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A Fuzzy Stopping Problem with the Concept of Perception

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Abstract

In this paper, we try the perceptive analysis of the optimal stopping problem in which both fuzzy and probabilistic uncertainty is accommodated. We establish the recursive equation for computing the expected value of the optimal stopped fuzzy perception reward. Also, a numerical example is given.

Keywords : Fuzzy stopping problem, fuzzy perception, fuzzy random variable, fuzzy perception reward, optimal stopping time.

1. Introduction and notation

Recently Zadeh [12] described an outline of the perception-based theory for probabilistic reasoning. This is a problem in the fields of the management of uncertainty and the decision-making in the knowledge-based systems. Traditionally these discussions have been the groundwork for enlargement of the application fields. In a discussion to widen the applicability of stochastic modeling in the real-world, for example, the stock market forecasting, the study of precise probabilities enhances the analysis of the modeling.

The usual stopping problem in a stochastic environment is described by real random variables. However, in practice, we often face with configuration of variables together with firmness of perception or measurement precision. Previously we have presented an evaluation method with a linear ranking function for fuzzy stochastic processes [5, 10]. Here, we alternatively discuss fuzzy expectation by way of the perception theory concerning probabilistic reasoning.

One of possible ways to handle such a case is the fuzzy set theory [11], where the grades of membership functions mean the perception values of the prices. Motivated by the above example, we try a perceptive analysis on stopping problems in which fuzzy perception is accommodated. We also investigate a method of computing the fuzzy perception reward when the processes are stopped optimally. As an typical example, the classical problem of selling an asset (cf. [4]) is extended to the case that the price of asset may be not observed exactly and linguistically and roughly perceived.

In the remainder of this section, we will give some notations and the definition of a fuzzy perception function referring to Baswell and Taylor[1], by which a perceptive stopping problem is formulated in the sequel.

For any set A, the fuzzy set on A will be denoted by its membership function $\widetilde{a} : A \to [0,1]$. The α-cut of $\widetilde{a}$ is given by $\widetilde{a}_\alpha := \{x \in A \mid \widetilde{a}(x) \geq \alpha\} (\alpha \in (0,1])$ and $\widetilde{a}_0 := \mathrm{cl}\{x \in A \mid \widetilde{a}(x) > 0\}$, where $\mathrm{cl}\{B\}$ is the closure of a set B. For the theory of fuzzy sets, we refer to Zadeh [11] and Dubois and Prade [3].

Let $\mathbb{R}$ be the set of all real numbers and $\widetilde{\mathbb{R}}$ the set of all fuzzy numbers, i.e., $\widetilde{r} \in \widetilde{\mathbb{R}}$ means that $\widetilde{r} : \mathbb{R} \to [0,1]$ is normal, upper-semicontinuous and fuzzy convex and has a compact support.

Let $\mathbb{C}$ be the set of all bounded and closed intervals of $\mathbb{R}$. Then, obviously for any $\widetilde{r} \in \widetilde{\mathbb{R}}$, it holds that $\widetilde{r}_\alpha \in \mathbb{C}$ $(\alpha \in [0,1])$. So, we write $\widetilde{r}_\alpha = [\widetilde{r}_\alpha^-, \widetilde{r}_\alpha^+]$ $(\alpha \in [0,1])$.

A partial order relation $\preccurlyeq$ on $\widetilde{\mathbb{R}}$, called the fuzzy max order (cf. [8]), is defined as follows: For $\widetilde{s}, \widetilde{r} \in \widetilde{\mathbb{R}}$, $\widetilde{s} \preccurlyeq \widetilde{r}$, if $\widetilde{s}_\alpha^- \leq \widetilde{r}_\alpha^-$ and $\widetilde{s}_\alpha^+ \leq \widetilde{r}_\alpha^+$ for all $\alpha \in [0,1]$, where $\widetilde{s}_\alpha = [\widetilde{s}_\alpha^-, \widetilde{s}_\alpha^+]$ and $\widetilde{r}_\alpha = [\widetilde{r}_\alpha^-, \widetilde{r}_\alpha^+]$.

