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共鳴管内の音響流の分岐について

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Abstract. The large amplitude standing wave excited in a resonator induces acoustic streaming of Rayleigh type outside the acoustic boundary layer on the wall of the resonator. For the case that the resonator is a two-dimensional rectangular box, the streaming motion with large Reynolds number is examined numerically. The two-dimensional incompressible Navier–Stokes equations with no external force are used as the governing equations for the streaming velocity, which is defined by a time-averaged mass flux density vector. The steady velocity component at the outer edge of the acoustic boundary layer, which induces Rayleigh type streaming, is employed as the boundary condition for the Navier–Stokes equations. By using a finite-difference method, we shall show the existence of multiple steady solutions.

INTRODUCTION

Steady streaming induced in acoustic standing wave fields is a classical topic in physics [1,2]. An active control of streaming in resonators becomes an important problem in some applications today (e.g., [3]). Some authors have recently carried out accurate measurements for slow streaming motions [4,5]. However, the behavior in the case of large Reynolds number remains unresolved.

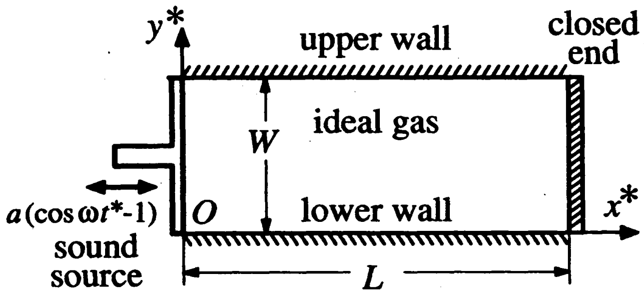

FIGURE 1. The schematic of the model.

Recently, one of the present authors has numerically studied the resonant gas oscillation with a periodic shock wave in a closed tube by solving the system of compressible Navier–Stokes equations [6]. The result has suggested the occurrence of turbulent acoustic streaming when a streaming Reynolds number is sufficiently large. However, the direct numerical simulation of viscous compressible flow is an extraordinarily hard task if one tries to resolve all phenomena from an initial state of uniform and at rest to an almost steady oscillation state throughout the flow field including the boundary layer. In the present paper, we shall adopt a simple model based on the linear standing wave solution and boundary layer theory, and thereby execute a number of numerical simulations to clarify how the classical symmetric streaming pattern changes into different flow patterns as the streaming Reynolds number increases.

FORMULATION

We shall consider the streaming motion induced by resonant gas oscillations in a two-dimensional rectangular box filled with an ideal gas (see Fig. 1). The box, whose length is L and width is W, is closed at one end by a solid plate and the other by a piston (sound source) oscillating harmonically with an amplitude a and angular frequency ω.

In the case that the excitation at the sound source is moderately weak and the dissipation effect is sufficiently small outside the boundary layer, the wave motion in the body of the fluid is a plane standing wave accompanied with small correction terms due to nonlinear and dissipation effects,

$$u = M\frac{\sin(x-b)}{\sin b}\sin t + \cdots, \quad \rho = 1 + M\frac{\cos(x-b)}{\sin b}\cos t + \cdots, \tag{1}$$

where $x = x^*\omega/c_0$ is a normalized axial coordinate, $t = \omega t^*$ is a normalized time, $u = u^*/c_0$ is the x component of normalized fluid velocity, $\rho = \rho^*/\rho_0$ is a normalized density, $M = a\omega/c_0$ ($\ll 1$) is the acoustic Mach number at the sound source, and $b = L\omega/c_0$ is a normalized box length or a normalized angular frequency (c_0 is the speed of sound in an initial undisturbed state of density ρ_0).

We shall assume that the oscillation is near the second mode resonance, i.e.,

$$b = 2\pi + \sqrt{M}\Delta, \tag{2}$$

where Δ is a nondimensional parameter which measures the detuning. If $|\Delta| = O(\sqrt{M})$, then the oscillation includes two periodical shock waves as long as the dissipation effect is sufficiently small. The formation of shock waves may induce the turbulent streaming motion as shown in [6]. In what follows we assume $\Delta \approx 1$. Equation (1) can then be rewritten into

$$u = \sqrt{M}\frac{1}{\Delta}\sin x \sin t + \cdots, \quad \rho = 1 + \sqrt{M}\frac{1}{\Delta}\cos x \cos t + \cdots. \tag{3}$$

From the standard boundary layer analysis, we have the so-called limiting velocity at the outer edge of the boundary layer,

$$\overline{u} = -U_0 \sin 2x, \quad \overline{v} = 0, \quad U_0 = \frac{M}{\Delta^2}\left[\frac{3}{8} + \frac{\sqrt{\mathrm{Pr}}(\gamma-1)}{4(\mathrm{Pr}+1)}\right], \tag{4}$$

where the bar denotes a time average, γ the ratio of specific heats, and Pr the Prandtl

