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Crossed products of Cuntz algebras by quasi-free actions of abelian groups

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1 Introduction

The crossed products of C^*-algebras give us plenty of interesting examples and the structures of them have been examined by several authors. In [KK1] and [KK2], A. Kishimoto and A. Kumjian dealt with, among others, the crossed products of Cuntz algebras by quasi-free actions of the real group $\mathbb{R}$. In [Ka1] and [Ka2], we examined the crossed products of Cuntz algebras by quasi-free actions of arbitrary locally compact, second countable, abelian groups. In this note, we summarize the results of [Ka1] and [Ka2], and discuss several examples.

2 Preliminaries

In this section, we review some basic objects and fix the notation.

For $n = 2, 3, \ldots$, the Cuntz algebra $\mathcal{O}_n$ is the universal C^*-algebra generated by n isometries $S_1, S_2, \ldots, S_n$, satisfying $\sum_{i=1}^{n} S_i S_i^* = 1$ [C1]. In this note, we only consider the case $n < \infty$. For similar results on the crossed products of $\mathcal{O}_\infty$, see [Ka3]. For $k \in \mathbb{N} = \{0, 1, \ldots\}$, we define the set $\mathcal{W}_n^{(k)}$ of k-tuples by $\mathcal{W}_n^{(0)} = \{\emptyset\}$ and

$$\mathcal{W}_n^{(k)} = \big\{(i_1, i_2, \ldots, i_k) \bigm| i_j \in \{1, 2, \ldots, n\}\big\}.$$

We set $\mathcal{W}_n = \bigcup_{k=0}^{\infty} \mathcal{W}_n^{(k)}$. For $\mu = (i_1, i_2, \ldots, i_k) \in \mathcal{W}_n$, we denote its length k by $|\mu|$, and set $S_\mu = S_{i_1} S_{i_2} \cdots S_{i_k} \in \mathcal{O}_n$. Note that $|\emptyset| = 0$, $S_\emptyset = 1$. For $\mu = (i_1, i_2, \ldots, i_k), \nu = (j_1, j_2, \ldots, j_l) \in \mathcal{W}_n$, we define their product $\mu\nu \in \mathcal{W}_n$ by $\mu\nu = (i_1, i_2, \ldots, i_k, j_1, j_2, \ldots, j_l)$.

Let G be a locally compact abelian group which satisfies the second axiom of countability and Γ be the dual group of G. We always use $+$ for multiplicative operations of abelian groups except for $\mathbb{T}$, which is the group of the unit circle in the complex plane $\mathbb{C}$. The pairing of $t \in G$ and $\gamma \in \Gamma$ is denoted by $\langle\, t \,|\, \gamma \,\rangle \in \mathbb{T}$.

Let us take $\omega = (\omega_1, \omega_2, \ldots, \omega_n) \in \Gamma^n$ and fix it. Since the n isometries $\langle\, t \,|\, \omega_1 \,\rangle S_1$, $\langle\, t \,|\, \omega_2 \,\rangle S_2$, $\ldots$, $\langle\, t \,|\, \omega_n \,\rangle S_n$ also satisfy the relation above for any $t \in G$, there is a $*$-automorphism $\alpha_t^\omega : \mathcal{O}_n \to \mathcal{O}_n$ such that $\alpha_t^\omega(S_i) = \langle\, t \,|\, \omega_i \,\rangle S_i$ for $i = 1, 2, \ldots, n$. One can see that $\alpha^\omega : G \ni t \mapsto \alpha_t^\omega \in \mathrm{Aut}(\mathcal{O}_n)$ is a strongly continuous group homomorphism.

