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Self-interacting particles - the quantized blowup mechanism

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1 Introduction

This paper is devoted to the following system of chemotaxis, where $\Omega \subset \mathbf{R}^n$ is a bounded domain with smooth boundary $\partial\Omega$, $a > 0$ is a constant, and ν is the outer unit vector on $\partial\Omega$:

$$\left.\begin{aligned} u_t &= \nabla \cdot (\nabla u - u\nabla v) \\ 0 &= \Delta v - av + u \end{aligned}\right\} \quad \text{in} \quad \Omega \times (0,T)$$

$$\frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0 \quad \text{on} \quad \partial\Omega \times (0,T) \tag{1}$$

$$u|_{t=0} = u_0(x) \quad \text{on} \quad \Omega \tag{2}$$

It is a system proposed by Nagai [14] as a simplified form of the ones given by Keller and Segel [13] and Nanjundiah [16]. Here, $u = u(x,t)$ and $v = v(x,t)$, respectively, stand for the density of cellular slime molds and the concentration of chemical substances secreted by themselves at the position $x \in \Omega$ and the time $t > 0$.

The first equation describes the conservation of the mass, where the flux of u is given by $\mathcal{F} = -\nabla u + u\nabla v$, as

$$\frac{d}{dt}\int_\omega u = -\int_{\partial\omega} \mathcal{F} \cdot \nu$$

holds for any subdomain $\omega \subset\subset \Omega$. Therefore, the effect of diffusion $-\nabla u$ and that of chemotaxis $u\nabla v$ are competing for u to vary. On the other hand, the microscopic derivation of this equation was done by Alt [1] from the biased random walk.

Nanjundiah [16] proposed

$$\left.\begin{aligned} u_t &= \nabla\cdot(\nabla u - \chi u\nabla v)\\ \tau v_t &= \nabla v - \gamma v + \alpha u \end{aligned}\right\} \quad \text{in} \quad \Omega\times(0,T)$$
$$\frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0 \quad \text{on} \quad \partial\Omega\times(0,T)$$
$$u|_{t=0} = u_0(x), \quad v|_{t=0} = v_0(x) \quad \text{in} \quad \Omega, \tag{3}$$

where $u_0 = u_0(x)$, $v_0 = v_0(x)$ are non-negative functions, and $\chi, \gamma, \alpha, \tau$ are positive constants. This system is called the *full system* in this paper. Because the time scales for u and v are different, the constant $\tau > 0$ is usually supposed to be small. Putting $\tau = 0$ gives system (2), as a normal form by the change of variables, that is, the dimensionless procedure.

Other *simplified systems* of parabolic-elliptic equations are proposed by Jäger and Luckhaus [12]:

$$\left.\begin{aligned} u_t &= \nabla\cdot(\nabla u - \chi u\nabla v)\\ 0 &= \Delta v + \alpha\left(u - \tfrac{1}{|\Omega|}\textstyle\int_\Omega u\right) \end{aligned}\right\} \quad \text{in} \quad \Omega\times(0,T)$$
$$\frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0 \quad \text{on} \quad \partial\Omega\times(0,T)$$
$$u|_{t=0} = u_0(x) \quad \text{in} \quad \Omega,$$

Diaz and Nagai [6] (in a modified form):

$$\left.\begin{aligned} u_t &= \nabla\cdot(\nabla u - \chi u\nabla v)\\ 0 &= \Delta v + \alpha u \end{aligned}\right\} \quad \text{in} \quad \Omega\times(0,T)$$
$$\frac{\partial u}{\partial \nu} - \chi u\frac{\partial v}{\partial \nu} = v = 0 \quad \text{on} \quad \partial\Omega\times(0,T)$$
$$u|_{t=0} = u_0(x) \quad \text{on} \quad \Omega,$$

and Senba and Suzuki [21]:

$$\left.\begin{aligned} u_t &= \nabla\cdot(\nabla u - \chi u\nabla v)\\ 0 &= \Delta v - \gamma v + \alpha u \end{aligned}\right\} \quad \text{on} \quad \mathcal{M}\times(0,T)$$
$$u|_{t=0} = u_0(x) \quad \text{on} \quad \mathcal{M},$$

where $\mathcal{M}$ denotes a compact Riemannian surface.

