数理解析研究所講究録 1339 巻 2003 年 83-92

Interaction between vortical structures and small heavy particles settling in turbulence

京工繊大・工芸 田中 満 (Mitsuru Tanaka)
京工繊大・工芸 三枝 大直 (Dainao Saegusa)
京工繊大・工芸 萩原 良道 (Yoshimichi Hagiwara)
Dept. of Mech. and System Eng., Kyoto Institute of Technology

1. Introduction

Turbulent gas flows laden with small heavy particles are encountered in a variety of natural and engineering applications. Examples include combustion of sprays and ash from volcanic eruptions settling in the atmospheric turbulent boundary layer. Many studies have been carried out to understand the flow characteristics, and the interaction between particles and gas flow turbulence.

One of the key elements of the interaction is preferential concentration or clustering of particles. Squires and Eaton (1991) showed in their numerical simulation of homogeneous isotropic turbulence that particles tend to accumulate in low-vorticity or high-strain regions due to the particle inertia. Wang and Maxey (1993) demonstrated that the particles in gravity are preferentially swept to the downward side of vortices, which leads to the significant increase in the mean settling velocity of particles.

Two-way coupling between carrier fluid and particles is another key factor. The effects of particles on the carrier fluid flow become important when the mean particle concentration exceeds $O(10^{-6})$. Numerical results show that the two-way coupling effects modify the settling velocity of particles and vortical structures as well as turbulence kinetic energy of carrier fluid (Squires and Eaton, 1990, Elghobashi and Truesdell, 1993). Recent experimental (Aliseda et al., 2002) and numerical (Tanaka et al., 2000) studies have demonstrated that the two-way coupling effect significantly increases the settling velocity of particles even in the case where the particles have only a minor effect on the turbulence kinetic energy of carrier fluid. However, it has not been clarified how the two-way coupling effects lead to the increase in the settling velocity.

In the present study, we investigate the interactions between particle clusters and vortical structures in decaying homogeneous isotropic turbulence by the use of a numerical simulation. We focus on the role which the interaction plays in the increase in the settling velocity.

2. Formulation

2.1. Fluid and Particle Motions

We consider the motions of small heavy spherical particles under the gravitational force in the negative x_2 direction. The particle diameter, d_p, was assumed to be small compared with the Kolmogorov length-scale, η, of turbulence. The particulate phase is assumed to be dilute enough that the effects of particle-particle interactions are neglected though the two-way coupling between two phases is considered. Taking account of the fact that the particle (solid) density, ρ_p, is much higher than the fluid (air) density, ρ_f, only the Stokes drag and the gravitational forces were assumed to exert on the particles. Under the assumption, the particle motions are governed by the following equations (Maxey and Riley, 1983),

$$\frac{\mathrm{d}v_i}{\mathrm{d}t} = \frac{1}{\tau_p}\left\{u_i(\boldsymbol{x}_p) - v_i - V_S\delta_{i2}\right\}, \ \frac{\mathrm{d}\boldsymbol{x}_p}{\mathrm{d}t} = \boldsymbol{v}, \ (i = 1, 2, 3) \tag{1}$$

where $\boldsymbol{v}$ and $\boldsymbol{x}_p$ denote the velocity and position of the particle. $\boldsymbol{u}$ and ν represent the velocity and kinematic viscosity of the fluid, respectively. Two parameters, $\tau_p = \rho_p d_p^2/18\rho_f\nu$ and $V_S = \tau_p g$, denote the particle inertia (or response time) and the still-fluid terminal velocity, respectively. g denotes the gravitational acceleration.

The motions of the carrier fluid are described by

$$\frac{\partial u_i}{\partial t} + u_k\frac{\partial u_i}{\partial x_k} = -\frac{1}{\rho_f}\frac{\partial p}{\partial x_i} + \nu\nabla^2 u_i + \frac{1}{\rho_f}f_i \tag{2}$$

with the solenoidal condition $\partial u_j/\partial x_j = 0$, where p is the pressure. f_i represents a body force which is the sum of the reaction forces exerted by the particles on the fluid. Here, we assume that the mean body force $\langle \boldsymbol{f} \rangle$ is balanced by the mean pressure gradient, where $\langle \ \rangle$ denotes the spatial average. Therefore, the mean fluid velocity remains zero. For later convenience, we introduce

$$\Delta V \equiv \langle -v_2 \rangle - V_S, \tag{3}$$

which represents the increase in the mean settling velocity of particles from their still-fluid terminal velocity.