Definition 2.1 Let $\omega = (\omega_1, \omega_2, \ldots, \omega_n) \in \Gamma^n$ be given. We define the action $\alpha^\omega : G \curvearrowright \mathcal{O}_n$ by

$$\alpha_t^\omega(S_i) = \langle t \,|\, \omega_i \rangle S_i \quad (i = 1, 2, \ldots, n, \ t \in G).$$

The action $\alpha^\omega : G \curvearrowright \mathcal{O}_n$ becomes quasi-free (for a definition of quasi-free actions on Cuntz algebras, see [E]). Conversely, any quasi-free action of the abelian group G on $\mathcal{O}_n$ is conjugate to α^ω for some $\omega \in \Gamma^n$.

Since the abelian group G is amenable, the reduced crossed product of the action $\alpha^\omega : G \curvearrowright \mathcal{O}_n$ coincides with the full crossed product of it. We denote it by $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ and call it the crossed product. The crossed product $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ has a C^*-subalgebra $\mathbb{C}1 \rtimes_{\alpha^\omega} G$ which is isomorphic to $C_0(\Gamma)$. Throughout this paper, we always consider $C_0(\Gamma)$ as a C^*-subalgebra of $\mathcal{O}_n \rtimes_{\alpha^\omega} G$, and use $f, g, \ldots$ for denoting elements of $C_0(\Gamma) \subset \mathcal{O}_n \rtimes_{\alpha^\omega} G$. The Cuntz algebra $\mathcal{O}_n$ is naturally embedded into the multiplier algebra $M(\mathcal{O}_n \rtimes_{\alpha^\omega} G)$ of $\mathcal{O}_n \rtimes_{\alpha^\omega} G$. For each $\mu = (i_1, i_2, \ldots, i_k)$ in $\mathcal{W}_n$, we define an element ω_μ of Γ by $\omega_\mu = \sum_{j=1}^k \omega_{i_j}$. For $\gamma_0 \in \Gamma$, we define a (reverse) shift automorphism $\sigma_{\gamma_0} : C_0(\Gamma) \to C_0(\Gamma)$ by $(\sigma_{\gamma_0} f)(\gamma) = f(\gamma + \gamma_0)$ for $f \in C_0(\Gamma)$. Once noting that $\alpha_t^\omega(S_\mu) = \langle t \,|\, \omega_\mu \rangle S_\mu$ for $\mu \in \mathcal{W}_n$, one can easily verify that $f S_\mu = S_\mu \sigma_{\omega_\mu} f$ for any $f \in C_0(\Gamma) \subset \mathcal{O}_n \rtimes_{\alpha^\omega} G$ and any $\mu \in \mathcal{W}_n$. From this fact, we have $\mathcal{O}_n \rtimes_{\alpha^\omega} G = \overline{\mathrm{span}}\{S_\mu f S_\nu^* \mid \mu, \nu \in \mathcal{W}_n, \ f \in C_0(\Gamma)\}$, where $\overline{\mathrm{span}}$ means the closure of the linear span.

3 The ideal structure of $\mathcal{O}_n \rtimes_{\alpha^\omega} G$

In [Ka1], we completely determined the ideal structures of the crossed product $\mathcal{O}_n \rtimes_{\alpha^\omega} G$. For an ideal I of the crossed product $\mathcal{O}_n \rtimes_{\alpha^\omega} G$, we define the closed subset X_I of Γ by $I \cap C_0(\Gamma) = C_0(\Gamma \setminus X_I)$. The closed subset X_I satisfies

(i) For any $\gamma \in X_I$ and any $i \in \{1, 2, \ldots, n\}$, we have $\gamma + \omega_i \in X_I$.

(ii) For any $\gamma \in X_I$, there exists $i \in \{1, 2, \ldots, n\}$ such that $\gamma - \omega_i \in X_I$.