Acoustic streaming velocity is usually defined as a time-averaged mass flux density vector. We introduce a streaming velocity normalized by U_0

$$U = \frac{1}{2\pi}\int_t^{t+2\pi} \frac{\rho u}{U_0}\,\mathrm{d}t, \quad V = \frac{1}{2\pi}\int_t^{t+2\pi} \frac{\rho v}{U_0}\,\mathrm{d}t. \tag{5}$$

Then, the governing equations for the streaming outside the boundary layer are

$$\left.\begin{aligned} &\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0, \qquad \frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}+\frac{\partial p}{\partial x}=\frac{1}{\mathrm{Re}}\Big(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\Big),\\ &\qquad \frac{\partial V}{\partial \tau}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}+\frac{\partial p}{\partial y}=\frac{1}{\mathrm{Re}}\Big(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\Big), \end{aligned}\right\} \tag{6}$$

where a slow scale time τ and the streaming Reynolds number Re are defined as

$$\tau = U_0 t, \quad \mathrm{Re} = \frac{M}{\epsilon^2\Delta^2}\left[\frac{3}{8}+\frac{\sqrt{\mathrm{Pr}}(\gamma-1)}{4(\mathrm{Pr}+1)}\right] \quad \left(\epsilon = \frac{\sqrt{\nu_0\omega}}{c_0}\right). \tag{7}$$

In the definition of Re, ϵ represents the ratio of acoustic boundary layer thickness to a typical wavelength. In the present study, we assume

$$\epsilon^2 \ll M \ll 1 \quad \text{and} \quad \epsilon \ll w = \frac{W\omega}{c_0}, \tag{8}$$

where W is the width of the box. Condition (8) supports Eq. (1) and enables large Reynolds number flows. We shall emphasize that the momentum equations in (6) have no driving force terms because the sound wave is unattenuated; this is also supported by condition (8).

NUMERICAL ANALYSIS

We shall numerically solve the incompressible Navier–Stokes equations in a two-dimensional box with the boundary condition (see Fig. 2),

$$U = -\sin 2x, \; V = 0 \text{ at } y = 0 \text{ and } y = w, \quad U = 0, \; V = 0 \text{ at } x = 0 \text{ and } x = 2\pi. \tag{9}$$

The solution method is the standard projection (MAC) method with accuracy of first order in time and second order in space. A regular even-spaced 512×128 grid points are mainly used. The time step is changed from 5×10^{-4} to 7×10^{-5}. The normalized box width w is fixed to $2\pi/5$.

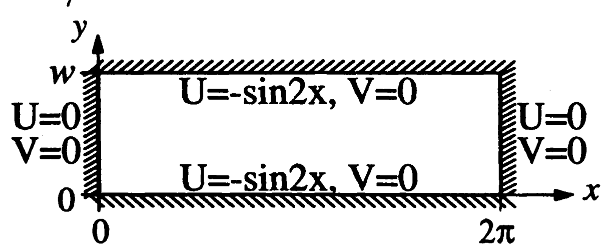

FIGURE 2. The 2D box for simulation.

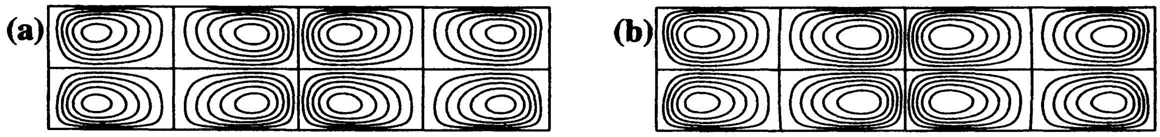

FIGURE 3. Streamlines for the classical symmetric flows. (a): Re = 100, (b): Re = 230.

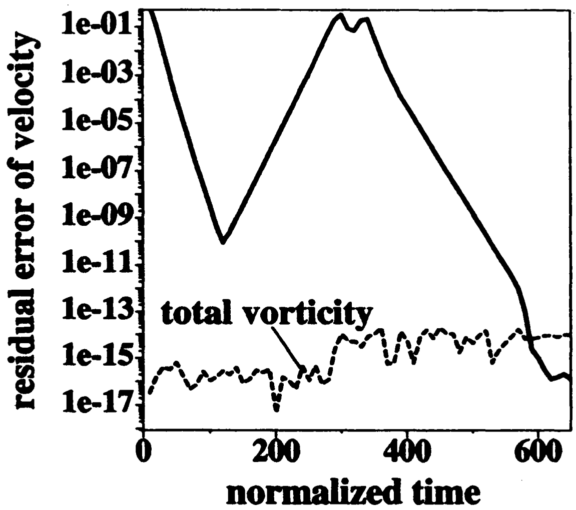

FIGURE 4. Residual error and total vorticity (Re = 280).