2.2. Numerical Method

The motions of the carrier fluid were solved on 64^3 grid points in a cubic box of sides of 2π by using the Fourier spectral/Runge-Kutta-Gill scheme. The initial velocity field was given by the Fourier coefficients with specified energy spectrum,

$$E(k) = c\,k\exp(-k/k_0), \tag{4}$$

and with random phase. Here, $k_0(= 4.2)$ is a wavenumber at which the energy spectrum takes the maximum and c is a normalization constant. We set c as $k_0^{-4}/6$ so that $\omega'(0) = 1$, where ω' is the vorticity magnitude. Hereafter, $t^* = \omega'(0)t = t$ is used as a dimensionless time. ηk_{max} varied from 0.98 at $t^* = 30$ to 2.66 at the end of the simulation ($t^* = 200$). The Taylor microscale Reynolds number was $20 \sim 24$ during the simulation except the initial transient period.

We introduced particles randomly throughout the computational domain at $t^* = 30$ after the flow attained a fully turbulent state. Many simulations were conducted with different values of the particle response time τ_p and the still-fluid settling velocity V_S. As in Aliseda et al., we focus on a case of $\tau_p \approx \tau_K$ and $V_S \approx v_K$, where a noticeable increase in settling velocity is expected due to preferential sweeping (Wang and Maxey, 1993). Here, τ_K and v_K are the Kolmogorov time and velocity scales of turbulence, respectively. In order to reduce the error that may result from point-force approximation (Boivin et al., 1998), the number of particles was increased α times by using the virtual particles which represent $1/\alpha$ real

 85

Table 1: Parameters for particles. Here, N_p denotes the number of particles. ϕ_v and ϕ_m represent the volume fraction and mass loading of particles, respectively.

N_p	ρ_p/ρ_f	τ_p/τ_K	V_S/v_K
2^{19}	1000	$2.18 \rightarrow 0.32$	$0.89 \rightarrow 2.3$
ϕ_v	ϕ_m	α	d_p
0.8×10^{-4}	0.8×10^{-1}	3.7	7.11×10^{-4}

ones. (Note that $\alpha^2 \rho_p/\rho_f = const.$ when τ_p, N_p and ϕ_m are fixed.) The parameters employed in this simulation are summarized in Table 1.

The particles were tracked in the Lagrangian frame. The initial particle velocity was set to be the same as the sum of the surrounding fluid velocity and the still-fluid terminal velocity. Cubic spline interpolation was used for the evaluation of fluid velocity at the particle position from its neighboring grid points, whereas Taylor series 13 points method (Yeung and Pope, 1988) was used for the distribution of the reaction force to the neighboring grid points.

For comparison a one-way coupling simulation was conducted with the same initial condition. Another one-way coupling simulation was performed starting from the data at $t^* = 100$ of the two-way coupling simulation in order to clarify the two-way coupling effects on the interaction between vortex structures and a particle cluster.

3. Results

3.1. Turbulence kinetic energy and enstrophy

The energy and the enstrophy evolves as t^{-1} and t^{-2}, respectively, in later times in the single-phase flow, as is expected from the form of the initial energy spectrum (Eq.(4)). Figure 1 shows the time evolution of turbulence kinetic energy and enstrophy of carrier fluid for the two-way coupling case. They are normalized by the counterparts in the single-phase flow to emphasize the two-way coupling effect. The relative magnitude of the energy slightly increases in later times in this simulation. It is found that the enstrophy is more effectively increased by the two-way coupling. The horizontal components of vorticity are found to become greater than the vertical one though all components increase with time relative to their counterparts in the single-phase flow. Examination of each term in the vorticity equations has revealed that the horizontal vorticity is directly generated by the particles, while the vertical vorticity is intensified by the stretching-and-tilting terms.

Figure 2 shows the time evolution of the mean settling velocity of particles for the one-way and two-way coupling simulations. In the figure, the increase in the settling velocity, ΔV (Eq.(3)), is normalized by the magnitude of the vertical component of velocity, u'_2. The increase in the mean settling velocity is about 10 percent of u'_2 in the one-way coupling simulation, which is consistent with the result obtained by Wang and Maxey (1993). In the two-way coupling simulation, the increase amounts to 35 percent of u'_2 at the end of the simulation. It is somewhat smaller than a increase (50%) in the experiment conducted by Aliseda et al. (2002). However, the additional increase due to the two-way coupling effects is found to be comparable to theirs.