Sometimes the first equation is replaced by

$$u_t = \nabla\cdot(\nabla A(u) - u\nabla\chi(v)) + f(u,v)$$

in order to derive more realistic spatial patterns such as the *streaming*. This case is referred to as the *generalized system*, where $\chi = \chi(v)$ acts as the *sensitive function*. Among many works, let me just refer to Harada, Senba, and Suzuki [8]. It says that if $f(u,v) = 0$, $A(u) = au^2 + u$ with $a > 0$, and $\chi(v) = v$, then the solution exists globally in time at least for $n \leq 7$.

This paper is concentrated on (2). The result stated below is valid to other simplified systems with minor changes. Furthermore, we take the case $n = 2$ only, although Herrero, Madina, and Velázquez [9], [10] obtained interesting families of blowup solutions for $n = 3$. We assume also that the initial value $u|_{t=0} = u_0(x) \geq 0$ is appropriately smooth. Then, we have a unique classical solution $u = u(x,t)$, $v = v(x,t)$ locally in time by the results of Yagi [30] and Biler [4]. Henceforth, $T_{\max} > 0$ denotes its existence time.

Let me recall the follwoing theorem by [20], where $\mathcal{M}(\overline{\Omega})$ denotes the set of measures on $\overline{\Omega}$, $\rightharpoonup$ the $*$-weak convergence there, and

$$m_*(x_0) \equiv \begin{cases} 8\pi & (x_0 \in \Omega) \\ 4\pi & (x_0 \in \partial\Omega)\,. \end{cases}$$

Theorem 1 *If $T_{\max} < +\infty$, then there exists a finite set $\mathcal{S} \subset \overline{\Omega}$ and a non-negative function $f = f(x) \in L^1(\Omega) \cap C(\overline{\Omega} \setminus \mathcal{S})$ such that*

$$u(x,t)dx \quad \rightharpoonup \quad \sum_{x_0 \in \mathcal{S}} m(x_0)\delta_{x_0}(dx) + f(x)dx \qquad \text{in} \quad \mathcal{M}(\overline{\Omega}) \tag{4}$$

holds with

$$m(x_0) \geq m_*(x_0) \qquad (x_0 \in \mathcal{S})\,. \tag{5}$$

We have $\|u(t)\|_\infty \to +\infty$ as $t \uparrow T_{\max} < +\infty$ and $\mathcal{S}$ is actually the blowup set of u. That is, $x_0 \in \mathcal{S}$ if and only if there exist $x_k \to x_0$ and $t_k \uparrow T_{\max}$ such that $u(x_k, t_k) \to +\infty$. Furthermore, we have

$$\|u(t)\|_1 = \|u_0\|_1 \qquad (t \in [0, T_{\max})) \tag{6}$$

and hence

$$2\sharp(\Omega \cap \mathcal{S}) + \sharp(\partial\Omega \cap \mathcal{S}) \leq \|u_0\|_1 / (4\pi) \tag{7}$$

follows from (4) and (5). Here and henceforth, $\|\cdot\|_p$ denotes the standard L^p norm on Ω for $p \in [1, \infty]$. In particular, we get the conclusion that $\|u_0\|_1 < 4\pi$ implies $T_{\max} = +\infty$. The final fact is related to the conjecture

by Childress and Percus [5] concerning the threshold in L^1 norm of the initial value for the blowup of the solution, and is proven independently by Nagai, Senba, and Yoshida [15], Biler [4], Gajewski and Zacharias [7].

On the other hand relation (4) was conjectured by Nanjundiah [16] and is referred to as the formation of *chemotactic collapses*. Inequality (7) indicates that the phenomenon of threshold in $\|u_0\|_1$ concerning the blowup of the solution can be a consequence of the formation of collapses in the blowup process. If equality holds in (5), then it means that the spore is formed with the normalized masses. We may call it the quantized of blowup mechanism. We have got the problem in Senba and Suzuki [19] by the study of stationary solutions. See also Ohtsuka and Suzuki [17]. Now we realize that this problem is related to the accuracy of concentration, or the blowup rate of local norms ([24]). Actually, [23] proved that the mass is quantized if the solution is continued after the blowup time. Along the same line, the mass quantization is proven if the solution blows-up in an infinite time.

