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Deflation in a Neoclassical Growth Model[1]

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1 Introduction

Recent years have witnessed the revival of theoretical studies into monetary policy and the surprising comeback of the liquidity traps and deflation in academia. In Japan, the average inflation rate was -0.5% in the 1995-2000 period. The study of deflation is no longer a theoretical curiosum; deflation is a real economic issue.

In this paper, I consider deflation in a neoclassical growth model. The cash-in-advance monetary model of Lucas-Stokey (1983) is introduced into an otherwise standard overlapping generations economy with productive capital and government debt. The main purpose of this study is to address the following question: how likely it is to observe deflationary (long-run) equilibrium in a neoclassical growth framework? I find that a *necessary* condition for a deflationary steady state to exist is that the economy grows at a positive rate. In other words, in contrast to the conventional wisdom, an economy with a negative growth rate cannot support deflation in the long run. In addition, a *sufficient* condition for a steady state equilibrium without government deficits to be deflationary is that the nominal interest rate is below the growth rate of the economy.

Recent theoretical studies reveal that how monetary and fiscal policies are conducted is of critical importance for uniqueness, determinacy, and stability of equilibria. Such study of monetary policy in a dynamic general equilibrium framework dates back at least to Sargent and Wallace (1981), who emphasized the importance of monetary and fiscal policy interactions in determining inflation rates. The recent revival of the theory of monetary policy is led by Woodford (1994, 1995, 2001), for example, who pushed Sargent and Wallace's argument further and popularized the fiscal theory of the price level.

Recent theoretical work on monetary policy mainly asks whether a particular monetary-fiscal policy rule introduces unintended instability such as sunspot fluctuations. To the best of my knowledge, there is little theoretical attempt in understanding deflation in the context of dynamic general equilibrium framework. In the textbook Keynesian framework, on the other hand, deflation or disinflation can easily occur when the aggregate demand declines or the aggregate supply goes up. Since such textbook explanation is derived from

[1] I thank the Seimeikai foundation for financial support.

a static, sticky price framework, it is not clear whether deflation that is obtained is a long-run phenomenon. This paper is intended to shed some light on deflationary long-run equilibria in a neoclassical growth model.

It is often documented that the standard neoclassical growth model fails to provide a theoretical framework that is consistent with the conventional wisdom that high inflation rates are associated with low nominal interest rates. In fact, the standard neoclassical growth model implies that an increase in the money growth rate drives interest rates up.[2] The textbook Keynesian IS-LM model, on the other hand, is conformed to the conventional wisdom. This, however, is due to the Keynesian presumption that prices are sticky so that any change in the nominal interest rate is equivalent to a change in the real rate of the same magnitude. The theoretical framework I offer in this paper is a simple neoclassical growth model with flexible prices in which higher nominal interest rates reduce capital and inflation.

For this is an attempt in understanding deflation in a neoclassical growth framework in general, the specific model I adopt here deliberately eliminates unnecessary complications. Thus, I extend Diamond's (1965) neoclassical growth framework to incorporate money and government bonds. In order to model money demand in a simple manner, I adopt the standard cash-in-advance model developed by Lucas and Stokey (1983). Although the cash-in-advance model of money is not commonly used in an overlapping generations framework, it provides a simple yet powerful tool to study monetary policy issues within a neoclassical production economy.

The primary focus of this paper is on deflation as an equilibrium phenomenon. This requires a model in which the money growth rate is endogenous. Thus, I consider an environment in which the monetary authority conducts its policy via nominal interest rate pegging. It is shown that if the government has no budget deficits, then there are in general two steady state equilibria. The steady state with a low capital stock has a low inflation rate and is deflationary if the nominal interest rate is set below the growth rate of the economy. It is shown that such a steady state is a saddle. The other steady state is associated with a high inflation rate, but the real bond holding at that steady state is negative. The steady state is shown to be asymptotically stable.

I extend the model by introducing government deficits. Introduction of such deficits changes the properties of the economy in a few respects. First, there are two steady state equilibria, both of which are dynamically inefficient. Second, the real bond holding at both steady states can be positive if the amount of deficits is sufficiently large. Finally, deflationary steady state is less likely to occur if the government has large deficits.