Here, we define $\widetilde{\max}\{\widetilde{s}, \widetilde{r}\} \in \widetilde{\mathbb{R}}$ by

$$(1.1) \qquad \widetilde{\max}\{\widetilde{s}, \widetilde{r}\}(y) := \sup_{\substack{x_1, x_2 \in \mathbb{R} \\ y = x_1 \vee x_2}} \{\widetilde{s}(x_1) \wedge \widetilde{r}(x_2)\} \quad (y \in \mathbb{R}),$$

where $a \wedge b = \min\{a, b\}$ and $a \vee b = \max\{a, b\}$ for any $a, b \in \mathbb{R}$.

Then, it is well-known (cf. [8]) that $\widetilde{s} \preccurlyeq \widetilde{r}$ if and only if $\widetilde{r} = \widetilde{\max}\{\widetilde{s}, \widetilde{r}\}$.

Let $(\Omega, \mathcal{M}, P)$ be a probability space. A map $\widetilde{X} : \Omega \to \widetilde{\mathbb{R}}$ is called a fuzzy perception function if for each $\alpha \in [0,1]$ the maps $\Omega \ni \omega \mapsto \widetilde{X}_\alpha^-(\omega)$ and $\Omega \ni \omega \mapsto \widetilde{X}_\alpha^+(\omega)$ are $\mathcal{M}$-measurable for all $\alpha \in [0,1]$, where $\widetilde{X}_\alpha(\omega) = [\widetilde{X}_\alpha^-(\omega), \widetilde{X}_\alpha^+(\omega)] := \{x \in \mathbb{R} \mid \widetilde{X}(\omega)(x) \geq \alpha\}$.

Let $\mathscr{X}$ be the set of all integrable random variables on $(\Omega, \mathcal{M}, P)$. For any fuzzy perception function $\widetilde{X}$, the expectation $E\widetilde{X} \in \widetilde{\mathbb{R}}$ is defined by

$$(1.2) \qquad E\widetilde{X}(x) = \sup_{\substack{X \in \mathscr{X} \\ EX = x}} \widetilde{\mu}(\widetilde{X})(X),$$

where $\widetilde{\mu}(\widetilde{X})$ is a fuzzy set of $\mathscr{X}$ and defined by

$$(1.3) \qquad \widetilde{\mu}(\widetilde{X})(X) = \inf_{\omega \in \Omega} \widetilde{X}(\omega)(X(\omega)) \quad \text{for all } X \in \mathscr{X}.$$

Obviously, we have

$$(1.4) \qquad E(\widetilde{X})_\alpha = [\int \widetilde{X}_\alpha^-(\omega) dP(\omega), \int \widetilde{X}_\alpha^-(\omega) dP(\omega)], \quad (\alpha \in [0,1]).$$

Note that a fuzzy set $\widetilde{\mu}(\widetilde{X})$ on $\mathscr{X}$ is called fuzzy random variable induced by $\widetilde{X}$ (cf. [1]). Regarding the another (equivalent) definition of fuzzy random variables, we refer to H. Kwakernaak [6] and Puri and Ralescu [7]. In this paper, the definition of fuzzy random variables from a perceptive stand point by Boswell and Taylor [1] is adopted for modeling a fuzzy perceptive stopping problem.

2. Stopped fuzzy perception rewards

Let $\mathscr{X}^n$ be the set of all n-dimensional row vectors whose elements are in $\mathscr{X}$, i.e.,

$$\mathscr{X}^n = \{\boldsymbol{X} = (X_1, X_2, \ldots, X_n) \mid X_t \in \mathscr{X}, t = 1, 2, \ldots, n\}.$$

The stopping time $\sigma : \Omega \to \mathbb{N}_n := \{1, 2, \ldots, n\}$ is said to be corresponding to $\boldsymbol{X} = (X_1, X_2, \ldots, X_n) \in \mathscr{X}^n$ if $\{\sigma = k\} \in \mathcal{B}(\boldsymbol{X}_k)$ $(k = 1, 2, \ldots, n)$ where $\boldsymbol{X}_k = (X_1, X_2, \ldots, X_k)$ and $\mathcal{B}(\boldsymbol{X}_k)$ is the σ-field on Ω generated by the random vector $\boldsymbol{X}_k$. The set of such stopping times will be denoted by $\Sigma\{\boldsymbol{X}\}$.