In this connection, we have got an important suggestion from the statistical physics. Here will be a good occasion to describe the underlying mathematical structures and physical backgrounds of this problem in order to promote the study of the blowup mechanism. Meanwhile we get the second conjecture that $f \in L\log L(\Omega)$ in (4), where $L\log L$ denotes the Zygmund space of Stein (see Rao and Ren [18]). This is related to the question on the movement of the collapses after the blowup time.

2 Mathematical Structures

Several mathematical structures are known to (2) and some of them are valid to the full system (3). For the moment, we describe them for (3) but they are valid for (2) if the initial value v_0 is taken as $(-\Delta_N + a)^{-1}u_0$ and τ is put to be zero.

First, the positivity of the solution is preserved so that $u_0(x) \geq 0$, $u_0(x) \not\equiv 0$, and $v_0(x) \geq 0$ imply $u(x,t) > 0$ and $v(x,t) > 0$ for $(x,t) \in \overline{\Omega} \times (0, T_{\max})$. This gives the total mass conservation (6) by

$$\frac{d}{dt}\int_\Omega u = \int_\Omega u_t = 0, \tag{8}$$

which follows from the first equation.

A more important feature is the existence of the Lyapunov function

$$W(u,v)=\int_\Omega\left(u\log u-uv+\frac{1}{2}|\nabla v|^2+\frac{a}{2}v^2\right).$$

To see this, for example let us write the first equation of (3) as

$$u_t=\nabla\cdot u\nabla\left(\log u-v\right).$$

Then, in use of the boundary conditions we obtain

$$\int_\Omega u_t\left(\log u-v\right)=-\int_\Omega u\left|\nabla\left(\log u-v\right)\right|^2,$$

where the left-hand side is equal to

$$\frac{d}{dt}\int_\Omega\left(u\log u-uv\right)-\int_\Omega u_t+\int_\Omega uv_t.$$

Here, we have (8) and

$$\int_\Omega uv_t=\int_\Omega\left(\tau v_t-\Delta v+av\right)v_t=\tau\left\|v_t\right\|_2^2+\frac{1}{2}\frac{d}{dt}\left(\left\|\nabla v\right\|_2^2+a\left\|v\right\|_2^2\right).$$

Therefore,

$$\frac{d}{dt}W\left(u,v\right)+\tau\left\|v_t\right\|_2^2+\int_\Omega u\left|\nabla\left(\log u-v\right)\right|^2=0\qquad\left(t\in[0,T_{\max})\right)\tag{9}$$

follows. In particular, $W(u,v)$ is a Lyapunov function and we have

$$W\left(u(t),v(t)\right)\leq W\left(u_0,v_0\right)\qquad\left(t\in[0,T_{\max})\right).$$

The first term of $W(u,v)$, that is $\int_\Omega u\log u$, is related to the Zygmund norm, as we have

$$\left\|w\right\|_{L\log L}\sim\int_\Omega|w|\log\left(e+\frac{|w|}{\left\|w\right\|_1}\right).$$

This relation is shown in Iwaniec and Verde [11]. We note that the Orlicz spaces $L\log L(\Omega)$ and $Exp(\Omega)$ form a duality. Actually, it is regarded as a local version of that between the Hardy space $\mathcal{H}^1$ and the BMO. We can regard the second term of $W(u,v)$, that is $\int_\Omega uv$, as a paring of this duality. This observation is useful, because the third term of $W(u,v)$, that is

$\frac{1}{2}\|\nabla v\|_2^2 + \frac{a}{2}\|v\|_2^2$, is associated with the H^1 norm and we have the inclusion $H^1 \subset BMO$ in the case of two space dimensions. See Suzuki [26] for an application of this observation.