[2]See, for example, Christiano and Eichenbaum (1992).

The rest of the paper is organized as follows. Section 2 presents a simple budget arithmetic using the steady state government budget constraint. Section 3 describes the model economy. Section 4 characterizes equilibrium conditions under interest rate pegging. Section 5 describes steady state equilibria. Section 6 discusses dynamic properties of the model. Section 7 concludes.

2 Budget Arithmetic Under Deflation

What are the characteristics of an economy that experiences deflation in the long run? In this section I consider implications of deflation for the government's budget constraint. The flow government budget constraint is given as

$$G_t = T_t + B_t - I_t B_{t-1} + M_t - M_{t-1}, \tag{1}$$

where G is the nominal government spending, T is the nominal tax receipt, B is the nominal bonds, M is the nominal money balance, and I is the gross nominal interest rate. (1) states that government expenditures are financed by taxes, bonds, and money. Suppose that the economy grows at the gross rate of $n > 0$. Let p denote the price level. Then one can rewrite (1) as

$$g_t = \tau_t + b_t - \frac{R_t}{n} b_{t-1} + m_t - \frac{1}{\Pi_t}\frac{1}{n} m_{t-1},$$

where g is the real government spending per capita, τ is the real tax per capita, b is the real bonds per capita, m is the real money balance per capita, R is the gross real interest rate, and Π is the gross inflation rate. Suppose that there is a steady state equilibrium in which all per capita real variables are constant over time. Then the steady state government budget constraint is

$$g = \tau + \left(1 - \frac{R}{n}\right) b + \left(1 - \frac{1}{\Pi n}\right) m. \tag{2}$$

Rewrite the government budget constraint, using the Fisher equation, $I = R\Pi$, as

$$g = \tau + \left(1 - \frac{I}{\Pi n}\right) b + \left(1 - \frac{1}{\Pi n}\right) m,$$

so the gross inflation rate is

$$\Pi = \frac{Ib + m}{(b + m + \tau - g)\, n}. \tag{3}$$

From (3), it is easy to show that, *ceteris paribus*, Π decreases as n increases. In addition, it is easy to establish that a steady state is deflationary, or equivalently, $\Pi < 1$ holds if and only if

$$n > \frac{Ib + m}{b + m + \tau - g} \equiv \phi,$$

In practice, governments run positive deficits, so one can safely assume that $g - \tau \geq 0$. Also, the nominal interest rate cannot be negative. Thus, $\phi > 1$ for all m and b. This implies that $n > 1$ must hold for a steady state to be deflationary. This supports the conventional view that the growth of the supply side causes inflation to go down and may even cause deflation. This implies that for a deflationary long-run equilibrium to exist, one must have a model which grows at a positive rate. Throughout, I consider a neoclassical growth model with a positive *exogenous* rate of population growth.

I now let $n > 1$. What are the characteristics of the monetary policy under deflation? Consider once again (3). It implies that a steady state equilibrium is deflationary if and only if

$$(g - \tau) n < (n - I) b + (n - 1) m.$$

In order to obtain a sharp result, assume for now that $g = \tau$. Then, it follows that in any long-run equilibrium with $I > 1$, setting the nominal interest rate at $I < n$ is sufficient for $\Pi < 1$.

To summarize,

Proposition 1 *a) A necessary condition for deflation is that an economy grows at a positive rate. b) At any steady state equilibrium without government deficits, a sufficient condition for deflation is that the nominal interest rate is below the growth rate of the economy.*

An economy that experiences deflation is one in which the nominal interest rate is below the growth rate of the economy. This is consistent with the historical observation that deflation occurs in an economy that is considered as being trapped in a so-called liquidity trap (i.e., the economy that hits the lower bound of the nominal interest rate), such as the US in the 1930's and Japan in the 1990's.

Although the structure presented in this section considers only the government budget constraint at a steady state, the analysis reveals that a necessary condition for a deflationary steady state to exist is that the economy grows at a positive rate. In other words, in contrast to the conventional wisdom, an economy with a negative growth rate cannot support deflation in the long-run. Further, setting the nominal interest rate below the growth rate of the economy is sufficient for deflation in an economy without deficits. In the following sections, I present a simple dynamic general equilibrium model that supports deflation at a steady state and discusses the properties of equilibria with deflation.