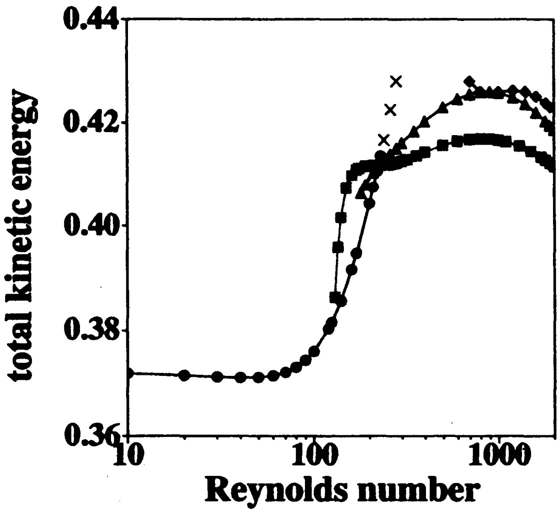

FIGURE 5. Total kinetic energy.

A typical example of convergence and accuracy check is shown in Fig. 4. The time evolution of a residual error denoted by the bold curve has a minimum at about $\tau = 120$, when the flow pattern is similar to that shown in Fig. 3(b). However, the classical symmetric flow is unstable at Re = 280, and hence the residual error grows until $\tau = 300$ and thereafter decreases again. Finally, at about $\tau = 600$, the flow field converges to a new steady state, where the flow pattern is similar to that shown in Fig. 6(b). The dashed curve in Fig. 4 represents the fluctuation of total vorticity, which should be zero due to the restriction from boundary condition (9).

Figure 5 shows the coexistence of various steady flows. For Re $<$ 130, the classical flow is the unique stable steady solution. Note that the classical flow has two types of symmetry, i.e., a mirror symmetry about $x = \pi$ and a rotational symmetry around $(x, y) = (\pi, w/2)$. Beyond Re = 130, another steady solution emerges, the flow pattern of which loses the rotational symmetry [see Figs. 6(a), 6(b), and 6(g)]. At Re = 240, the classical flow becomes unstable but even after that it continues to be a steady (unstable) solution of the problem. At Re = 180, another new steady solution appears, whose flow pattern no longer has symmetry [see Figs. 6(c), 6(d), and 6(h)]. At Re = 700, a further new steady solution appears, which possesses the rotational symmetry but loses the mirror symmetry [see Figs. 6(e) and 6(f)].

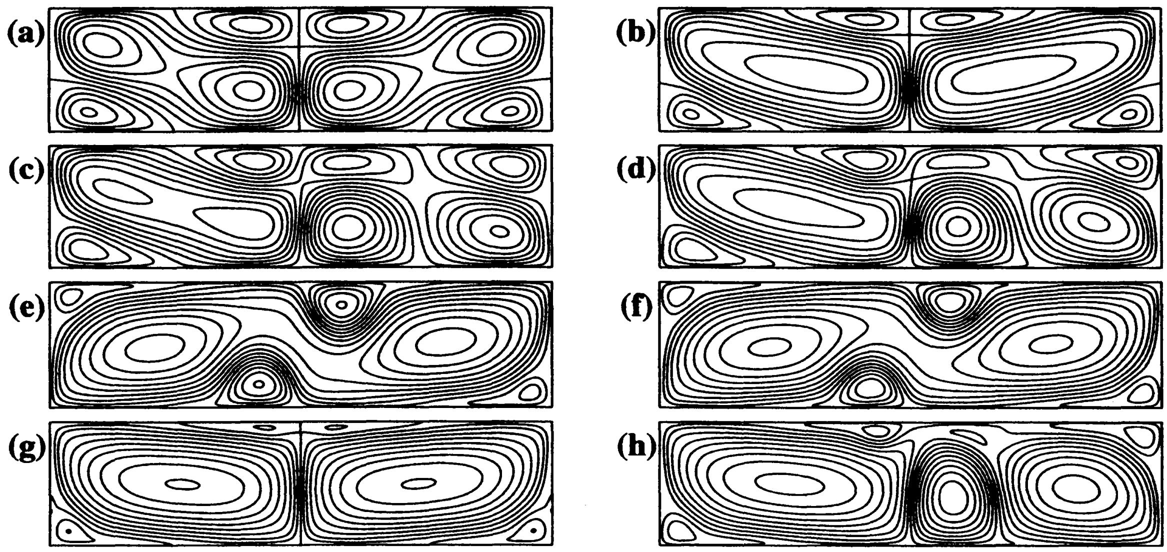

FIGURE 6. Streamlines of various steady flows. (a): Re = 140, (b): Re = 230, (c): Re = 180, (d): Re = 240, (e): Re = 700, (f): Re = 1000, (g): Re = 1000, (h): Re = 1000.

The mirror image and the rotational image of a solution of boundary value problem (6) and (9) are also solutions of the same problem. Accordingly, the number of stable steady solutions changes as $1 \to 3 \to 7 \to 6 \to 8$ with the increase of Re (< 2000). The type of bifurcation seems to be subcritical, although we have not yet verified.

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