The map δ on $\mathscr{X}^n$ with $\delta(\boldsymbol{X}) \in \Sigma\{\boldsymbol{X}\}$ for all $\boldsymbol{X} \in \mathscr{X}^n$ is called a stopping time function. A stopping time function δ is called monotone if for any $\boldsymbol{X} = (X_1, X_2, \ldots, X_n), \boldsymbol{Y} = (Y_1, Y_2, \ldots, Y_n) \in \mathscr{X}^n$ with $\boldsymbol{X} \leq \boldsymbol{Y}$, i.e., $X_t \leq Y_t$ $(t = 1, 2, \ldots, n)$ P-a.s., it holds that $E\boldsymbol{X}_\delta \leq E\boldsymbol{Y}_\delta$, where $\boldsymbol{X}_\delta := X_{\delta(\boldsymbol{X})}$ and $\boldsymbol{Y}_\delta := Y_{\delta(\boldsymbol{Y})}$.

For any $\boldsymbol{X} = (X_1, X_2, \ldots, X_n), \boldsymbol{Y} = (Y_1, Y_2, \ldots, Y_n) \in \mathscr{X}^n$ and $\beta \in [0, 1]$, let $\boldsymbol{Z} := \beta\boldsymbol{X} + (1-\beta)\boldsymbol{Y} = (\beta X_1 + (1-\beta)Y_1, \beta X_2 + (1-\beta)Y_2, \ldots, \beta X_n + (1-\beta)Y_n) \in \mathscr{X}^n$. Then δ is called convex if $E\boldsymbol{Z}_\delta \leq \beta E\boldsymbol{X}_\delta + (1-\beta)E\boldsymbol{Y}_\delta$ for all $\beta \in [0, 1]$, where $\boldsymbol{Z} = (Z_1, Z_2, \ldots, Z_n)$ and $\boldsymbol{Z}_\delta := Z_{\delta(\boldsymbol{Z})}$. The set of all monotone and convex stopping time functions will be denoted by Δ.

Let $\widetilde{\boldsymbol{X}} = (\widetilde{X}_1, \widetilde{X}_2, \ldots, \widetilde{X}_n)$ be a sequence of fuzzy perception functions. For any $\delta \in \Delta$, the δ-stopped fuzzy perception reward $\widetilde{\boldsymbol{X}}_\delta$ is defined by,

$$\widetilde{\boldsymbol{X}}_\delta(\omega)(x) := \sup_{\substack{\boldsymbol{X}_\delta(\omega)=x \\ \boldsymbol{X}=(X_1,\ldots,X_n)\in\mathscr{X}^n}} \{\widetilde{X}_1(\omega)(X_1(\omega)) \wedge \ldots \wedge \widetilde{X}_n(\omega)(X_n(\omega))\}. \tag{2.1}$$

Note that $\widetilde{\boldsymbol{X}}_\delta(\omega)(x)$ is a fuzzy set on $\mathbb{R}$ but not necessarily a fuzzy perception function.

Similarly as (1.2), we define the expected value of $\widetilde{\boldsymbol{X}}_\delta(\omega)(x)$ by

$$E\widetilde{\boldsymbol{X}}_\delta(x) := \sup_{\substack{E(X)=x \\ X\in\mathscr{X}}} \inf_{\omega\in\Omega} \{\widetilde{\boldsymbol{X}}_\delta(\omega)(X(\omega))\}. \tag{2.2}$$

Let, for each $\alpha \in [0, 1]$, $\widetilde{\boldsymbol{X}}_\alpha^- := (\widetilde{X}_{1,\alpha}^-, \ldots, \widetilde{X}_{n,\alpha}^-) \in \mathscr{X}^n$ and $\widetilde{\boldsymbol{X}}_\alpha^+ := (\widetilde{X}_{1,\alpha}^+, \ldots, \widetilde{X}_{n,\alpha}^+) \in \mathscr{X}^n$, where the α-cut of $\widetilde{X}_k$ is described by $\widetilde{X}_{k,\alpha} = [\widetilde{X}_{k,\alpha}^-, \widetilde{X}_{k,\alpha}^+]$.