Relation (9) is also useful in the formulation of the stationary problem: $u = u(x)$, $v = v(x)$. Because we are interested in the non-trivial case $u > 0$, it gives that $\log u - v =$ constant on $\overline{\Omega}$. This unknon constant is prescribed by $\|u\|_1 = \lambda$, which is reasonable from relation (6) concerning the non-stationary problem. Consequently, the relation

$$u = \lambda e^v / \int_\Omega e^v$$

is obtained, and thus the stationary problem of (3) arises from the second equation as

$$-\Delta v + av = \lambda e^v / \int_\Omega e^v \quad \text{in} \quad \Omega, \qquad \frac{\partial v}{\partial \nu} = 0 \quad \text{on} \quad \partial\Omega, \tag{10}$$

where $\lambda = \|u_0\|_1$. This is actually the formulation of Childress and Percus [5]. On the other hand, problem (10) has several relatives such as the mean field equation of vortex points, the prescribed Gaussian curvature equation on compact Riemannian manifolds, the limiting equation in the gauge theory of Chern-Simons-Higgs, and so forth. See [17] and the references therein for their details.

The stationary problem (10) has a variational structure. Namely, $v = v(x)$ is a solution if and only if it is a critical value of

$$J_\lambda(v) = \frac{1}{2}\left(\|\nabla v\|_2^2 + a\|v\|_2^2\right) - \lambda \log\left(\int_\Omega e^v\right) \qquad \left(v \in H^1(\Omega)\right),$$

where the Trudinger-Moser inequality takes a fundamental role. Furthermore, the linearized operator around the stationary solution $v = v(x)$ is associated with the bi-linear form

$$\mathcal{A}(\varphi, \varphi) = \int_\Omega \left(|\nabla\varphi|^2 + a\varphi^2 - p\varphi^2\right) + \frac{1}{\lambda}\left\{\int_\Omega p\varphi\right\}^2 \qquad \left(\varphi \in H^1(\Omega)\right),$$

where $p = \lambda e^v / \int_\Omega e^v$. In this way, the methods developed by Suzuki [25], use of the complex variables, spectral analysis combined with the isoperimetric inequalities on surfaces, control of Palais-Smale sequences by Struwe's argument, and so on, are applicable to (10). See [19] and [17] concerning the structure of the solution set obtained in those ways.

Here is a key identity controlling the stability of stationary solutions:

$$W\left(\lambda e^v / \int_\Omega e^v, v\right) = J_\lambda(v) + \lambda \log \lambda$$

For more details, see Suzuki [26] and Senba and Suzuki [23].

Simplified system (2) has one more remarkable structure, which may be referred to as the *compensated compactness via the symmetrization.* In fact, in use of the Green's function $G(x,y)$ for $-\Delta_N + a$ the second equation is converted to

$$v(x,t) = \int_\Omega G(x,y)u(y,t)dy.$$

Then, taking $\psi \in C^2(\overline{\Omega})$ satisyfing $\frac{\partial \psi}{\partial \nu}\Big|_{\partial\Omega} = 0$ as a test function, we get the weak formulation,

$$\begin{aligned}
&\frac{d}{dt}\int_\Omega \psi(x)u(x,t)dx - \int_\Omega \Delta\psi(x)u(x,t)dx \\
&\quad = \int_\Omega u(x,t)\nabla v(x,t)\cdot\nabla\psi(x)dx \\
&\quad = \int\int_{\Omega\times\Omega} \nabla\psi(x)\cdot\nabla_x G(x,y)u(x,t)u(y,t)dxdy \\
&\quad = \frac{1}{2}\int\int_{\Omega\times\Omega} \rho_\psi(x,y)u(x,t)u(y,t)dxdy,
\end{aligned}$$

where

$$\rho_\psi(x,y) = \nabla\psi(x)\cdot\nabla_x G(x,y) + \nabla\psi(y)\cdot\nabla_y G(x,y).$$

If we apply

$$G(x,y) = \frac{1}{2\pi}\log\frac{1}{|x-y|} + K(x,y)$$

with $K \in C^{1,\theta}(\Omega\times\Omega)$, we know that

$$\rho_\psi(x,y) = -\frac{(\nabla\psi(x)-\nabla\psi(y))\cdot(x-y)}{2\pi|x-y|^2} + C^\theta(\Omega\times\Omega),$$

where the first term of the right-hand side is in L^∞ in $\Omega\times\Omega$ although it is not continuous. More delicate analysis is necessary near $\partial\Omega$, but an important consequence of the above expression is that the local L^1 norm of u has a bounded variation in $t \in [0, T_{\max})$. This actually gives the finiteness of blowup points to the simplified system. See [20] for details.