The closed subset of Γ satisfying two conditions above is said to be ω-invariant. A closed set X is ω-invariant if and only if $X = \bigcup_{i=1}^n (X + \omega_i)$. For a closed ω-invariant subset X of Γ, we define $I_X \subset \mathcal{O}_n \rtimes_{\alpha^\omega} G$ by

$$I_X = \overline{\mathrm{span}}\{S_\mu f S_\nu^* \mid \mu, \nu \in \mathcal{W}_n, \ f \in C_0(\Gamma \setminus X)\}.$$

One can see that I_X is an ideal of $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ and invariant under the gauge action β of $\mathbb{T}$ on $\mathcal{O}_n \rtimes_{\alpha^\omega} G$, which is defined by $\beta_t(S_\mu f S_\nu^*) = t^{|\mu| - |\nu|} S_\mu f S_\nu^*$ for $\mu, \nu \in \mathcal{W}_n$, $f \in C_0(\Gamma)$ and $t \in \mathbb{T}$. With a technique using conditional expectations, we can prove the following.

Proposition 3.1 ([Ka1, Theorem 3.14]) *The two maps $I \mapsto X_I$ and $X \mapsto I$ between the set of gauge invariant ideals of $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ and the set of closed ω-invariant subsets of Γ are the inverses of each other.*

The ideal structure of $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ depends on whether $\omega \in \Gamma^n$ satisfies the following condition:

Condition 3.2 For each $i \in \{1, 2, \ldots, n\}$, one of the following two conditions is satisfied:

(i) For any positive integer k, $k\omega_i \neq 0$.

(ii) There exists $j \neq i$ such that $-\omega_j$ is in the closed semigroup generated by $\omega_1, \omega_2, \ldots, \omega_n$ and $-\omega_i$.

This condition is an analogue of Condition (II) in the case of Cuntz-Krieger algebras [C2] or Condition (K) in the case of graph algebras [KPRR].

Theorem 3.3 ([Ka1, Theorem 5.2]) *When ω satisfies Condition 3.2, any ideal is gauge invariant. Hence there is a one-to-one correspondence between the set of ideals of $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ and the set of closed ω-invariant subsets of Γ.*

When ω does not satisfy Condition 3.2, there exists $i_0 \in \{1, 2, \ldots, n\}$ such that $k\omega_{i_0} = 0$ for some positive integer k, and that $-\omega_i$ is not in the closed semigroup generated by $\omega_1, \omega_2, \ldots, \omega_n$ and $-\omega_{i_0}$ for any $i \neq i_0$. Note that such i_0 is unique. Let Γ' be the quotient group of Γ by the subgroup generated by ω_1 and denote by $[\gamma]$ the image in Γ' of $\gamma \in \Gamma$. Define a C^*-subalgebra A of $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ by $A = \overline{\mathrm{span}}\{S_{i_0}^k f S_{i_0}^{*\,l} \mid f \in C_0(\Gamma), k, l \in \mathbb{N}\}$. The C^*-algebra A is isomorphic to the Toeplitz algebra of the Hilbert module coming from the automorphism $\sigma_{\omega_{i_0}}$ of $C_0(\Gamma)$, hence there is a surjective map $\pi : A \to C_0(\Gamma) \rtimes_{\sigma_{\omega_{i_0}}} \mathbb{Z}$. It is not hard to see that there is a one-to-one correspondence between the set of ideals of $C_0(\Gamma) \rtimes_{\sigma_{\omega_{i_0}}} \mathbb{Z}$ and the set of closed subset of $\Gamma' \times \mathbb{T}$. For an ideal I of $\mathcal{O}_n \rtimes_{\alpha^\omega} G$, we define the closed subset Y_I of $\Gamma' \times \mathbb{T}$ which corresponds to the ideal $\pi(I \cap A)$. The closed set Y_I satisfies that $([\gamma + \omega_i], \theta') \in Y_I$ for any $i \neq i_0$ any $\theta' \in \mathbb{T}$ and any $([\gamma], \theta) \in Y_I$. Conversely, for any closed set Y of $\Gamma' \times \mathbb{T}$ satisfying the condition above, we can construct the ideal I_Y of $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ so that $Y_{I_Y} = Y$ (see Definition 5.17 and Proposition 5.23 of [Ka1]).