Then, we have the following.

Theorem 2.1 *For any $\delta \in \Delta$, it holds that*
(i) $E\widetilde{\boldsymbol{X}}_\delta \in \widetilde{\mathbb{R}}$ *and*
(ii) $(E\widetilde{\boldsymbol{X}}_\delta)_\alpha = [E((\widetilde{\boldsymbol{X}}_\alpha^-)_\delta), E((\widetilde{\boldsymbol{X}}_\alpha^+)_\delta)]$ *for* $\alpha \in [0, 1]$.

For the proof of Theorem 2.1, we need the several preliminary lemmas.

Here, we put, for each $\alpha \in [0, 1]$,

$$Z^\alpha(\beta) := \beta\widetilde{\boldsymbol{X}}_\alpha^+ + (1-\beta)\widetilde{\boldsymbol{X}}_\alpha^- \quad (\beta \in [0, 1]). \tag{2.3}$$

Lemma 2.1 *For any $\delta \in \Delta$, $E(Z^\alpha(\beta)_\delta)$ is continuous with respect to $\beta \in [0, 1]$.*

Proof. For any β, β' with $0 \leq \beta < \beta' < 1$,

$$Z^\alpha(\beta') = \frac{\beta'-\beta}{1-\beta}\widetilde{\boldsymbol{X}}_\alpha^+ + \Big(1 - \frac{\beta'-\beta}{1-\beta}\Big)Z^\alpha(\beta).$$

So, from the monotonicity and convexity of $\delta \in \Delta$, we have for $0 \leq \beta < \beta' < 1$,

$$\begin{aligned} E(Z^\alpha(\beta)_\delta) &\leq E(Z^\alpha(\beta')_\delta) \\ &\leq \frac{\beta'-\beta}{1-\beta}E((\widetilde{\boldsymbol{X}}_\alpha^+)_\delta) + \Big(1 - \frac{\beta'-\beta}{1-\beta}\Big)E(Z^\alpha(\beta)_\delta), \end{aligned}$$

which implies that $\lim_{\beta' \downarrow \beta} E(Z^\alpha(\beta')_\delta) = E(Z^\alpha(\beta)_\delta)$.

Similarly, we have for $0 \leq \beta'' < \beta < 1$,

$$E(Z^\alpha(\beta)_\delta) \leq \frac{\beta - \beta''}{1 - \beta''} E((\widetilde{\boldsymbol{X}}_\alpha^+)_\delta) + (1 - \frac{\beta - \beta''}{1 - \beta''}) E(Z^\alpha(\beta'')_\delta).$$

Thus, it holds that

$$\begin{aligned} 0 &\leq E(Z^\alpha(\beta)_\delta) - E(Z^\alpha(\beta'')_\delta) \leq \frac{\beta - \beta''}{1 - \beta''}(E((\widetilde{\boldsymbol{X}}_\alpha^+)_\delta) - E(Z^\alpha(\beta'')_\delta)) \\ &\leq \frac{\beta - \beta''}{1 - \beta''}(E((\widetilde{\boldsymbol{X}}_\alpha^+)_\delta) - E((\widetilde{\boldsymbol{X}}_\alpha^-)_\delta)). \end{aligned}$$

Thus we get $\lim_{\beta'' \uparrow \beta} E(Z^\alpha(\beta'')_\delta) = E(Z^\alpha(\beta)_\delta)$. □

The following lemma follows easily from (2.1) and (2.2).