3 Physical Backgrounds

Parabolic - elliptic systems of cross diffusion are found in several areas. Here, we mention two of them, the semi-conductor device equation and vortex formulation of the Navier-Stokes equation. The first one is written as

$$\left.\begin{array}{l} n_t = \nabla \cdot (\nabla n - n\nabla\varphi) \\ p_t = \nabla \cdot (\nabla p + p\nabla\varphi) \\ \Delta\phi = n - p \end{array}\right\} \quad \text{in} \quad \Omega \times (0,T)$$

$$\left.\begin{array}{l} \frac{\partial n}{\partial \nu} - n\frac{\partial \varphi}{\partial \nu} = 0 \\ \frac{\partial p}{\partial \nu} + p\frac{\partial p}{\partial \nu} = 0 \\ \varphi = 0 \end{array}\right\} \quad \text{on} \quad \partial\Omega \times (0,T),$$

where $n = n(x,t)$ and $p = p(x,t)$ are the densities of electron and positron, respectively, and $\varphi = \varphi(x,t)$ is the electric charge field. The case $p = 0$ is easy to treat. Then, we see that the electrons are subject to the self-repulsive force, which makes the system to be dissipative. See Bank [2] for more details.

The second one is given, for example, by

$$\left.\begin{array}{l} \omega_t = \nabla \cdot \left(\nabla\omega - \omega\nabla^{\perp}\psi\right) \\ -\Delta\psi = \omega \end{array}\right\} \quad \text{in} \quad \mathbf{R}^2 \times (0,T),$$

where

$$\nabla^{\perp} = \begin{pmatrix} -\frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_1} \end{pmatrix}$$

for $x = (x_1, x_2)$. It comes from the Navier-Stokes system

$$\left.\begin{array}{c} u_t - \Delta u + u \cdot \nabla u = \nabla p \\ \nabla \cdot u = 0 \end{array}\right\} \quad \text{in} \quad \mathbf{R}^3 \times (0,T),$$

where

$$u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \qquad \text{and} \qquad \nabla = \begin{pmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_3} \end{pmatrix}$$

denote the velocity and the gradient operator, respectively. If we take the two dimensional model with $x = (x_1, x_2, 0)$ and $u_3 = 0$, then we get

$$\nabla \times u = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \qquad \text{for} \qquad \omega = \omega(x_1, x_2).$$

This system is also dissipative but some underlying chaotic features are observed.

Directions of self-interacting forces of those systems, chemotaxis, semiconductor device, and vortices are different, but some common structures are noticed. Let me recall that the principle of thermodynamics is that the mean field of many particles is governed by the free energy in such a way that it always decreases. Its local minimum is the equilibrium state, while transient dynamics are controlled by the critical points, especially, the non-local minima.

We note that the free energy is given by the total energy minus the entropy. If $\rho = \rho(x) \geq 0$ denotes the density of particles, the entropy on the domain $\Omega \subset \mathbf{R}^n$ is given as

$$-\int_\Omega \rho \log \rho.$$

On the other hand, the total energy is composed of the kinetic and the potential energies so that is given as

$$\frac{1}{2}\int\int_{\Omega\times\Omega} K(x,y)\rho(x)\rho(y)dxdy + \int_\Omega \rho V,$$

where $K = K(x,y)$ and $V = V(x)$ denote the potentials of self-interactions and external force, respectively. Note that Newton's third law implies

$$K(x,y) = K(y,x).$$

If the self-interaction is caused by the gravitational force, we have

$$K(x,y) = \begin{cases} \frac{1}{2}\log|x-y| & (n=1) \\ \frac{1}{2\pi}\log|x-y| & (n=2) \\ -\frac{1}{4\pi|x-y|} & (n=3). \end{cases} \tag{11}$$

Thus, we get a physical question: what is the mean field equation of which free energy is given by

$$\mathcal{F}(\rho) = \int_\Omega \rho\log\rho + \frac{1}{2}\int\int_{\Omega\times\Omega} K(x,y)\rho(x)\rho(y)dxdy + \int_\Omega \rho V \quad ?$$

It has been known that such a system is realized by introducing friction and fluctuations of particles. Actually, we have mathematical papers such as Bavaud [3] and Wolanski [28], [29].

