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DENSITY OF GEOMETRICALLY FINITE KLEINIAN GROUPS

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1. INTRODUCTION

In 1988, Thurston published a list of unsolved problems on Kleinian groups and hyperbolic 3-manifolds in [19]. One of the problems there asks if every finitely generated Kleinian group is an algebraic limit of geometrically finite ones. This problem can be regarded as a generalization of Bers' density conjecture asserting that every b-group would be a limit of quasi-Fuchsian groups with the same conformal structure on invariant components of the regions of discontinuity. Recently, Bromberg [5], using a brilliant idea involving a deformation theory of geometrically finite cone manifolds, solved most part of Bers' density conjecture. To be more precise, he proved that every totally degenerate b-group without parabolic elements such that the corresponding hyperbolic 3-manifold has a simply-degenerate end whose neighbourhood has unbounded injectivity radii is an algebraic limit of quasi-Fuchsian groups. The case of bounded injectivity radii had been already solved by Minsky [9], [11]. In the present paper, we shall prove Thurston's conjecture above for freely indecomposable Kleinian groups that are not isomorphic to surface groups, using the result of Bromberg, but not his technique.

Theorem 1. *Let G be a finitely generated freely indecomposable Kleinian group without parabolic elements that is not isomorphic to a surface group. Then, there are a geometrically finite Kleinian group Γ such that $\mathbf{H}^3/\Gamma$ is homeomorphic to $\mathbf{H}^3/G$ and its quasi-conformal deformations Γ_i with isomorphisms $\phi_i : \Gamma \to \Gamma_i$ which converge to an isomorphism $\psi : \Gamma \to G$ as representations into $PSL_2\mathbf{C}$.*

We have been informed that Brock and Bromberg [4] also proved this theorem using a similar argument previous to our work. They also dealt with the case of doubly degenerate group. Thus, the only remaining case of Thurston's conjecture for freely indecomposable Kleinian groups is that of groups with parabolic elements. We are also informed that Brock and Bromberg are working on such Kleinian groups.

We should also note that our main theorem follows from the main theorem of [15] if the ending lamination conjecture is solved affirmatively. Minsky has been working on the conjecture (see [9], [10], [11]), and we expect that he will achieve the goal in quite near future.

2. Preliminaries

In this paper, all Kleinian groups are assumed to be finitely generated and torsion free. For a Kleinian group G, we consider the corresponding complete hyperbolic 3-manifold $\mathbf{H}^3/G$. The convex submanifold of $\mathbf{H}^3/G$ that is minimal among the convex deformation retracts is called the convex core of $\mathbf{H}^3/G$, and is denoted by $C(\mathbf{H}^3/G)$. A Kleinian group G is said to be geometrically finite when the convex core $C(\mathbf{H}^3/G)$ has finite volume.

By Scott's theorem [17], for a Kleinian group G, there is a codimension-0 compact submanifold C in $\mathbf{H}^3/G$ such that the inclusion from C to $\mathbf{H}^3/G$ is a homotopy equivalence. The boundary components of C correspond bijectively to the ends of $\mathbf{H}^3/G$ since for each boundary component S of C, there is a unique end e contained in the component of the complement attached to S. We say then that the end e faces the boundary component S, and also that S faces e. An end e of $\mathbf{H}^3/G$ is said to be geometrically finite when there is a neighbourhood of e that contains no closed geodesics. If an end e facing a boundary component S of a compact core is geometrically finite, then e has a neighbourhood homeomorphic to $S \times \mathbf{R}$. The Kleinian group G is geometrically finite if and only if all the ends of $\mathbf{H}^3/G$ are geometrically finite.

A Kleinian group G is said to be freely indecomposable when there is no non-trivial free-product decomposition of G. Bonahon showed in [2] that when G is freely indecomposable, every end of $\mathbf{H}^3/G$ is either geometrically finite or simply degenerate in the following sense. An end e facing an incompressible boundary component S of a compact core is said to be simply degenerate when there is a sequence of simple closed curves $\{\gamma_i\}$ on S such that the closed geodesic γ_i^* homotopic to γ_i in $\mathbf{H}^3/G$ tends to e as $i \to \infty$. It was also proved that if e facing S is simply degenerate, then e has a neighbourhood homeomorphic to $S \times \mathbf{R}$. Regarding simple closed curves $[\gamma_i]$ as projective laminations on S, we can consider their limit, after taking a subsequence, in the projective lamination space, which is compact. Such a limit projective lamination or a measured lamination in that class is said to represent the ending lamination of e. If two projective laminations represent the ending lamination of the same end, then their supports must coincide. Moreover, a measured lamination representing an ending lamination is known to be maximal and connected. Here we say that a measured lamination is maximal when it is not a proper sub-lamination of another measured lamination.