Lemma 2.2 *For any $\delta \in \Delta$ and $\alpha \in [0, 1]$, it holds that*

$$\begin{aligned} (E\widetilde{\boldsymbol{X}}_\delta)_\alpha &= \{E\boldsymbol{X}_\delta \mid \boldsymbol{X} = (X_1, X_2, \ldots, X_n) \in \mathscr{X}^n, \\ & \quad X_t(\omega) \in [\widetilde{X}_{t,\alpha}^-(\omega), \widetilde{X}_{t,\alpha}^+(\omega)] \quad \text{for } t = 1, 2, \ldots, n\}. \end{aligned}$$

The proof of Theorem 2.1. Since (ii) means (i), it suffices to show that (ii) holds. By Lemma 2.2 and monotonicity of δ, the inclusion $\subset$ of (ii) is immediate. Also, the inclusion $\supset$ follows from the observation that $Z^\alpha(1) = \widetilde{\boldsymbol{X}}_\alpha^+$, $Z^\alpha(0) = \widetilde{\boldsymbol{X}}_\alpha^-$ and Lemma 2.1. □

By Theorem 2.1, we observe that $E\widetilde{\boldsymbol{X}}_\delta \in \widetilde{\mathbb{R}}$ for all $\delta \in \Delta$. Here we can specify the perceptive fuzzy stopping problem investigated in the next section: The problem is to maximize $E\widetilde{\boldsymbol{X}}_\delta$ for all $\delta \in \Delta$ with respect to the fuzzy max order $\preccurlyeq$ on $\widetilde{\mathbb{R}}$.

3. Perceptive optimization and recursive equations

In this section, for any given sequence of fuzzy perception functions $\widetilde{\boldsymbol{X}} = (\widetilde{X}_1, \widetilde{X}_2, \ldots, \widetilde{X}_n)$, we find the optimal stopping time function δ^* and to characterize the optimal fuzzy perception value $E\widetilde{\boldsymbol{X}}_{\delta^*}$.

For each sequence of random variables $\boldsymbol{X} = (X_1, X_2, \ldots, X_n) \in \mathscr{X}^n$, we denote by $\delta^*(\boldsymbol{X})$ the optimal stopping time for $\boldsymbol{X}$ (cf. [2]), which is thought as a stopping time function.

Lemma 3.1 $\delta^* \in \Delta$.

Proof. For $\boldsymbol{X} = (X_1, X_2, \ldots, X_n) \in \mathscr{X}^n$, involving $\boldsymbol{X}$, we define the sequence $\{\gamma_k^n = \gamma_k^n(\boldsymbol{X})\}_{k=1}^n$ by

(3.1) $\quad \gamma_n^n(\boldsymbol{X}) = X_n, \quad \gamma_k^n(\boldsymbol{X}) = \max\{X_k, E[\gamma_{k+1}^n \mid \mathcal{B}(\boldsymbol{X}_k)]\} \quad (k = n-1, \ldots, 1),$

where $\boldsymbol{X}_k = (X_1, X_2, \dots, X_k)$. Then, by the usual theory of optimal stopping problems (cf. [2]), we have $E(\boldsymbol{X}_{\delta^*}) = E\gamma_1^n(\boldsymbol{X})$.

Let $\boldsymbol{X} = (X_1, X_2, \dots, X_n), \boldsymbol{Y} = (Y_1, Y_2, \dots, Y_n) \in \mathscr{X}^n$ with $X_t \leq Y_t$ $(t = 1, 2, \dots, n)$ P-a.s.. Then, by induction on k, we can easily prove that $\gamma_k^n(\boldsymbol{X}) \leq \gamma_k^n(\boldsymbol{X})$ for $k = n, n-1, \dots, 1$. Thus, we get

$$E(\boldsymbol{X}_{\delta^*}) = E(\gamma_1^n(\boldsymbol{X})) \leq E(\gamma_1^n(\boldsymbol{X})) = E(\boldsymbol{Y}_{\delta^*}),$$

which shows the monotonicity of δ^*.