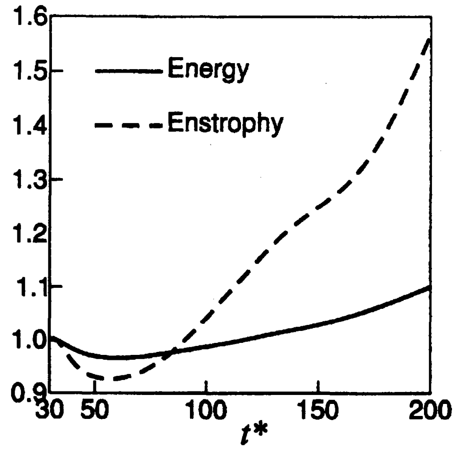


Fig. 1 The effect of particles on the time evolution of turbulence kinetic energy and enstrophy of carrier fluid. The values were normalized by those in the single-phase flow.

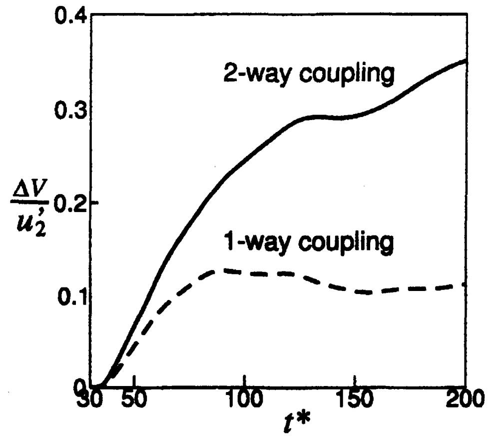


Fig. 2 Time evolution of the increase in the settling velocity of particles.

3.2. Counter-rotating vortex pair

We show a typical example of the interaction between particle clusters and vortical structures to understand the mechanism leading to the increase in the settling velocity. It is found that counter-rotating vortex pairs play an essential roll in the increase of settling velocity. We focus on a vortex pair (Fig. 3), which is found to make the largest contribution to the increase. In Figs. 3, the vortex tubes are represented by their central axes, which were extracted by tracing the loci of sectional local minimum of the pressure (Kida and Miura 1998). The light one is inclined at about $-30°$ to the $+x_3$ direction, while the dark one at about 30° to the $-x_3$ direction. These two vortices induce a downward fluid flow between them.

Figure 4 shows the time evolution of the vortex pair. Thick and thin lines denote the vortex pairs in the two-way simulation and the one-way coupling simulation starting from the data of the former at $t^* = 100$, respectively. As time elapses, the central part of the vortex pair (indicated by the vertical lines) moves in the gravitational direction due to the self-induction of the vortex pair. It is interesting to note that the downward motion of the vortex pair is enhanced by about 30% due to the two-way coupling effects, indicating that the vortices are activated by the particle cluster. It is also interesting that the lower parts of the vortex tubes are stretched in the gravitational direction as a result of the two-way coupling (see the bottom panel of Fig. 4). Generation of vertical vorticity was also observed in a homogeneous turbulent shear flow laden with small heavy particles (Tanaka et al., 2002).

3.3. Interaction between the vortex pair and a particle cluster

Figure 5 shows how the vortex pair interacts with the particles in the two-way coupling case. It is found that a mushroom of high particle concentration is created at the end of the interaction (see Fig. 8 below). The particles composing the cluster were traced back to the beginning of the interaction to examine how the particle cluster was generated. The counterpart in the one-way coupling case is shown in Fig. 6 for comparison. It turns out that these particles represent the region of high particle concentration in the one-way coupling case as well as that in the two-way coupling.

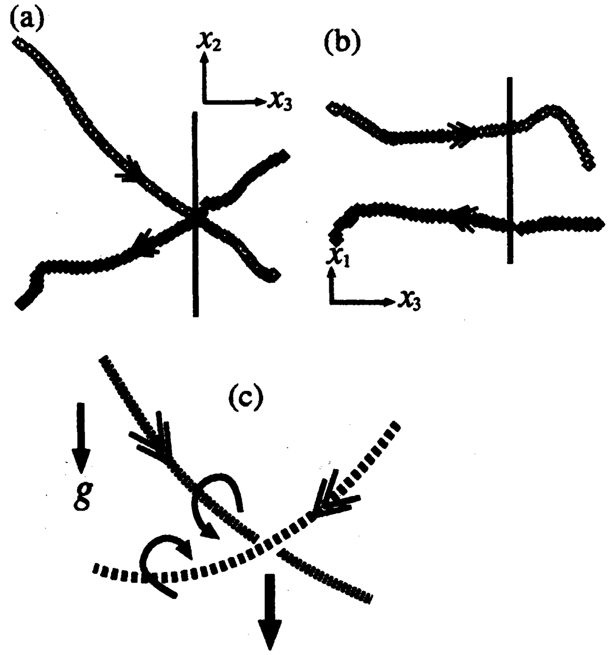


Fig. 3 Vortex pair which is focused on ($t^* = 140$). (a) side view, (b) top view, (c) sketch of the vortex pair.