3 The Model

3.1 Environment

Consider a growing economy consisting of an infinite sequence of two-period lived overlapping generations, an initial old generation, and an infinitely-lived government. Let $t = 1, 2, \ldots$ index time. At each date t, a new generation comprised of N_t identical members appears where N_t evolves according to $N_{t+1} = nN_t$. I assume that population growth is the only source of (exogenous) growth. Each agent is endowed with one unit of labor when young and is retired when old. In addition, the initial old agents are endowed with $M_0 > 0$ units of fiat currency and $k_1 > 0$ units of capital.

There is a single final good produced using a standard neoclassical production function $F(K_t, L_t)$ where K_t denotes the capital input and L_t denotes the labor input at t. Let $k_t \equiv K_t/L_t$ denote the capital-labor ratio. Then, output per worker at time t may be expressed as $f(k_t)$ where $f(k_t) \equiv F(K_t/L_t, 1)$ is the intensive production function. I assume that $f(0) = 0$, $f' > 0 > f''$, and that Inada conditions hold. The final good can either be consumed in the period it is produced, or it can be stored to yield capital the following period. For reasons of analytical tractability, capital is assumed to depreciate 100% between periods.

3.2 Factor Markets

Factor markets are perfectly competitive. Thus, factors of production receive their marginal product. Let r_{t+1} denote the gross return on capital, and let w_t denote the real wage rate. Young agents supply their labor endowment inelastically in the labor market. Then, firms' profit maximization requires

$$\begin{aligned} r_{t+1} &= f'(k_{t+1}), & (4) \\ w_t &= f(k_t) - k_t f'(k_t) \equiv w(k_t). & (5) \end{aligned}$$

Note that $w'(k) = -kf''(k) > 0$ for all k. It will be useful to introduce a restriction on the production technology. In particular, I assume throughout that $kw'(k)/w(k) < 1$ holds. Cobb-Douglas production function, for example, satisfies this condition.

3.3 Consumers

Let c_{1t} (c_{2t}) denote the consumption of the final good by a young (old) agent born at date t. In order to simplify the analysis as much as possible, I assume that agents care consumption only when old. This immediately follows that $c_{1t} = 0$ for all t so all income will be saved. Following Lucas and Stokey (1983) and more recently Woodford (1994), I assume that consumption goods are divided into two types: "cash goods" and "credit goods." Cash goods must be purchased by cash, so agents wishing to consume cash goods need cash in advance. On the other hand, agents do not need cash to purchase credit goods. Let c_{mt} (c_{nt}) denote the amount of cash (credit) goods consumed when old. Then, $c_{2t} = c_{mt} + c_{nt}$ holds. I assume that the marginal rate of transformation in production is unity between the two goods so the price of the two goods is identical and denoted by P_t. The cash-in-advance constraint is

$$p_{t+1}c_{mt} \leq \frac{M_t}{N_t}, \tag{6}$$

where p_t denotes the time t price level and M_t/N_t denotes the nominal money balance per young. According to (6), a young agent must set aside cash in advance in order to purchase cash goods when old.

It is assumed that agents may hold money and non-monetary assets. The non-monetary assets, denoted by A_t, are assumed to yield the gross nominal return of $I_{t+1} \geq 1$ in the next period. I assume that agents do not have access to any other storage technology. The budget constraint for a young agent born at date t is therefore

$$\frac{M_t}{N_t} + \frac{A_t}{N_t} \leq p_t w_t, \tag{7}$$

where A_t/N_t is the non-monetary asset holding per capita. (7) states that a young agent of generation t receives nominal wage income and allocates all income to monetary and non-monetary assets (because no one consumes when young). Throughout, I consider only symmetric equilibria in which all agents of the same generation have the same amount of assets. Since the nominal interest rate on money is zero, the budget constraint when old is