Theorem 3.4 ([Ka1, Theorem 5.49]) *In the above setting, we have $I_{Y_I} = I$ for any ideal I of $\mathcal{O}_n \rtimes_{\alpha^\omega} G$. Thus there is a one-to-one correspondence between the set of ideals of $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ and the set of closed subsets of $\Gamma' \times \mathbb{T}$ satisfying the condition above.*

On the way to prove the two theorems above, we get another proofs of the following known facts (see [Ki] and [OP]):

- $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ is simple if and only if the closed semigroup generated by $\omega_1, \omega_2, \ldots, \omega_n$ and $-\omega_i$ is equal to Γ for any $i = 1, 2, \ldots, n$ [Ka1, Theorem 4.8].

- $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ is primitive if and only if the closed group generated by $\omega_1, \omega_2, \ldots, \omega_n$ is equal to Γ [Ka1, Theorem 4.12],

By Theorem 3.3 and Theorem 3.4, we can show that the strong Connes spectrum $\widetilde{\Gamma}(\alpha^\omega)$ of the action α^ω is the intersection of the n closed semigroups generated by $\omega_1, \omega_2, \ldots, \omega_n$ and $-\omega_i$ where $i = 1, 2, \ldots, n$ [Ka1, Proposition 6.2]. The crossed product $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ is isomorphic to the Cuntz Pimsner algebra of a certain Hilbert bimodule. From this fact, we have the following exact sequence.

$$\begin{array}{ccccc} K_0(C_0(\Gamma)) & \xrightarrow{\mathrm{id}-\sum_{i=1}^{n}(\sigma_{\omega_i})_*} & K_0(C_0(\Gamma)) & \xrightarrow{\iota_*} & K_0(\mathcal{O}_n \rtimes_{\alpha^\omega} G) \\ \uparrow & & & & \downarrow \\ K_1(\mathcal{O}_n \rtimes_{\alpha^\omega} G) & \xleftarrow{\iota_*} & K_1(C_0(\Gamma)) & \xleftarrow{\mathrm{id}-\sum_{i=1}^{n}(\sigma_{\omega_i})_*} & K_1(C_0(\Gamma)) \end{array}$$

where ι is the embedding $\iota : C_0(\Gamma) \hookrightarrow \mathcal{O}_n \rtimes_{\alpha^\omega} G$ [Ka1, Proposition 6.5].

4 AF-embeddability and pure infiniteness of $\mathcal{O}_n \rtimes_{\alpha^\omega} G$

In [Ka2], we gave a sufficient condition for the crossed products $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ to be AF-embeddable. To the best of the author's knowledge, this is the first case to have succeeded in embedding crossed products of purely infinite C^*-algebras into AF-algebras except trivial cases.

Theorem 4.1 ([Ka2, Theorem 3.8]) *If $-\omega_i \notin \overline{\{\omega_\mu \mid \mu \in \mathcal{W}_n\}}$ for any $i = 1, 2, \ldots, n$, then the crossed product $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ is AF-embeddable.*

In [KK1], Kishimoto and Kumjian proved that $\mathcal{O}_n \rtimes_{\alpha^\omega} \mathbb{R}$ becomes stable and projectionless when $\omega \in \mathbb{R}^n$ satisfies $-\omega_i \notin \overline{\{\omega_\mu \mid \mu \in \mathcal{W}_n\}}$. Hence $\mathcal{O}_n \rtimes_{\alpha^\omega} \mathbb{R}$ is stably finite in this case. Theorem 4.1 gives another proof of this fact.

In [KK2], they gave a necessary and sufficient condition that $\mathcal{O}_n \rtimes_{\alpha^\omega} \mathbb{R}$ becomes simple and purely infinite. Here, we generalize their result.

Theorem 4.2 ([Ka2, Corollary 4.9]) *The crossed product $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ is simple and purely infinite if and only if $\Gamma = \overline{\{\omega_\mu \mid \mu \in \mathcal{W}_n\}}$.*

By the two theorems above and the characterization of simplicity, we have the following dichotomy.