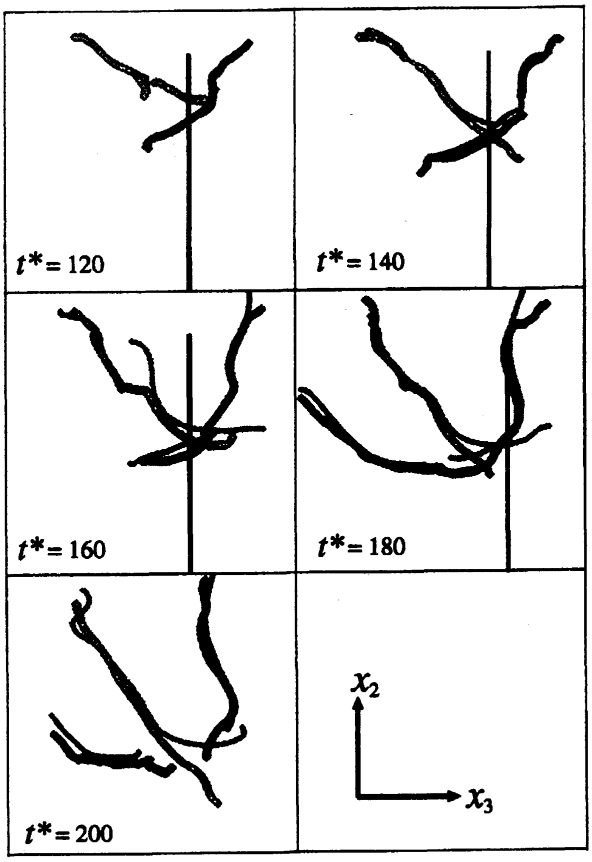
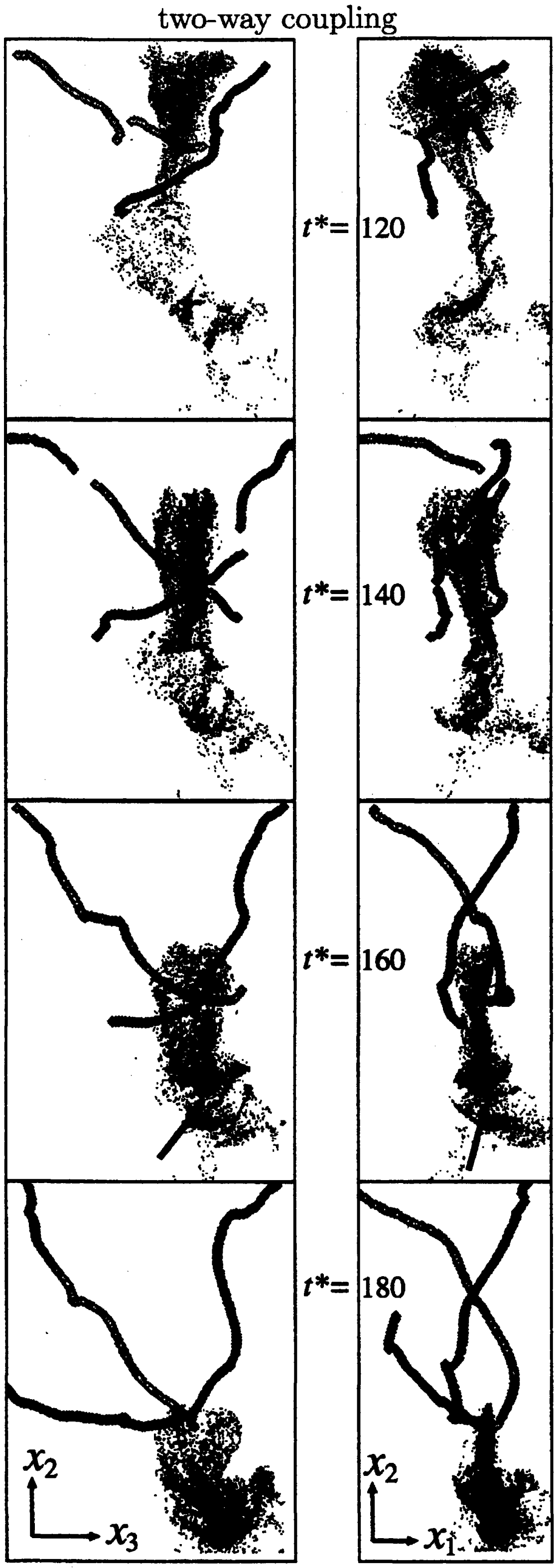


Fig. 4 Time evolution of the vortex pair. Thick and thin lines for the two-way and one-way coupling simulations, respectively.