$$p_{t+1}c_{2t} \leq \frac{M_t}{N_t} + I_{t+1}\frac{A_t}{N_t}. \tag{8}$$

Divide both sides of (7) and (8) by p_t and p_{t+1}, respectively, to obtain

$$m_t + a_t \leq w_t \tag{9}$$

and

$$c_{2t} \leq \frac{p_t}{p_{t+1}}m_t + R_{t+1}a_t, \tag{10}$$

where $m_t \equiv M_t/N_t p_t$, $a_t \equiv A_t/N_t p_t$, and the gross real interest rate satisfies the Fisher equation,

$$I_{t+1} \equiv R_{t+1}\frac{p_{t+1}}{p_t}. \tag{11}$$

Individual rationality implies that (9) and (10) hold at equality. Eliminate a_t from (9) and (10) to obtain the intertemporal budget constraint,

$$\frac{c_{2t}}{R_{t+1}} + \frac{I_{t+1}-1}{I_{t+1}} m_t = w_t. \tag{12}$$

The cash-in-advance constraint binds if and only if money is dominated by non-monetary assets in rates of return, which is equivalent to $I_{t+1} \geq 1$. In other words, the cash-in-advance constraint binds as long as the nominal interest rate is non-negative, which is plausible. Under biding cash-in-advance constraint, (6) holds at equality. Then, (6) and (10) imply that

$$c_{mt} = \frac{p_t}{p_{t+1}} m_t, \tag{13}$$

$$c_{nt} = R_{t+1} a_t \tag{14}$$

must hold in equilibrium.

Following Chari, Christiano, and Kehoe (1991), I specify the utility function as[3]

$$U(c_{mt}, c_{nt}) = \ln c_t, \tag{15}$$

$$c_t \equiv \left[(1-\sigma) c_{mt}^{1-\rho} + \sigma c_{nt}^{1-\rho}\right]^{\frac{1}{1-\rho}}. \tag{16}$$

where $0 < \sigma < 1$ and $0 < \rho < 1$. Each young agent chooses c_{mt} and c_{nt} to maximize (15) subject to (9), (13), and (14). This problem is equivalent to maximizing

$$\ln\left\{\left[(1-\sigma)\left(\frac{p_t}{p_{t+1}} m_t\right)^{1-\rho} + \sigma (R_{t+1} a_t)^{1-\rho}\right]^{\frac{1}{1-\rho}}\right\}$$

with respect to m_t and a_t subject to (9). The first order necessary condition for the maximization problem gives the real money demand function,

$$m_t = \gamma(I_{t+1}) w_t, \tag{17}$$

where

$$\gamma(I) \equiv \left[1 + \left(\frac{\sigma}{1-\sigma}\right)^{\frac{1}{\rho}} I^{\frac{1-\rho}{\rho}}\right]^{-1}. \tag{18}$$

It is important to check the properties of the money demand function just derived.

Lemma 2 $\gamma(I)$ *satisfies (a)* $\gamma'(I) < 0$ *for* $0 < \rho < 1$, *(b)* $\lim_{I\to\infty} \gamma(I) = 0$ *for* $0 < \rho < 1$, *(c)* $0 < \gamma(I) < 1$, *and (d)*

$$\frac{I\gamma'(I)}{\gamma(I)} = -\frac{1-\rho}{\rho}[1-\gamma(I)].$$

[3]According to Chari, Christiano, and Kehoe (1991), $\sigma = 0.57, \rho = 0.17$ for the U.S. economy. Note, however, that the parameter values are for their model economy in which there is an infinitely lived agent, rather than a series of overlapping

Proof. See Kudoh (2001). ∎

Lemma 2 (a) states the condition under which the real money demand is decreasing in the nominal interest rate. As the nominal interest rate increases, the household substitutes away from money, which reduces money demand. An increase in the nominal rates, at the same time, raises earning from bond holding, which raises money demand through income effect. The former dominates the latter if $0 < \rho < 1$, which I assume to hold throughout. In addition, I assume that $(1 - \rho) I < 1$ holds, which is plausible and easily satisfied.