Corollary 4.3 ([Ka2, Corollary 4.8]) *The crossed product $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ is either purely infinite or AF-embeddable when it is simple.*

5 Examples

5.1 When G is compact

When G is compact, its dual group Γ becomes discrete. In this case, for any $\omega \in \Gamma^n$ the crossed product $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ is a graph algebra of some skew product graph which is row-finite (see [KP]) and a part of our results here has been already proved in, for example, [BPRS]. Particularly, we have the following.

Proposition 5.1 ([Ka2, Proposition 3.9]) *When G is compact, the following are equiv-*

(i) $-\omega_i \notin \overline{\{\omega_\mu \mid \mu \in \mathcal{W}_n\}}$ *for any* $i = 1, 2, \ldots, n$.

(ii) *The crossed product* $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ *is stably finite.*

(iii) *The crossed product* $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ *is AF-embeddable.*

(iv) *The crossed product* $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ *itself is an AF-algebra.*

5.2 When G is discrete

When G is discrete, its dual group Γ becomes compact. Let us denote by Λ_ω a closed semigroup generated by $\omega_1, \omega_2, \ldots, \omega_n$. One can see that $-\omega_i \in \Lambda_\omega$ for $i = 1, 2, \ldots, n$. Hence any $\omega \in \Gamma^n$ satisfies Condition 3.2. Since the closed set X is ω-invariant if and only if $X + \Lambda_\omega = X$, the set of all closed ω-invariant subsets of Γ is one-to-one correspondent to the set of all closed subset of Γ/Λ_ω. Here note that Λ_ω is a closed subgroup of Γ. By Theorem 3.3, the set of all ideals of $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ corresponds bijectively to the set of all closed subset of Γ/Λ_ω.

We can examine the ideal structures of $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ directly as well as other structures of it. Let G' be the quotient of G by the closed subgroup

$$\begin{aligned}\{t \in G \mid \alpha_t^\omega = \mathrm{id}\} &= \{t \in G \mid \langle t \,|\, \omega_i \rangle = 1 \text{ for } i = 1, 2, \ldots, n\} \\ &= \{t \in G \mid \langle t \,|\, \gamma \rangle = 1 \text{ for any } \gamma \in \Lambda_\omega\}.\end{aligned}$$

The dual group of G' is naturally isomorphic to Λ_ω. Since $\omega \in \Lambda_\omega^n \subset \Gamma^n$, we can define an action $\alpha^\omega : G' \curvearrowright \mathcal{O}_n$. The crossed product $\mathcal{O}_n \rtimes_{\alpha^\omega} G'$ is simple and purely infinite by Theorem 4.2. The crossed product $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ becomes a continuous field over the compact space Γ/Λ_ω whose fiber of any point is isomorphic to $\mathcal{O}_n \rtimes_{\alpha^\omega} G'$. From this observation, we can easily see that the set of all ideals of $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ corresponds bijectively to the set of all closed subset of Γ/Λ_ω.

When G is discrete, the crossed product $\mathcal{O}_n \rtimes_{\alpha^\omega} G$ has an infinite projection, hence is never AF-embeddable.

5.3 When $G = \mathbb{R}^m$

When $G = \mathbb{R}^m$, its dual group Γ is also $\mathbb{R}^m$. For $\omega \in (\mathbb{R}^m)^n$, we define the following.

Definition 5.2 Let $\omega = (\omega_1, \omega_2, \ldots, \omega_n) \in (\mathbb{R}^m)^n$. We denote the affine space generated by $\omega_1, \omega_2, \ldots, \omega_n \in \mathbb{R}^m$ and their convex hull by

$$L_\omega = \left\{ \sum_{i=1}^n t_i \omega_i \in \mathbb{R}^m \;\middle|\; \sum_{i=1}^n t_i = 1 \right\}, \quad C_\omega = \left\{ \sum_{i=1}^n t_i \omega_i \in \mathbb{R}^m \;\middle|\; t_i \geq 0, \sum_{i=1}^n t_i = 1 \right\},$$

respectively. The set C_ω is a closed subset of L_ω. We denote by O_ω the interior of C_ω in L_ω.

