distribution function $P(T)$ of the characteristic time T for the diffusion was derived many years ago(Aizawa, 1989) in the following form; $P(T) \propto \frac{1}{T(\log T)^c}$ $(T \gg 1)$. It is easily obtained that the distribution function $P(T)$ for the Arnold diffusion is nothing but the Log-Weibull distribution function demonstrated in Fig.1; if we put $c = 2$, the intrinsic long time tails in cluster formation (in 2-dimensional simulations) are completely understood in terms of the Arnold diffusion (Aizawa, 2000).

3. Quasi-structure surviving as a Whole-body

Clusters appear almost always in the transitional regime between two different thermodynamical phases as a stable kinetic phase. Only difference from the ordinary phase in thermodynamic limit is that the cluster is an extremely small open system with finite scales in time as well as in space, where microscopic fluctuations influence prominently on the whole processes extending from the birth to the death of the quasi-structure. The cluster discussed here behaves like a giant particle composed of many microscopic particles. The internal structure of the cluster has been explored for long time, but no one succeeded to find out any rigid structures in the inside of the cluster. However, our simulations elucidates that the internal structures are clearly understood in terms of the coexistence of two different kinetic laws embedded in a cluster. The self-organization of these two different types of kinetic phases is essential in order that the cluster can survive for long period, and the stability seems to be protected by its own internal mechanisms, which are inherent to the cluster itself. We can say that a cluster should be understood as an entity with active nature, and it is never a passive entity only adapting to the environment.

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