Before $t^* = 100$, the particles are distributed in a large area above the vortex pair. As time goes on, they are pulled down into the region between the pair vortices. Because of particle inertia, they accumulate in a narrow region of high downward fluid velocity between the vortices. This accelerates the particles in the gravitational direction. In addition to the sweep from the region above the vortex pair to the region below, the downward motion of the vortex pair due to the self-induction also contributes to the increase in the settling velocity. The descending velocity of the cluster is surprisingly high (about four times higher than u_2' in the two-way coupling case) during the interaction.

By comparing Fig. 5 and 6, it is found that a region of very high concentration is generated at the bottom of the cluster due to the two-way coupling effect (indicated by arrows in Fig. 5). As will be mentioned later, the fluid pressure is lowered behind (or above) the head (or bottom) of the cluster, which enhances the accumulation of the particles. After passing through the vortex pair, the descending velocity of the cluster rapidly decreases in the one-way coupling case. The cluster is stretched in the horizontal directions due to the swirling motions around the vortices. In the two-way coupling case, on the other hand, the cluster continues to move downward while it is transformed into a mushroom-like structure (see Fig. 8 below). The descending velocity of the cluster is gradually lowered as the mushroom becomes larger with time.



g. 5 Interaction between a vortex pair and a ırticle cluster in the two-way coupling case.

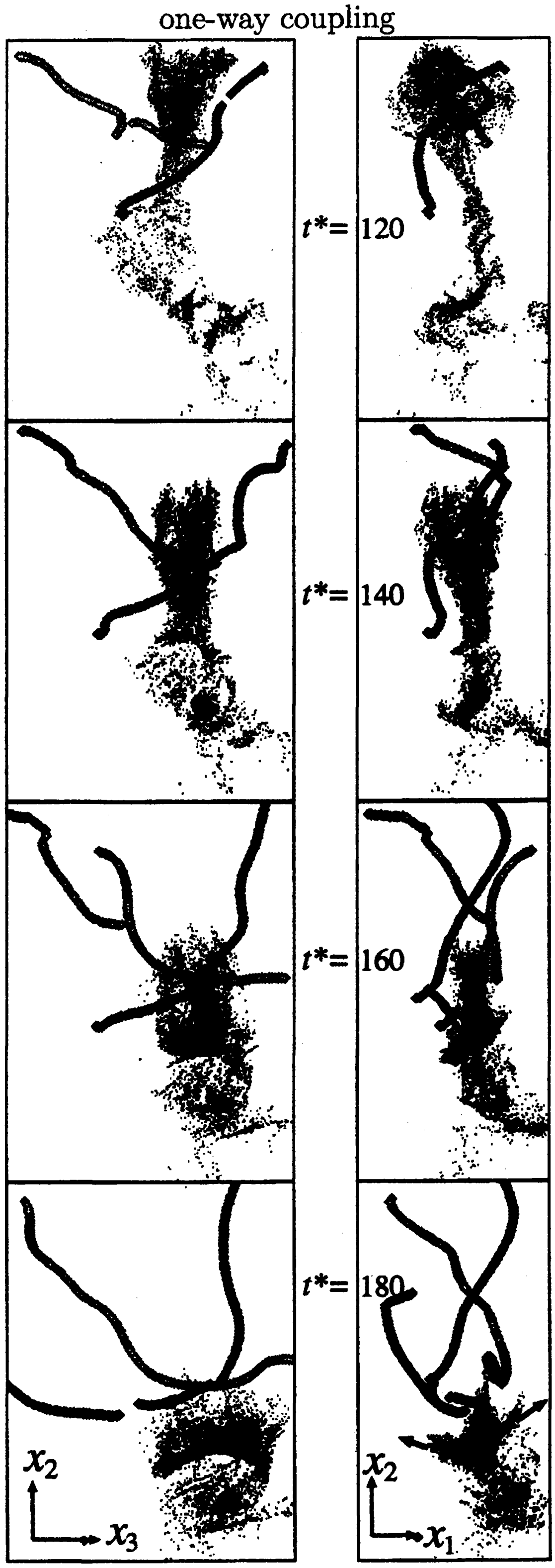
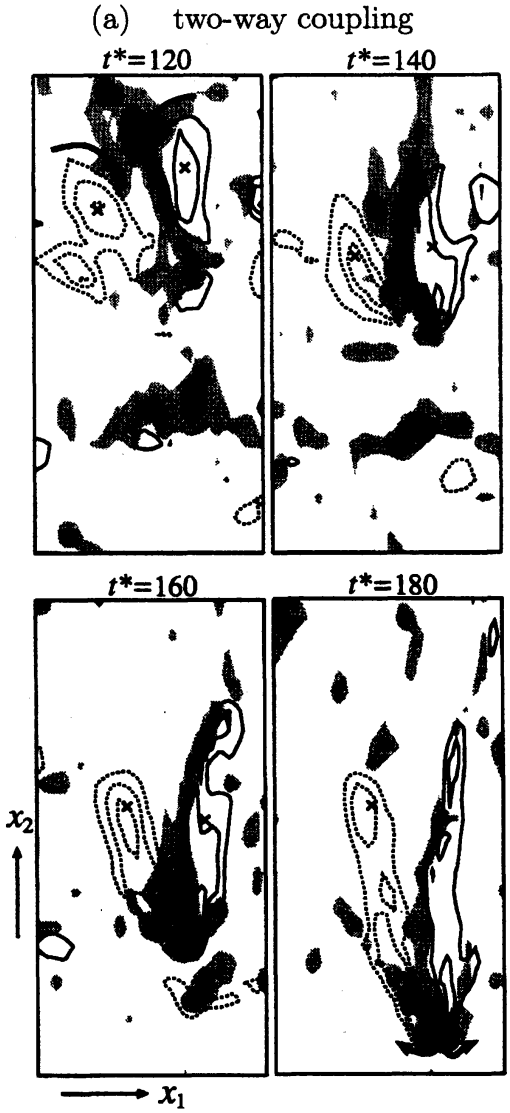