We define the three types for elements of $(\mathbb{R}^m)^n$.

Definition 5.3 Let $\omega = (\omega_1, \omega_2, \ldots, \omega_n) \in (\mathbb{R}^m)^n$. The element ω is said to be of type $(+)$ if $0 \notin C_\omega$, to be of type (0) if $0 \in C_\omega \setminus O_\omega$, and to be of type $(-)$ if $0 \in O_\omega$.

On this type, we can prove the following. We omit proofs.

Lemma 5.4 *If ω is of type $(+)$, then there exists $v \in \mathbb{R}^m \setminus \{0\}$ such that the inner product $\omega_i \cdot v$ of ω_i and v is non-negative for any $i = 1, 2, \ldots, n$. Moreover when $m \geq 2$, we can find such v so that there exists i_0 with $\omega_{i_0} \cdot v = 0$.*

Lemma 5.5 *If ω is of type (0), then there exists $v \in \mathbb{R}^m \setminus \{0\}$ such that $\omega_i \cdot v \geq 0$ for any $i = 1, 2, \ldots, n$, and there exists i_0 with $\omega_{i_0} \cdot v = 0$.*

From these two lemmas, we get the following characterizations of type $(-)$ and type $(+)$.

Proposition 5.6 *An element ω is of type $(-)$ if and only if the closed semigroup generated by $\omega_1, \omega_2, \ldots, \omega_n$ is a group. An element ω is of type $(+)$ if and only if $-\omega_i \notin \overline{\{\omega_\mu \mid \mu \in \mathcal{W}_n\}}$ for any $i = 1, 2, \ldots, n$.*

Combining this proposition with Theorem 4.1 and Theorem 4.2, we have the following. An element ω is called aperiodic if the closed group generated by $\omega_1, \omega_2, \ldots, \omega_n$ is $\mathbb{R}^m$.

Proposition 5.7 *The crossed product $\mathcal{O}_n \rtimes_{\alpha^\omega} \mathbb{R}^m$ is AF-embeddable if ω is of type $(+)$. The crossed product $\mathcal{O}_n \rtimes_{\alpha^\omega} \mathbb{R}^m$ is simple and purely infinite if and only if ω is of type $(-)$ and aperiodic.*

It is easy to see that an element ω does not satisfy Condition 3.2 if and only if 0 is an extreme point of C_ω and there is only one $i \in \{1, 2, \ldots, n\}$ with $\omega_i = 0$. In this case, ω is of type (0). The following is a consequence of Lemma 5.4 and Lemma 5.5.

Proposition 5.8 *If ω is of type (0) or if ω is of type $(+)$ and $m \geq 2$, then there exists $i_0 \in \{1, 2, \ldots, n\}$ such that the closed semigroup generated by $\omega_1, \omega_2, \ldots, \omega_n$ and $-\omega_{i_0}$ is not $\mathbb{R}^m$. Hence in this case, the crossed product $\mathcal{O}_n \rtimes_{\alpha^\omega} \mathbb{R}^m$ is not simple.*

The condition for simplicity follows from the proposition above.

Proposition 5.9 *When $m = 1$, the crossed product $\mathcal{O}_n \rtimes_{\alpha^\omega} \mathbb{R}^m$ is simple if and only if ω is of type $(+)$ or $(-)$ and aperiodic.*

When $m \geq 2$, the crossed product $\mathcal{O}_n \rtimes_{\alpha^\omega} \mathbb{R}^m$ is simple if and only if ω is of type $(-)$ and aperiodic.

When $m \geq 2$, the crossed product $\mathcal{O}_n \rtimes_{\alpha^\omega} \mathbb{R}^m$ is purely infinite if it is simple.

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