For $\boldsymbol{Z} = \beta\boldsymbol{X} + (1-\beta)\boldsymbol{Y}$ $(\beta \in [0,1])$, we have

$$\begin{aligned} E[Z_{\delta^*(\boldsymbol{Z})}] &= \beta E[X_{\delta^*(\boldsymbol{Z})}] + (1-\beta)E[Y_{\delta^*(\boldsymbol{Z})}] \\ &\leq \beta E[X_{\delta^*(\boldsymbol{X})}] + (1-\beta)E[Y_{\delta^*(\boldsymbol{Y})}], \end{aligned}$$

where $\boldsymbol{Z} = (Z_1, Z_2, \dots, Z_n)$. This shows the convexity of δ^*. $\square$

By Lemma 3.1, we observe that δ^* is an optimal stopping time function.

For simplicity, we assume the sequence of perception functions $\widetilde{\boldsymbol{X}} = (\widetilde{X}_1, \widetilde{X}_2, \dots, \widetilde{X}_n)$ is independent with each $\widetilde{X}_t$ $(t = 1, 2, \dots, n)$. Then, in the following theorem it will be shown that the optimal fuzzy perception value $E\widetilde{\boldsymbol{X}}_{\delta^*}$ is given by the backward recursive equation:

$$\widetilde{\gamma}_n^n = E\widetilde{X}_n, \quad \widetilde{\gamma}_k^n = E\widetilde{\max}\{\widetilde{X}_k, \widetilde{\gamma}_{k+1}^n\} \ (k = n-1, \dots, 2, 1). \tag{3.2}$$

Let the α-cut of $\widetilde{\gamma}_k^n$ in (3.2) be

$$\widetilde{\gamma}_{k,\alpha}^n = [\widetilde{\gamma}_{k,\alpha}^{n,-}, \widetilde{\gamma}_{k,\alpha}^{n,+}] \quad (k = 1, 2, \dots, n).$$

Then, the α-cut expression of (3.2) is as follows:

$$\begin{aligned} &\widetilde{\gamma}_{n,\alpha}^{n,-} = E(\widetilde{X}_{n,\alpha}^-), \quad \widetilde{\gamma}_{n,\alpha}^{n,+} = E(\widetilde{X}_{n,\alpha}^+) \\ &\widetilde{\gamma}_k^{n,-} = E\max\{\widetilde{X}_{k,\alpha}^-, \widetilde{\gamma}_{(k+1),\alpha}^{n,-}\}, \quad \widetilde{\gamma}_k^{n,+} = E\max\{\widetilde{X}_{k,\alpha}^+, \widetilde{\gamma}_{(k+1),\alpha}^{n,+}\} \\ &(\alpha \in [0,1], \quad k = n-1, n-2, \dots, 1). \end{aligned} \tag{3.3}$$

Theorem 3.1 $E\widetilde{\boldsymbol{X}}_{\delta^*} = \widetilde{\gamma}_n^1$.

Proof. By (3.2) and (3.3), we have that, for $\alpha \in [0,1]$,

$$\begin{aligned} \widetilde{\gamma}_{k,\alpha}^n &= [E\max\{\widetilde{X}_{k,\alpha}^-, \gamma_{(k+1),\alpha}^{n,-}\}, E\max\{\widetilde{X}_{k,\alpha}^+, \gamma_{(k+1),\alpha}^{n,+}\}] \\ &= [E\max\{\widetilde{X}_{k,\alpha}^-, \widetilde{\gamma}_{(k+1)}^n(\widetilde{\boldsymbol{X}}_\alpha^-)\}, E\max\{\widetilde{X}_{k,\alpha}^+, \widetilde{\gamma}_{(k+1)}^n(\widetilde{\boldsymbol{X}}_\alpha^+)\}], \end{aligned}$$

where $\gamma_{(k+1)}^n(\widetilde{\boldsymbol{X}}_\alpha^-)$ and $\gamma_{(k+1)}^n(\widetilde{\boldsymbol{X}}_\alpha^+)$ are defined in (3.1). Applying Theorem 2.1, we get $(E\widetilde{\boldsymbol{X}}_{\delta^*})_\alpha = (\widetilde{\gamma}_n^1)_\alpha$. Thus, $E\widetilde{\boldsymbol{X}}_{\delta^*} = \widetilde{\gamma}_n^1$, as required. $\square$

4. A numerical example

In this section, we will compute the optimal fuzzy perception value for the perception stopping problem described by simple triangular fuzzy numbers.