Fig. 6 The same as Fig. 5 but for the one-way coupling case.



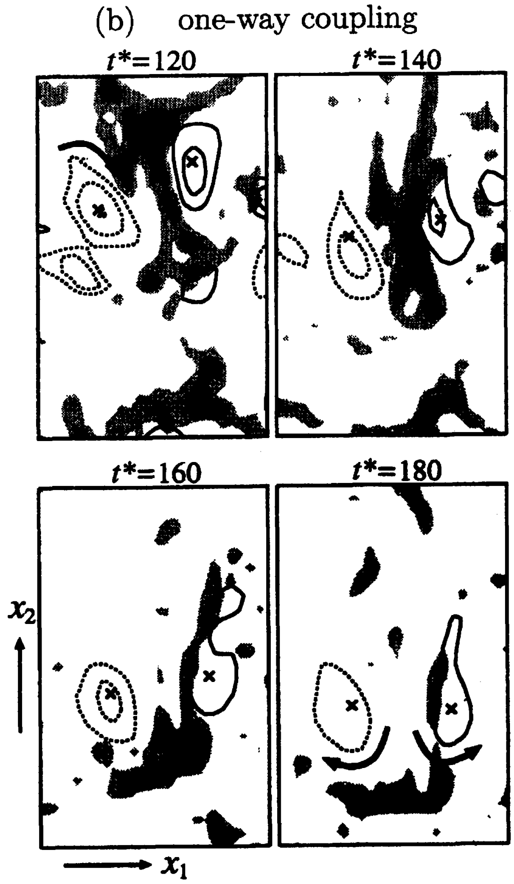
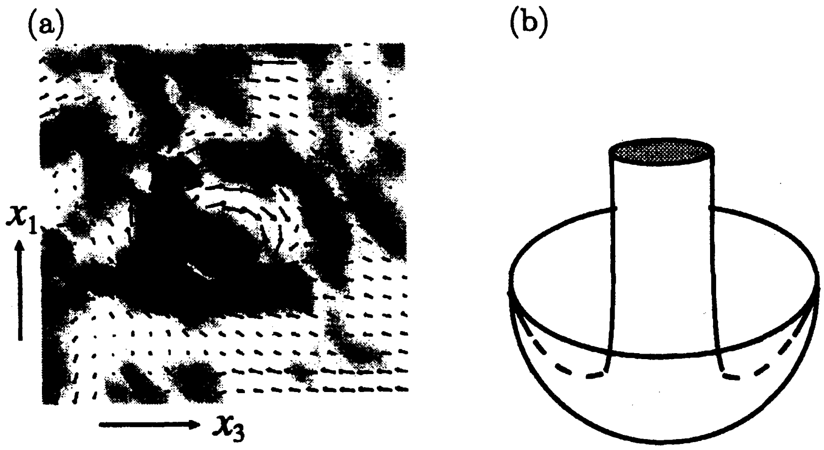


