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RENORMALIZATION IN COMPLEX DYNAMICS

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The renormalization has been one of important tools and objectives in the study of (low-dimensional) dynamical systems, since it was introduced by Feigenbaum and Coullet-Tresser. Thier goal was to explain the universality in the bifurcation phenomena of families of unimodal mappings on the interval. For this purpose, they defined a renormalization "operator" (instead of group) on the space of unimodal maps and hypothesized the existence of its fixed point and the hyperbolicity of its derivative. This was proved by Lanford in 1982 by a computer-assisted proof. In 1980's, there were works towards a *non* computer-assisted proof, and this created a new movement in the study of low-dimensional dynamics.

In this talk, we discuss the relationship between the renormalization and the problem of *rigidity*. The rigidity means that with a certain class of mathematical objects, a weak equivalence automatically implies a stronger equivalence. For example, in the case of Feigenbaum-Coullet-Tresser renormalization, Lanford's theorem implies that two Feigenbaum renoramalaizable maps with certain smoothness are smoothly (C^1) conjugate on their limit Cantor sets. There are various questions related to the rigidity of real or complex one-dimensional dynamical systems.

The main result we discuss will be

Theorem. *Let f and g be polynomial-like mappings with the same renormalization type which is not satellite type. Then they are quasiconformally conjugate outside the renormalizing Yoccoz puzzle piece and the quasiconformal dilatation depends only on the combinatorial type of the renoramalization and the moduli of the fundamental annuli of f and g.*

Moreover if both f and g are real, the dilatation depends only on the moduli of the fundamental annuli.

Applying this theorem to the seuqence of renormalizations, we obtain a new proof of the following:

Theorem. *Hyperbolic maps are dense among real quadratic polynomials.*