The triangular fuzzy number (a, m, b) with $a > 0$ and $b > 0$ is given by

$$(a,m,b)(x) = \begin{cases} \max\{(x-m+a)/a, 0\} & \text{if } x \le m \\ \max\{(x-m-b)/b, 0\} & \text{if } x > m. \end{cases}$$

Obviously, the α-cut of (a, m, b) is

$$(a,m,b)_\alpha = [m - a(1-\alpha), m + b(1-\alpha)] \quad \alpha \in [0,1].$$

Let $\widetilde{\boldsymbol{X}} = (\widetilde{X}_1, \widetilde{X}_2, \ldots, \widetilde{X}_n)$ be independent and identically distributed sequence of fuzzy perception functions with $\widetilde{X}_t = (Y_t, X_t, Z_t)$ $(t = 1, 2, \ldots, n)$. (See Fig.1). We assume that $X_t \sim U[0,1]$ and $Y_t,\ Z_t \sim U[0, 1/2]$ $(t = 1, 2, \ldots, n)$, where $X \sim U[a, b]$ $(a < b)$ means that the distribution of X is a uniform distribution on $[a, b]$.

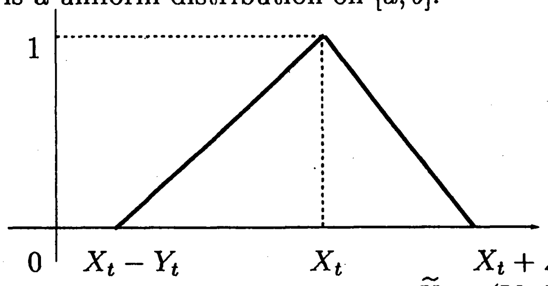

Figure 1. The fuzzy perception $\widetilde{X}_t = (Y_t, X_t, Z_t)$

The optimal fuzzy perception value $E\widetilde{\boldsymbol{X}}_{\delta^*} = \widetilde{\gamma}_1^n$ is computed recursively by (3.3), which is given as follows.

$$\begin{aligned} &\widetilde{\gamma}_{n,\alpha}^{n,-} = \tfrac{1+\alpha}{2}, \quad \widetilde{\gamma}_{n,\alpha}^{n,+} = \tfrac{3-\alpha}{2} \\ &\widetilde{\gamma}_{k,\alpha}^{n,-} = E\max\{X_k - (1-\alpha)Y_k, \widetilde{\gamma}_{(k+1),\alpha}^{n,-}\} \\ &\widetilde{\gamma}_{k,\alpha}^{n,+} = E\max\{X_k + (1-\alpha)Z_k, \widetilde{\gamma}_{(k+1),\alpha}^{n,+}\} \\ &\qquad (\alpha \in [0,1],\ k = n-1, n-2, \ldots, 1). \end{aligned}$$

Fig.2 gives the $\widetilde{\gamma}_1^n$ $(n = 1, 5, 20)$, by which we observe that $\widetilde{\gamma}_1^{20}$ is concave on its left-side slope and convex on its right-side slope.

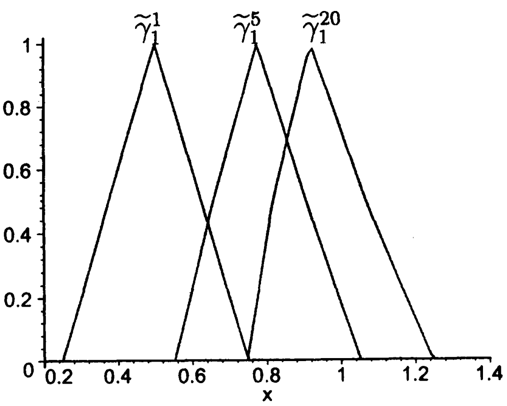

Figure 2. The graph of $\widetilde{\gamma}_1^n$ $(n = 1, 5, 20)$

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