It is important to compare competing models of money demand in a dynamic general equilibrium environment. Schreft and Smith (1997, 2000) develop an environment in which spatial separation and limited communication give rise to the role of banking sector in providing liquidity.[4] As is clear, the money demand function obtained in this paper is virtually identical to the one obtained in Schreft and Smith (1997, 2000). That is, the cash-in-advance model of Lucas-Stokey (1983) and the random relocations model of Schreft-Smith (1997, 1998, 2000) are qualitatively the same. An advantage of the present approach is its simple model environment.

3.4 Monetary and Fiscal Policy Rules

Recent theoretical studies of monetary policy reveal that how fiscal and monetary policies are conducted is of crucial importance for determinacy, multiplicity, and stability of equilibria.[5] In this paper, I consider monetary and fiscal policy rules that are simple yet plausible. In particular, I assume that the fiscal authority sets the sequence of the real primary deficits per capita, and that the monetary authority conducts its policy through targeting the nominal interest rate.

To simplify matters, I let $T_t = 0$ for all t. Then, from (5) the government's flow budget constraint becomes

$$G_t + I_t B_{t-1} = B_t + M_t - M_{t-1} \tag{19}$$

for $t \geq 2$ and $G_1 + M_0 = M_1 + B_1$ for $t = 1$, where the initial stock of bonds is assumed to be zero. I assume that the government simply consumes G_t and that it does not affect utility of any generation or the production process at any date. In order to simplify the analysis, I further assume that $G_t/N_t p_t = g \geq 0$

[4] Wallace (1984) and Bhattacharya and Kudoh (2001) are examples of the model in which money is held by the banking sector just to meet the legal reserve requirement.

[5] The study of monetary-fiscal policy interactions in a dynamic general equilibrium setup dates back at least to Sargent and Wallace (1981), who first linked monetary and fiscal policies by a single government's budget constraint. Examples of recent theoretical work on monetary policy include Leeper (1991), Woodford (1994, 1995), and Schmitt-Grohe and Uribe (2000).

for all t. That is, the real government spending per young is assumed to be constant over time. Then, it is easy to rewrite (19) as

$$g+\frac{R_t}{n}b_{t-1}=m_t-\frac{p_{t-1}}{p_t}\frac{1}{n}m_{t-1}+b_t, \tag{20}$$

where $b_t \equiv B_t/N_t p_t$. The absence of arbitrage opportunity in the capital market requires that capital and bonds yield the same rate of return. That is, $r_{t+1}=R_{t+1}$ holds. Therefore, I let R_{t+1} denote the gross rate of return on capital, bonds and non-monetary asset (a_t) interchangeably.

The primary interest of this paper is deflation as an equilibrium phenomenon. For this reason, the money growth rate and the inflation rate must be endogenous in the model. In order to describe determination of the inflation rate in a simple manner, I assume that the monetary authority conducts its policy via nominal interest rate pegging. That is, I assume that $I_t = I > 1$ for all dates.

4 Equilibrium

This section characterizes equilibrium conditions of the model.

Definition 3 *A monetary equilibrium is a set of sequences for allocations $\{m_t\}$, $\{a_t\}$, $\{k_t\}$, $\{b_t\}$, prices $\{r_t\}$, $\{w_t\}$, $\{p_t\}$, and the initial conditions $M_0 > 0$, $k_1 > 0$, $B_0 = 0$ such that (a) factor markets clear, i.e., (4) and (5) hold, (b) asset market clears: $K_{t+1}+N_t b_t = N_t a_t$, (c) the allocations solve agents' utility maximization problem, (d) the cash-in-advance constraint (6) binds, or equivalently, $I_t > 1$ holds, (e) the government's budget constraints $g+M_0=M_1+B_1$ for $t=1$ and (19) for $t \geq 2$ hold, and (f) $I_t = I$ and $g_t = g$ for all t.*

The money market equilibrium requires that

$$\frac{M_t}{p_t}=\gamma(I)\,w(k_t). \tag{21}$$

The asset market equilibrium requires $k_{t+1}+b_t=a_t=w(k_t)-m_t$, which can be rewritten as

$$k_{t+1}+b_t=[1-\gamma(I)]\,w(k_t). \tag{22}$$

The Fisher equation implies that the gross inflation rate is determined by

$$\frac{p_{t+1}}{p_t}=\frac{I}{f'(k_{t+1})}. \tag{23}$$

Substitute (23) and (21) into (20) to obtain

$$b_t=g+\frac{f'(k_t)}{n}b_{t-1}-\gamma(I)\,w(k_t)+\frac{f'(k_t)}{nI}\gamma(I)\,w(k_{t-1}). \tag{24}$$

(22) and (24) describe the laws of motion for capital and bonds.