For a Kleinian group G, we denote by $AH(G)$ the set of faithful discrete representations of G into $PSL_2\mathbf{C}$ modulo conjugacy. We endow $AH(G)$ with the quotient topology induced from the space of representations with the topology of point-wise convergence. We represent an element of $AH(G)$ as a pair (Γ, ϕ) where ϕ is a representation representing the element, and Γ is its image in $PSL_2\mathbf{C}$, which is a Kleinian group. An element (Γ, ϕ) in $AH(G)$

is said to be a quasi-conformal deformation of G when there is a quasi-conformal homeomorphism $f : S^2_\infty \to S^2_\infty$ such that $\phi(g) = fgf^{-1}$ for all $g \in G$. The subspace of $AH(G)$ consisting of all quasi-conformal deformations of G is denoted by $QH(G)$. It is known, by work of Ahlfors, Bers, Kra, Maskit and Sullivan, that if G is freely indecomposable, then $QH(G)$ is homeomorphic to the Teichmüller space of Ω_G/G, where Ω_G denotes the region of discontinuity of G on S^2_∞. This correspondence is given by taking $(\Gamma, \phi) \in QH(G)$ to the conformal structure on Ω_G/G induced from the natural conformal structure on Ω_Γ/Γ using a homeomorphism from Ω_G/G to Ω_Γ/Γ given by a quasi-conformal homeomorphism f as above. We denote the inverse of this correspondence by $qc : \mathcal{T}(\Omega_G/G) \to QH(G)$ and call it the Ahlfors-Bers map.

Let S be a closed surface of genus at least 2. Since S admits a complete hyperbolic metric, there is a faithful discrete representation of $\pi_1(S)$ into $PSL_2\mathbf{R}$. The image of such a representation is called a Fuchsian group, and its quasi-conformal deformations, regarded as representations to $PSL_2\mathbf{C}$, are called quasi-Fuchsian groups. A Kleinian group G is quasi-Fuchsian if and only if Ω_G consists of two components both of which are invariant by G. For a Fuchsian group H, as was explained in the last paragraph, the space of quasi-Fuchsian groups modulo conjugacy is homeomorphic to $\mathcal{T}(\Omega_H/H) \cong \mathcal{T}(S) \times \mathcal{T}(\overline{S})$, where $\mathcal{T}(\overline{S})$ denotes the Teichmüller space of the marked conformal structures with orientation reversing markings. The Kleinian group G which is the image of a faithful discrete representation of $\pi_1(S)$ into $PSL_2\mathbf{C}$ is said to be a b-group when Ω_G has only one invariant component. Furthermore, if Ω_G is connected, then G is said to be a totally degenerate b-group. We should note that any b-group without parabolic elements must be totally degenerate. Similarly a Kleinian group G as above is said to be doubly degenerate if Ω_G is empty. If such G without parabolic elements is neither quasi-Fuchsian nor a totally degenerate b-group, then it must be doubly degenerate.

3. Proof of the main theorem

Let C be a compact core of $\mathbf{H}^3/G$. By Bonahon's theorem, each end of $\mathbf{H}^3/G$ is either geometrically finite or simply degenerate. In particular, $\mathbf{H}^3/G$ is homeomorphic to the interior of C. By Thurston's uniformization theorem for atoroidal compact 3-manifolds with boundary (see Thurston [20] and Morgan [12]), there is a geometrically finite Kleinian group without parabolic elements Γ such that $\mathbf{H}^3/\Gamma$ is homeomorphic to $\mathbf{H}^3/G$. Let $\psi : \Gamma \to G$ be an isomorphism induced by a homeomorphism as above, and regard (G, ψ) as an element of $AH(\Gamma)$. We are to define a sequence of quasi-conformal deformations $\{(\Gamma_i, \phi_i)\}$, which will be proved to converge algebraically to (G, ψ) eventually.