Recall that the classical theory starts with the Newton equation

$$\frac{dx_i}{dt} = v_i, \quad m\frac{dv_i}{dt} = -\nabla_{x_i}\left\{V(x_i) + m\sum_{j\neq i} K(x_j, x_i)\right\}$$

for $1 \leq i \leq N$. Now, letting $N \to \infty$ with $M = mN$ preserved, we get the kinetic model, referred to as the Jeans-Vlasov equation. In the normal form, it is given as

$$f_t = -\nabla_x \cdot (vf) + \gamma \nabla_v \cdot [f\nabla_x (U + V)]$$
$$U(x,t) = \int\int G(x,y) f(y,v,t) dv dt$$

Here, making $\gamma \to \infty$ corresponds to $(dv_i)/(dt) \to 0$. This process is called the adiabatic limit and f is supposed to approach the Maxwell distribution. This implies the Euler equation; in the vorticity formulation we have

$$-\Delta\psi = \omega, \qquad \omega_t = -\nabla \cdot \left(\nabla^{\perp}\omega\right).$$

The stationary state of this equation, $\omega = \omega(x)$ is given as the elliptic problem

$$-\Delta\psi = g(\psi)$$

with the nonlinearity g unknown. If the mass is concentrated as

$$\omega = \sum \delta_{x_j(t)}(dx),$$

then it is reduced to the Hamiltonian system

$$\frac{dx_i}{dt} = \nabla^{\perp}_{x_i} H(x_1, x_2, \cdots, x_N) \qquad (i = 1, 2, \cdots, N),$$

where

$$H(x_1, x_2, \cdots, x_N) = \frac{1}{2}\sum_i R(x_i) + \sum_{j\neq i} K(x_i, x_j),$$

with $R(x)$ being the regular part of $K(x,y)$. (We have $R(x) = 0$ if $K(x,y)$ is given as in (11).) However, the Newton equation is time reversible and this hierarchy of systems is not subject to the free energy. This line is governed by at least three laws of conservation, that is, those of mass, momentum, and energy. As a consequence, it has a feature of some chaotic motion of particles.

The answer that we now know is to replace it by the Langevin equation, under the assumption that the N-particles are subject to the friction and random fluctuations:

$$dx_i = v_i dt$$
$$mdv_i = -\nabla_{x_i}\left(V(x_i) + m\sum_{j\neq i} G(x_j, x_i)\right) - \beta v dt + (2\beta kT)^{1/2} dW_t$$

Here, k, T, and β are Boltzmann constant, temperature, friction coefficient, respectively, and W_t denotes the white noise. Its kinetic model, referred to as the Fokker-Planck equation is given as

$$f_t = -\nabla_x \cdot (vf) + \nabla_v \cdot [f\nabla_x (U + \beta V)] + \beta kT\Delta_v f$$
$$U(x,t) = \int\int G(x,y) f(y,v,t) dydv,$$

where

$$\rho(x,t) = \int f(x,v,t)dv \qquad \text{and} \qquad M = \int \rho(x,t)dx$$

stand for the density and the total mass, respectively. Then, in the adiabatic limit, we have

$$\beta\rho_t = \nabla \cdot (\rho\nabla U) + \nabla \cdot (\rho\nabla V) + kT\Delta\rho.$$

It is regarded as a simplified system of chemotaxis.

As we have seen, its stationary state is described by the elliptic problem with the exponential nonlinearity, and finally, we expect that the localized densities are to be subject to a gradient flow. In this way, this hierarchy of equations starts with the free energy as a physical principle, and as we are convinced, is characterized by the quantization of blowup mechanism mathematically.

Let me come back to the problem of mass quantizatioin in (2). First, we have shown in [21] that any collapse is quantized if the post-blowup continuation of the solution is possible. Next, it is known that the Fokker-Planck equation has a weak solution globally in time if the initial value is $L^1 \cap L^\infty$ and has a finite second moment. See Victory, Jr. [27], and so forth. Therefore, as a physical suggestion, it seems that the mass quantization of collapses always holds. To approach the problem, we take the scheme of [27] and construct a family of approximate solutions globally in time. For that

approximate solutions, we can derive some inequality involving the localized second moment. Then, in way of the limiting processes, we can derive some informations. In this way, features of the Fokker-Planck equation and those of its adiabatic limit are rather different, but still share some underlying structures.

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