Fig. 7 Spatial distributions of particle concentration and x_3 component of vorticity on the $x_1 - x_2$ plane for the (a) two-way and (b) one-way coupling cases. × denotes the position of the vortex axis. Solid (broken) lines denote the contour lines of $\omega_3 = 0.1, 0.15, 0.2$ $(-0.1, -0.15, -0.2)$. The local particle concentration changes as $C/\langle C\rangle = 2, 4, 8, 16$ from the lightest to darkest shades.



(a) Concentrated region of particles and vorticity vectors on an $x_3 - x_1$ plane at $t^* = 180$ lack for $C/\langle C\rangle \geq 10$ and white for $C = 0$. (b) Sketch of the cluster.

3.4. Modification of vortical structures

Figure 7(a) shows a time series of the vortical structure and the local concentration of particles on the x_1-x_2 planes indicated by the vertical lines in Figs. 3 and 4. The distribution of the x_3 component of vorticity is represented with solid lines for $\omega_3 > 0$ (counter-clockwise rotation) and broken lines for $\omega_3 < 0$ (clockwise rotation). Shades denote the regions of high particle concentration with darker shades representing higher concentration. The crossing point of the vortex axis is represented by $\times$. In Fig. 8, a cross section of the mushroom at $t^* = 180$ is shown with a sketch. For comparison, the vorticity and particle concentration fields are shown for the one-way coupling case in Fig. 7(b).

As was shown before, the particles accumulate in the region between the pair vortices. As the particle concentration becomes high, the particle cluster begins to accelerate downward due to gravity, inducing the downward fluid flow locally between the vortices. Then, a region of low fluid pressure is created behind (or above) the cluster. Because of the low pressure, the vortices of positive and negative vorticities approach each other. This in turn enhances the accumulation of particles, resulting in high particle concentration at the head (or bottom) of the cluster.

As was found in Ferrante and Elghobashi (2003), elongated regions of high horizontal vorticity appear on both sides of the downward flow. It is found that the vorticity vectors are almost parallel to the iso-concentration surface around the mushroom (Fig. 8).

4. Conclusions

Numerical simulations have been conducted for decaying homogeneous isotropic turbulence laden with small heavy particles settling under the effect of gravity. We have focused on the interactions between vortical structures and particle clusters, and the role which they play in the increase in the settling velocity of particles. It is found that the counter-rotating vortex pairs descending due to their self-induction play an essential role in the interaction. We have observed the following scenario of the interaction (see Fig. 9).

(1) Particles accumulate in the downward fluid flow between the pair vortices to form a particle cluster. The settling velocity of the particles is increased through this process.
(2) The vortex pair moves downward owing to its self-induction. The particle cluster is transfered downward not only by the sweep from the region above the vortex pair to the region below but also by the vortex motion itself.
(3) The downward fluid flow is locally accelerated by the cluster, which further increases the settling velocity of particles.
(4) The fluid pressure is lowered behind the particle cluster. This reduces the separation between the pair vortices and enhances the accumulation of particles in the cluster.
(5) The particle cluster is transformed into a mushroom-like structure after passing through the vortex pair.
(6) The particle cluster activates the vortex pair and its downward motion. The lower parts of the vortices are finally stretched in the vertical direction.
(7) The regions of high horizontal vorticity are generated by particle clusters. They are elongated in the vertical direction.