Let $S_1, \ldots, S_k$ be the boundary components of C, which are incompressible since G is freely indecomposable. We number them so that $S_1, \ldots, S_\kappa$

among them face simply-degenerate ends, whereas the rest face geometrically finite ends. We can assume that $\kappa \geq 1$, for otherwise G itself is geometrically finite. Consider a component S_j with $j \leq \kappa$, and let e^{S_j} be the end of $\mathbf{H}^3/G$ facing S_j.

Take a subgroup G^{S_j} of G corresponding to the subgroup $\pi_1(S_j)$ of $\pi_1(C) \cong G$.

Lemma 2. *For $j \leq \kappa$, the Kleinian group G^{S_j} is a totally degenerate b-group.*

Proof. Since we assumed that G has no parabolic elements, G^{S_j} has no parabolic elements either. Therefore, G^{S_j} is either quasi-Fuchsian or a totally degenerate b-group or a doubly degenerate group. Since there is a neighbourhood of the end e^{S_j} that can be lifted homeomorphically to $\mathbf{H}^3/G^{S_j}$, which we denote by $\tilde{e}^{S_j}$, the manifold $\mathbf{H}^3/G^{S_j}$ must have at least one simply-degenerate end; hence G^{S_j} cannot be quasi-Fuchsian.

Suppose now, seeking a contradiction, that G^{S_j} is doubly degenerate. Then $\mathbf{H}^3/G^{S_j}$ has a simply-degenerate end $\tilde{e}'$ other than $\tilde{e}^{S_j}$. Let $p^{S_j} : \mathbf{H}^3/G^{S_j} \to \mathbf{H}^3/G$ be the covering projection associated to the inclusion. By the covering theorem due to Thurston (see also Canary [6]), there is a neighbourhood E' of $\tilde{e}'$ such that $p^{S_j}|E'$ is proper and a covering map to its image since $\mathbf{H}^3/G$ is not a surface bundle over S^1. Also, it is impossible that E' covers a neighbourhood of e^{S_j} since the ending laminations for $\tilde{e}'$ and for $\tilde{e}^{S_j}$ differ. It follows that $\mathbf{H}^3/G$ has a simply-degenerate end e' distinct from e^{S_j}, whose neighbourhood is covered by E'. Let S' be the boundary component of C facing e'. Then e' has a neighbourhood homeomorphic to $S' \times I$. It follows that pleated surfaces $f_i : S \to \mathbf{H}^3/G$ tending to e', which can be obtained by projecting pleated surfaces in $\mathbf{H}^3/G^{S_j}$ tending to $\tilde{e}'$, are homotopic to a finite-sheeted covering by S'. Therefore, S' must be homotopic to a finite-sheeted covering of S. This is impossible, as can be seen by elementary 3-dimensional topology, since we assumed that C is not homeomorphic to $S \times I$. □

Thus, we have shown that G^{S_j} is a totally degenerate b-group. By the main theorem of Bromberg [5], there is a sequence of quasi-Fuchsian groups H_i with isomorphisms $\rho_i : \pi_1(S_j) \to H_i$ converging to the isomorphism $\iota : \pi_1(S_j) \to G^{S_j}$ induced by the inclusion of S_j to $\mathbf{H}^3/G$. Let (n_i^j, m^j) be a point in $\mathcal{T}(S_j) \times \mathcal{T}(\overline{S}_j)$ corresponding to (H_i, ρ_i) by the Ahlfors-Bers map. Here, $\{n_i^j\}$ converges in the Thurston compactification of the Teichmüller space to a projective lamination $[\lambda_j]$ representing the ending lamination of e^{S_j}, and m^j is constant with respect to i.

Recall that we have a parametrization of the quasi-conformal deformations of Γ by $qc : \mathcal{T}(S_1) \times \cdots \times \mathcal{T}(S_k) \to QH(\Gamma)$ since Ω_Γ/Γ is homeomorphic to ∂C by a homeomorphism inducing the isomorphism ψ from $\Gamma \cong \pi_1((\mathbf{H}^3 \cup \Omega_\Gamma)/\Gamma)$ to $G \cong \pi_1(C)$. We numbered $S_1, \ldots, S_k$ so that

$S_{\kappa+1}, \ldots, S_k$ face geometrically finite ends. The conformal structure at infinity of $\mathbf{H}^3/G$ facing them, i.e., that of Ω_G/G, determines points $n^j \in \mathcal{T}(S_j)$ for $j = \kappa+1, \ldots, k$. We define a sequence of quasi-conformal deformations $\{(\Gamma_i, \phi_i)\}$ by setting $(\Gamma_i, \phi_i) = qc(n_i^1, \ldots, n_i^\kappa, n^{\kappa+1}, \ldots, n^k)$.