5 Steady State Equilibria

5.1 Existence

This section considers steady state equilibria in which the government has no primary deficits, that is, $g = 0$. From (22) and (24), steady state equilibria must satisfy

$$b = [1 - \gamma(I)]\, w(k) - k \equiv \Gamma(k) \tag{25}$$

and

$$b = -\frac{1 - \frac{f'(k)}{nI}}{1 - \frac{f'(k)}{n}} \gamma(I)\, w(k) \equiv H(k). \tag{26}$$

It is important to establish the shape of the function H, which is spelt out below.

Lemma 4 *Let k_g solve $f'(k) = n$ and let k_b solve $f'(k) = nI$. Then, the function H satisfies (a) $H(0) = H(k_b) = 0$, (b) $H(k) > 0$ if and only if $k_b < k < k_g$. (c) $\lim_{k \to k_g} H'(k) = \infty$.*

Proof. Omitted. ■

Given the shape of H, it is easy to show that steady state equilibria, characterized by (25) and (26), are shown in figure 1. Lemma 4, combined with figure 1, implies that there are in general two non-trivial steady state equilibria. One is located in the region $k_b < k < k_g$ and $b > 0$ holds at that steady state. The other one is found in the region $k_g < k$ and b is negative at that steady state.

Note that any steady state solves

$$k = [1 - \gamma(I)]\, w(k) - H(k) \equiv \Omega(k). \tag{27}$$

For future reference, it is important to know some properties of the function Ω.

Lemma 5 *Define $h(I) \equiv 1 - \gamma(I) + \gamma(I)/I > 0$. Then, the function Ω satisfies (a)*

$$\Omega(k) = \left[1 - h(I)\frac{f'(k)}{n}\right] \frac{w(k)}{1 - \frac{f'(k)}{n}},$$

and (b) $\Omega'(k) < 1$ holds at any steady state.

Proof. Scc Kudoh (2001). ■

Lemma 6 *Let k_I solve $f'(k) = I$. Then, any steady state with $k < k_I$ satisfies $\Pi < 1$.*

Proof. From the Fisher equation, it is easy to show that $\Pi = I/f'(k)$. Thus, $\Pi < 1$ holds if and only if $f'(k) > I$. ■

The expression for the steady state inflation rate is obtained from (20) as

$$\Pi = \Pi(k) \equiv \frac{\gamma(I)\, w(k)/n}{\left(1 - \frac{f'(k)}{n}\right) b + \gamma(I)\, w(k)}.$$

Proposition 7 *A necessary condition for* $\Pi < 1$ *at a steady state with* $b > 0$ *is* $n > 1$.

Proof. It is easy to show that

$$\Pi < 1 \Leftrightarrow n > \frac{\gamma(I)\, w(k) + f'(k)\, b}{b + \gamma(I)\, w(k)} \equiv \phi(k, b).$$

$\phi(k, b) > 1$ since $f'(k) > 1$ at a steady state with $b > 0$. ■

Proposition 8 *If* $I < n$ *holds at a steady state equilibrium with* $g = 0$ *and* $b > 0$, *then such a steady state is unique and is deflationary.*

It is easy to see that in the model with $g = 0$ and $I < n$, any non-trivial steady state is considered as being in deflation. In other words, if the monetary authority sets the net nominal interest rate close to zero, then the economy is necessarily deflationary.

Example 9 *Suppose that the production function is* $3k^{0.33}$, *and let* $\sigma = 0.6$, $\rho = 0.2$, $n = 1.03$, $g = 0$, $I = 1.01$. *Then, there are two non-trivial steady states at* $k_l = 0.94$, $k_h = 2.85$. *The associated inflation rates and real bond holdings are, respectively,* $\Pi_l = 0.98$, $\Pi_h = 2.05$, $b_l = 0.80$, $b_h = -0.32$.

