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Blocks of Central p-Group Extensions of Finite Groups

中心的 p- 群拡大である有限群のブロックについて

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This is a joint work with **Naoko Kunugi**. A result stated here will be published with a complete proof, see [**1**]. The result in [**1**] is, actually, inspired by a result stated in their paper [**3**] of Usami and Nakabayashi, where they prove our theorem for principal block algebras.

Here, we consider the following setting-up.

First of all, let G and G' be finite groups which have a common central p-subgroup Z for a prime number p, and let $\overline{A}$ and $\overline{A'}$ respectively be p-blocks of G/Z and G'/Z induced by p-blocks A and A' respectively of G and G', both of which have the same defect group. Let $(\mathcal{O}, K, k)$ be a splitting p-modular system for all subgroups of G and G', that is, $\mathcal{O}$ is a complete discrete valuation ring of rank one with its quotient field K of characteristic zero and with its residue field k of characteristic p, and both K and k are splitting fields for all subgroups of G and G'.

Then, we may have the following natural question. Namely,

Question. If $\overline{A}$ and $\overline{A'}$ are of a certain equivalence, then so are A and A'?

Our main result is in fact the following.

Theorem (Koshitani-Kunugi). *Keep the notation above. Assume that G and G' have a common subgroup H satisfying $H \supseteq P \supseteq Z$ for a p-subgroup P of H and a central p-subgroup Z of G and G'. Let A and A', respectively, be block algebras of $\mathcal{O}G$ and $\mathcal{O}G'$ such that P is a defect group of A and A'. Set $\overline{G} = G/Z$, $\overline{G'} = G'/Z$, $\overline{P} = P/Z$ and $\overline{H} = H/Z$, and let $\pi : \mathcal{O}G \twoheadrightarrow \mathcal{O}\overline{G}$ and $\pi' : \mathcal{O}G' \twoheadrightarrow \mathcal{O}\overline{G'}$ be the canonical $\mathcal{O}$-algebra-epimorphisms induced by the canonical group-epimorphisms $G \twoheadrightarrow \overline{G}$ and*

$G' \twoheadrightarrow \overline{G'}$, respectively. Write $\overline{A} = \pi(A)$ and $\overline{A'} = \pi'(A')$. Then, it is well-known that $\overline{A}$ and $\overline{A'}$, respectively, are again block algebras of $\mathcal{O}\overline{G}$ and $\mathcal{O}\overline{G'}$ such that $\overline{P}$ is a defect group of $\overline{A}$ and $\overline{A'}$.

If there is an $(\overline{A}, \overline{A'})$-bimodule $\overline{M}$ such that $\overline{A} \otimes_{\mathcal{O}\overline{H}} \overline{A'} = \overline{M} \oplus (\text{projective})$ and $\overline{M}$ realizes a Morita equivalence between $\overline{A}$ and $\overline{A'}$, then A and A' are also Morita equivalent via an (A, A')-bimodule M such that $M | A \otimes_{\mathcal{O}H} A'$.

Remark. Theorem above is, actually, pretty much usable to prove Broué's abelian defect group conjecture for non-principal block algebras. For instance, see [**2**].

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References

[1] S. Koshitani and N. Kunugi, Blocks of central p-group extensions, to appear in Proc. Amer. Math. Soc.

[2] S. Koshitani, N. Kunugi and K. Waki, Broué's abelian defect group conjecture for the Held group and the sporadic Suzuki group, preprint October 2003.

[3] Y. Usami and M. Nakabayashi, Morita equivalent principal 3-blocks of the Chevalley group $G_2(q)$, Proc. London Math. Soc.(**3**) **86** (2003), 397–434.