Lemma 3. *The sequence $\{(\Gamma_i, \phi_i)\}$ converges in $AH(\Gamma)$, after passing through a subsequence if necessary.*

Proof. This lemma follows from Theorem 3.5 and Lemma 3.6 of Ohshika [13]. We have only to verify that our sequence $\{(\Gamma_i, \phi_i)\}$ satisfies the assumption of Theorem 3.5 using Lemma 3.5 there. Let A be an essential annulus in $\overline{\mathbf{H}^3/\Gamma} = (\mathbf{H}^3 \cup \Omega_\Gamma)/\Gamma$. Then, each component of ∂A is contained either in a component of Ω_Γ/Γ corresponding to S_j with $j > \kappa$ on which the conformal structure is constant with respect to i, or in a component corresponding to S_j with $j \leq \kappa$, hence intersects essentially the limit projective lamination to which $\{n_i^j\}$ converges since the lamination is maximal and connected. By Lemma 3.6 in [13], in the latter case, there exists a sequence of measured laminations $\{\lambda_i\}$ on $\partial\overline{\mathbf{H}^3/\Gamma}$ with bounded $\text{length}_{n_i^j}(\lambda_i)$, whose limit measured lamination intersects A essentially. Therefore, we can apply Theorem 3.5 in [13] to the quasi-conformal deformations Γ_i, and see that $\{(\Gamma_i, \phi_i)\}$ has a convergent subsequence. □

Let $(G', \psi') \in AH(\Gamma)$ be an algebraic limit of (a subsequence) of $\{(\Gamma_i, \phi_i)\}$.

Lemma 4. *The limit group G' has no parabolic elements. Let $\Psi' : \mathbf{H}^3/\Gamma \to \mathbf{H}^3/G'$ be a homotopy equivalence inducing the isomorphism $\psi' : \Gamma \to G'$. Then, Ψ' is homotopic to a homeomorphism from $\mathbf{H}^3/\Gamma$ to $\mathbf{H}^3/G'$.*

Proof. Let C' be a compact core of $\mathbf{H}^3/G'$. We shall prove that $\Psi'|C$ is homotopic to a homeomorphism to C'. This is sufficient to prove the second sentence of our lemma since the inclusions of both C and C' are homotopic to homeomorphisms.

Consider a component S_j of ∂C. If $j \leq \kappa$, then $\{n_i^j\}$ converges to the projective lamination $[\lambda_j]$ as $i \to \infty$. By the continuity of the length function (see Thurston [21], Ohshika [16], and Brock [3]) together with Theorem 2.2 in Thurston [21], whose proof can be found in [23], as was shown in the proof of Theorem 8 in Ohshika [15], there is a simply-degenerate end of $\mathbf{H}^3/G'$ whose ending lamination is represented by a measured lamination homotopic to $\Psi'(\lambda_j)$, and there is a boundary component of C' homotopic to $\Psi'(S_j)$.

On the other hand, by Lemma 3 in Abikoff [1], for each S_j with $j > \kappa$, there is a geometrically finite end of $\mathbf{H}^3/G'$ facing a boundary component of C' which is homotopic to $\Psi'(S_j)$. Combining these, we can apply Corollary 13.7 in Hempel [8] (originally due to Waldhausen [24]) to conclude that $\Psi'|C$ is homotopic to a homeomorphism to C'. Moreover, since each boundary component of C' faces a unique end as above, there is no room for a simple closed curve representing a parabolic element of G', which must reside on the boundary of C'. □

We shall next show that the limit is a quasi-conformal deformation of G.

Proposition 5. *The limit group (G', ψ') is a quasi-conformal deformation of G.*

To prove this proposition, we consider first subgroups corresponding to the boundary components of ∂C. Let Γ^{S_j} be a subgroup of Γ corresponding to G^{S_j} by $\psi : \Gamma \to G$. Then, Γ_i contains a quasi-conformal deformation $(\Gamma_i^{S_j}, \phi_i^{S_j})$ of Γ^{S_j} converging as $i \to \infty$ to a subgroup of (G', ψ'), which we denote by (G'^{S_j}, ψ'_j).