The above example computes steady state equilibria when $I < n$. It demonstrates that there are indeed two steady state equilibria and that the low-k steady state is deflationary.

5.2 Comparative Statics

It is now possible to study the effects of a change in I on capital accumulation and inflation.

Lemma 10 $h'(I) < 0$ *holds.*

Proof. See Kudoh (2001). ■

Proposition 11

$$\left.\frac{dk}{dI}\right|_{k=k_l} < 0, \text{ and } \left.\frac{dk}{dI}\right|_{k=k_h} > 0$$

Proof. Totally differentiate (27) to obtain

$$\frac{dk}{dI}=\frac{-h'(I)\frac{f'(k)}{n}w(k)}{\left(1-\frac{f'(k)}{n}\right)(1-\Omega'(k))}.$$

From Lemma 5, $\Omega'(k)-1<0$ holds at any steady state. Further, From lemma 10, $h'(I)<0$ holds. It is therefore easy to establish that $dk/dI<0$ if and only if $f'(k)>n$. The rest of the proof is immediate. ■

Proposition 11 asserts that an increase in the nominal interest rate reduces the capital labor ratio if and only if the economy is dynamically efficient at the steady state. Since k and Π are positively related, an increase in the nominal interest rate reduces the inflation rate at a dynamically efficient steady state. Since any steady state equilibrium with $b>0$ is dynamically efficient in the model without government deficits, it is possible to conclude that a tight money policy through interest rate targeting reduces capital stock and inflation in the long run. An increase in I reduces the real money balance, which, *ceteris paribus*, raises capital stock. At the same time, an increase in I raises the demand for bonds because the return on bonds goes up. At the low-k steady state, the latter effect dominates the former so capital investment is reduced and so is inflation.

The result obtained here is consistent with the conventional wisdom that high nominal interest rates reduce inflation. In fact, the textbook IS-LM model predicts that an increase in the nominal interest rate raises the cost of capital and reduces investment, which has a negative impact on the aggregate income and the inflation rate. Such predictions are based upon the Keynesian presumption that the price level is sticky. It is easy to see that changes in the nominal rates cause one-to-one changes in the real rates under sticky prices. It is well-known, however, that getting such predictions in a neoclassical, flexible price framework is not a trivial matter. As Christiano and Eichenbaum (1992) note, the standard dynamic general equilibrium model predicts in general that the growth rate of money is *positively* related with the nominal interest rate.

Proposition 12 *$dk/dn<0$ holds at any steady state.*

Proof. See Kudoh (2001). ■

Proposition 12 asserts that an increase in the growth rate of the economy reduces the steady state capital-labor ratio and the inflation rate. This result supports the conventional wisdom that the capacity growth causes inflation to go down.

6 Dynamics

This section describes dynamic properties of the model. From (22), $k_t > k_{t-1} \Leftrightarrow$

$$b_{t-1} < [1-\gamma(I)]\, w(k_t) - k_t. \tag{28}$$

From (22) and (24), it is easy to establish that $b_t > b_{t-1} \Leftrightarrow$

$$\frac{f'(k_t)}{n} b_{t-1} - \gamma(I)\, w(k_t) + \frac{f'(k_t)}{n}\frac{\gamma(I)}{I}\frac{k_t + b_{t-1}}{1-\gamma(I)} > b_{t-1},$$

which can be rewritten as

$$\left[h(I)\frac{f'(k_t)}{n} - [1-\gamma(I)]\right] b_{t-1} > [1-\gamma(I)]\,\gamma(I)\, w(k_t) - \frac{\gamma(I)\, k_t f'(k_t)}{nI} \tag{29}$$

Suppose

$$\frac{f'(k_t)}{n} > \frac{1-\gamma(I)}{h(I)} \equiv \Phi(I), \tag{30}$$

where $0 < \Phi(I) < 1$ holds for any $I > 0$. Then $b_t > b_{t-1} \Leftrightarrow$

$$b_{t-1} > \frac{[1-\gamma(I)]\,\gamma(I)\, w(k_t) - \frac{\gamma(I) k_t f'(k_t)}{nI}}{h(I)\frac{f'(k_t)}{n} - [1-\gamma(I)]} \equiv \Lambda(k_t). \tag{31}$$

Figure 2 shows a typical configuration of the phase diagram of the system, where I let k_Φ solve $f'(k) = n\Phi(I)$. According to figure 2, the low-k steady state is a saddle, while the high-k steady state is asymptotically stable.