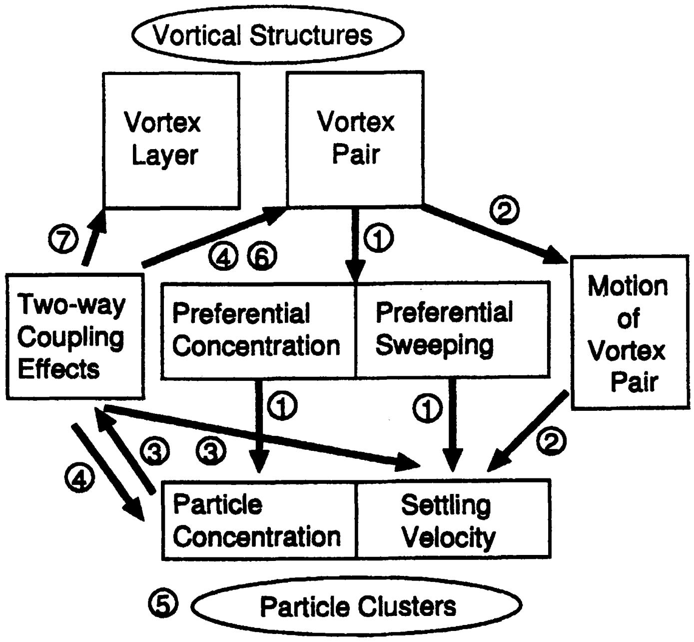


Fig. 9 Mechanism of the increase in the settling velocity of particles.

The response time of the cluster is generally much longer than the particle response time. This means that the lifetime of the cluster needs to be long so that the two-way coupling can be effective. Our results indicate that the vortex pair and its downward motion are important to sustain the cluster for a long time.

Aliseda et al. (2002) have found that the additional increase in settling velocity due to the two-way coupling effects can be estimated by a simple model where the particle cluster is regarded as one larger solid particle descending in still fluid. If we employ the diameter of the mushroom at $t^* = 180$ as the dimension of the cluster, the descending velocity is estimated as $1.3V_S$ by using the following formula for the drag coefficient, $C_D = 24/Re_p(1 + 0.15Re_p^{0.687})$. Here, Re_p is the particle Reynolds number for the cluster ($Re_p \approx 50$ in this case). This is close to the actual increase in settling velocity due to the two-way coupling effect (about $1.7V_S$). Judging from the complexity of the interaction between the vortices and the particle cluster, however, this may be a coincidence. Further studies are needed to discuss the two-way coupling effect quantitatively.

References

Aliseda, A., Cartellier, A., Hainaux, F., and Lasheras, J.C, 2002, "Effect of preferential concentration on the settling velocity of heavy particles in homogeneous isotropic turbulence", *J. Fluid Mech.*, Vol. 468, pp. 77-105.

Boivin, M., Simonin, O., and Squires, K.D., 1998, "Direct numerical simulation of turbulencce modification by particles in isotropic turbulence", *J. Fluid Mech.*, Vol. 375, pp. 235-263.

Elghobashi, S., and Truesdell, G.C., 1993, "On the two-way interaction between homogeneous turbulence and dispersed solid particles. I: Turbulence modification", *Phys. Fluids A*, Vol. 5, pp. 1790-1801.

Ferrante, A, and Elghobashi, S., 2003, "On the physical mechanisms of two-way coupling in particle-laden isotropic turbulence", *Phys. Fluids*, Vol. 15, pp. 315-329.

Kida, S,. and Miura, H., 1998, "Swirl condition in low-pressure vortices", *J. Phys. Soc. Japan*, Vol. 67, pp. 2166-2169.

Maxey, M.R., and Riley, J.J., 1983, "Equation of motion for a small rigid sphere in a nonuniform flow", *Phys. Fluids*, Vol. 26, pp. 883-889.

Squires, K.D., and Eaton, J.K., 1990, "Particle response and turbulence modification in isotropic turbulence", *Phys. Fluids A*, Vol. 2, pp. 1191-1203.

Squires, K.D., and Eaton, J.K., 1991, "Preferential concentration of particles by turbulence", *Phys. Fluids A*, Vol. 3, pp. 1169-1178.

Tanaka, M., Komai, N., Maeda, Y., and Hagiwara, Y., 2000, "Two-way coupling effect on settling velocity of small heavy particles in homogeneous turbulence", *Advances in Turbulence VIII*, C. Dopazo et al., ed., CIMNE, Barcelona, pp. 557-560.

Tanaka, M., Maeda, Y., and Hagiwara, Y., 2002, "Turbulence modification in a homogeneous turbulent shear flow laden with small heavy particles", *Int. J. Heat and Fluid Flow*, Vol. 23, pp. 615-626.

Wang, L.P., and Maxey, M.R., 1993, "Settling velocity and concentration distribution of heavy particles in homogeneous isotropic turbulence", *J. Fluid Mech.*, Vol. 256, pp. 27-68.

Yeung, P.K., and Pope, S.B., 1988, "An Algorithm for Tracking Fluid Particles in Numerical Simulations of Homogeneous Turbulence", *J. Comput. Phys.*, Vol. 79, pp.