Lemma 6. *For $j \leq \kappa$, the limit group G'^{S_j} is a quasi-conformal deformation of G^{S_j}.*

Proof. There is a pair $(n'_i, r'_i) \in \mathcal{T}(S_j) \times \mathcal{T}(\overline{S_j})$ such that $qc(n'_i, r'_i) = (\Gamma_i^{S_j}, \phi_i^{S_j})$, where we regard the first factor as corresponding to the component of $\Omega_{\Gamma^{S_j}}/\Gamma^{S_j}$ that is a homeomorphic lift of a component of Ω_Γ/Γ. Since n'_i here is equal to n_i^j, which we defined before, $\{n'_i\}$ converges to the projective lamination $[\lambda_j]$ in the Thurston compactification of the Teichmüller space $\mathcal{T}(S_j)$. We shall show that $\{r'_i\}$ stays in a compact set of $\mathcal{T}(S_j)$. Suppose not. Then, there is a projective lamination $[\mu_j]$ to which a subsequence of $\{r'_i\}$ converges in the Thurston compactification. Since λ_j represents an ending lamination, it is maximal and connected on S_j. Hence, by the main theorem of Ohshika [16], if $i(\mu_j, \lambda_j) = 0$, then $\{(\Gamma_i^{S_j}, \phi_i^{S_j})\}$ has no convergent subsequences. Therefore, we have $i(\mu_j, \lambda_j) > 0$.

On the other hand, if μ_j is either not maximal or disconnected, each boundary component of the minimal hyperbolic subsurface of S_j containing a component of μ_j represents a parabolic element of G'^{S_j}. (This fact is originally due to Thurston [22]. See also Lemma 4.1 in [13].) Since G'^{S_j}, which is a subgroup of G', has no parabolic elements, this is impossible. Hence μ_j must also be maximal and connected.

By Theorem 2.2 in Thurston [21] (and a remark there), there is a sequence of measured laminations ν_k^j converging to μ_j as $k \to \infty$ such that $\text{length}_{r'_i}(\nu_i^j) \to 0$ as $i \to \infty$. It follows from Sullivan's theorem proved by Epstein-Marden [7] or Proposition 2.1 in [14] that the length of the realization of ν_i^j in $\mathbf{H}^3/\Gamma_i$ by a pleated surface inducing the isomorphism $\phi_i^{S_j}$ also goes to 0 as $i \to \infty$. By the continuity of the length function, we see that μ_j represents an ending lamination of $\mathbf{H}^3/G'^{S_j}$. Since $i(\lambda_j, \mu_j) > 0$, the end with ending lamination represented by μ_j is distinct from the one with ending lamination represented by λ_j. Therefore, $\mathbf{H}^3/G^{S_j}$ has two simply-degenerate ends. As before, using the covering theorem, we get a contradiction. Thus we have proved that $\{r'_i\}$ stays in a compact set of $\mathcal{T}(S_j)$.

By taking a subsequence, we can assume that $\{r'_i\}$ converges to a point $r'_\infty \in \mathcal{T}(S_j)$ as $i \to \infty$. Therefore $qc(n'_i, r'_i)$ is a K_i-quasi-conformal deformation of $qc(n^j_i, m^j)$ with bounded K_i. (Recall that $n'_i = n^j_i$.) This implies that its limit (G'^{S_j}, ψ'_j) is a quasi-conformal deformation of $(G^{S_j}, \psi|\Gamma^{S_j})$. □

Proof of Proposition 5. The lemma above implies that a neighbourhood of the end of $\mathbf{H}^3/G'$ facing $\Psi'(S_j)$ is quasi-isometric to a neighbourhood of the end e^{S_j} by a homeomorphism in the right homotopy class. Since this holds for each S_j facing a simply-degenerate end, this means that there is a quasi-isometry in the right homotopy class from $\mathbf{H}^3/G$ to $\mathbf{H}^3/G'$; hence (G', ψ') is a quasi-conformal deformation of (G, ψ). □

Let $f : S^2_\infty \to S^2_\infty$ be a quasi-conformal homeomorphism such that $f\psi f^{-1} = \psi'$. Since the conformal structures of $\Omega_{G'}/G'$ corresponding to the geometrically finite ends are the same as those on Ω_G/G by our definition of Γ_i, this map f is taken to be conformal on Ω_G. By Sullivan's rigidity theorem ([18]), this implies that f is in fact a conformal homeomorphism; hence $(G, \psi) = (G', \psi')$. Thus, we have proved that (G, ψ) is also a limit of geometrically finite groups (Γ_i, ϕ_i), and completed the proof of Theorem 1.

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