7 Conclusion

This paper has considered equilibria with deflation in a neoclassical growth model. The cash-in-advance monetary model of Lucas-Stokey (1983) is introduced into a standard overlapping generations economy with productive capital and government debt. Monetary policy is conducted via interest rate targeting so the inflation rate is endogenous. In contract to the standard monetary growth model with an infinitely lived agent, the model developed in this paper predicts that higher nominal interest rates reduce inflation in the long-run. This proves that the model developed in this paper is a reasonable platform for studying monetary and fiscal policy issues.

Simple budget arithmetic reveals that the necessary condition for a long-run equilibrium with deflation to arise is that the economy grows at a positive rate. Further, if the nominal interest rate is set below the growth rate of the economy, then the economy without deficits is deflationary.

Figure 1. Steady state equilibria without deficits.

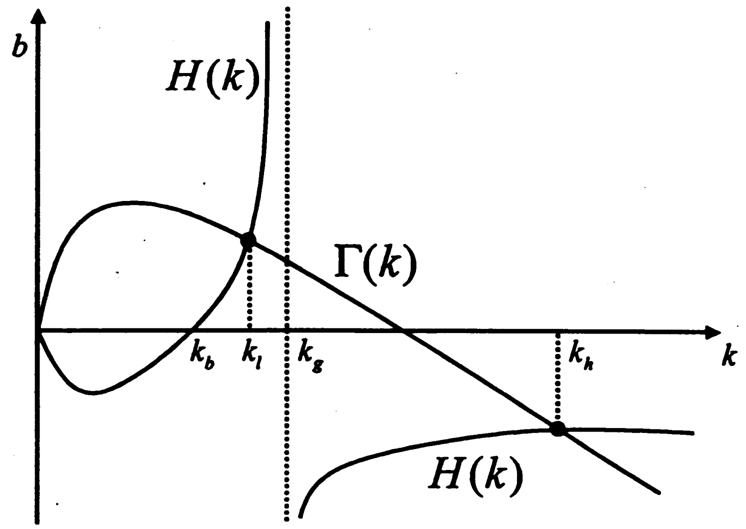
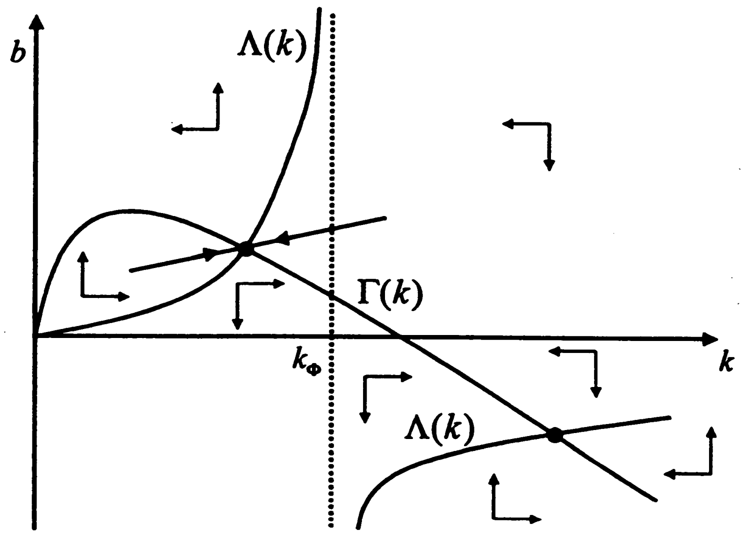


Figure 2. Dynamical equilibria without deficits.



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