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Numerical study of acoustic wave scattering from assemblies of cylinders

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1. Introduction

Wave motions in inhomogeneous media, such as bubbly liquids, biological tissue, etc, have been extensively studied in physics and engineering. The multiple scattering of acoustic waves is one of typical examples. Averaging techniques and continuum approximation may resolve these problems. However, understanding of real phenomena will require the direct simulation of the problem. In the present study, we shall consider the problem of acoustic wave scattering from assemblies of cylinders by the numerical method PHYSALIS, originally devised for incompressible potential flows by Prosperetti and Oguz.[1]

2. Formulation of the problem

We shall consider a two-dimensional acoustic wave propagation in an ideal gas. The geometry of the problem considered is shown in Fig. 1.

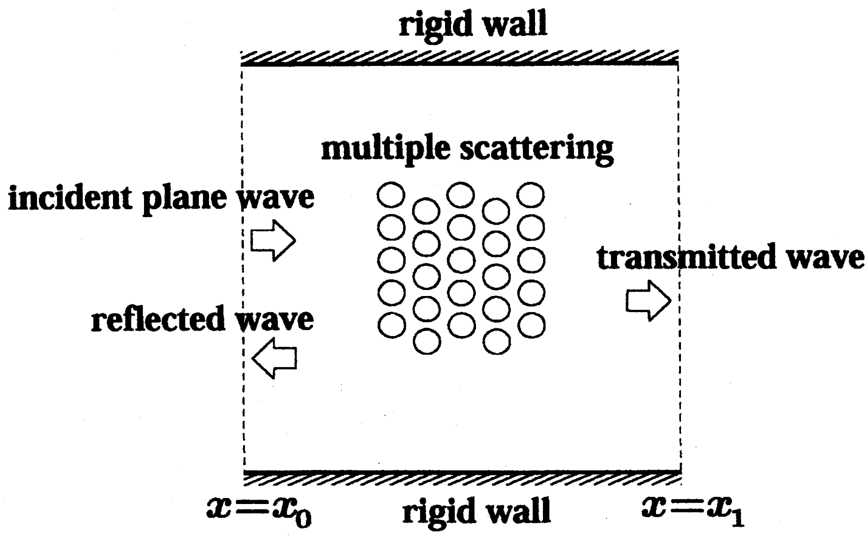

Fig. 1 Schematic of the model.

As shown in Fig. 1, a plane acoustic wave $\phi^i = e^{i[\kappa(x-t)+\varphi_0]}$ propagates from the left-side of a two-dimensional channel with rigid walls, where κ is a nondimensional wave number, x is a nondimensional coordinate along the channel, t is a nondimensional time, and φ_0 is an initial phase. There are a number of circular cylinders with the same radius a in the channel. The incident plane acoustic wave is scattered by the cylinders, and as a result, a part of which propagates forward as a transmitted wave and the other propagates backward as a reflected wave.

The acoustic wave motion is governed by the Helmholtz equation

$$\Delta\phi + \kappa^2\phi = 0, \quad \phi = \phi_R + i\phi_I \tag{1}$$

where ϕ is a complex velocity potential. The boundary condition on the sidewall of the channel is

$$\frac{\partial\phi}{\partial n} = 0 \tag{2}$$

and at the entrance of the channel ($x = x_0$) and at the exit ($x = x_1$),

$$\frac{\partial\phi}{\partial x} = -i\kappa\phi + 2i\kappa e^{i(\kappa x_0 + \varphi_0)} \quad (x = x_0) \tag{3}$$

$$\frac{\partial\phi}{\partial x} = i\kappa\phi \quad (x = x_1) \tag{4}$$

Equations (3) and (4) are a kind of non-reflecting boundary condition based on a local one-dimensional approximation.

The acoustic pressure p and the x component of fluid velocity u can be retrieved from the velocity potential ϕ by the following relations:

$$p = -\frac{\partial}{\partial t}(e^{-i\kappa t}\phi) = i\kappa e^{-i\kappa t}\phi \tag{5}$$

$$u = \frac{\partial\phi}{\partial x}e^{-i\kappa t} \tag{6}$$

The acoustic energy flux (acoustic intensity) based on the local one-dimensional approximation can be given as

$$q = \begin{cases} -\frac{1}{2}\kappa^2|\phi - \phi^i|^2 & x = x_0 \\ \frac{1}{2}\kappa^2|\phi - \phi^i|^2 & x = x_1 \end{cases} \tag{7}$$

3. Numerical method

Numerical approach is based on the method of PHYSALIS originally devised for incompressible potential flows by Prosperetti and Oguz.[1] We here extend the method to the Helmholtz equation.

In the neighborhood of each cylinder, the method utilizes a local analytical representation of a general solution of the Helmholtz equation,

$$\phi = \sum_{n=0}^{\infty}[J_n(\kappa r)Y_n'(\kappa a) - Y_n(\kappa r)J_n'(\kappa a)](A_n \cos n\theta + B_n \sin n\theta) \tag{8}$$

where J_n and Y_n are the Bessel functions of the first and second kinds, r is the distance from a cylinder, a is the radius of the cylinder, θ is the azimuthal angle measured from the x axis, and the prime denotes the differentiation with respect to the argument.

The coefficient of expansion, A_n and B_n, are determined in an iteration:

(0) Start with an appropriate initial expansion coefficients.

(1) From the local analytical representation (truncated suitably), we evaluate the normal derivative of the velocity potential along a closed curve enclosing each cylinder.

(2) We solve the Helmholtz equation with the finite-difference method with the second-order central difference. The finite-difference solution is directly obtained with the help of FFT, without any iterative procedure.

(3) The expansion coefficients are updated by the finite-difference solution near each cylinder.

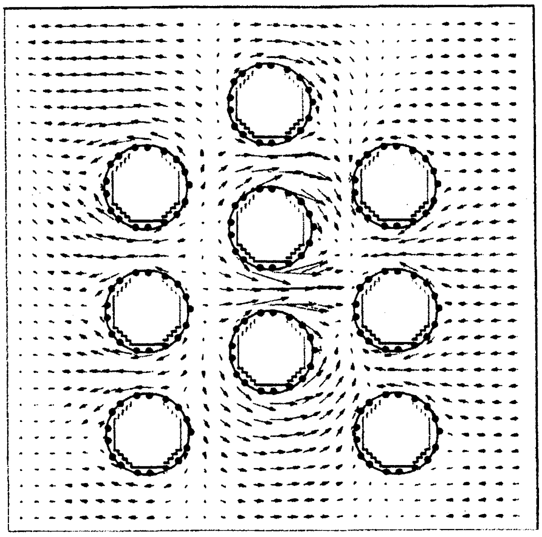

Fig. 2 Numerical example.

In Fig. 2, we present an example as an illustration of the numerical method, where one can see nine cylinders in the channel, The boundary of each cylinder is denoted by a circle plotted by a thin solid curve. Along a closed line like octagon in the circle, we evaluate the normal derivative of the velocity potential, with which the exterior boundary-value problem of the Helmholtz equation is solved. At the 17 black spots around each cylinder, we update the values of expansion coefficients A_n and B_n with the help of the finite-difference solution. In this example, we use 17×2 coefficients. The small arrows are the fluid velocity obtained from the finite-difference solution.

4. Results

Figure 3 shows the pressure wave fields for the case that the incident wave is scattered by a column of cylinders and by three columns of cylinders. One can easily see that the amplitude of transmitted wave depends on the number of columns and the parameter κa.

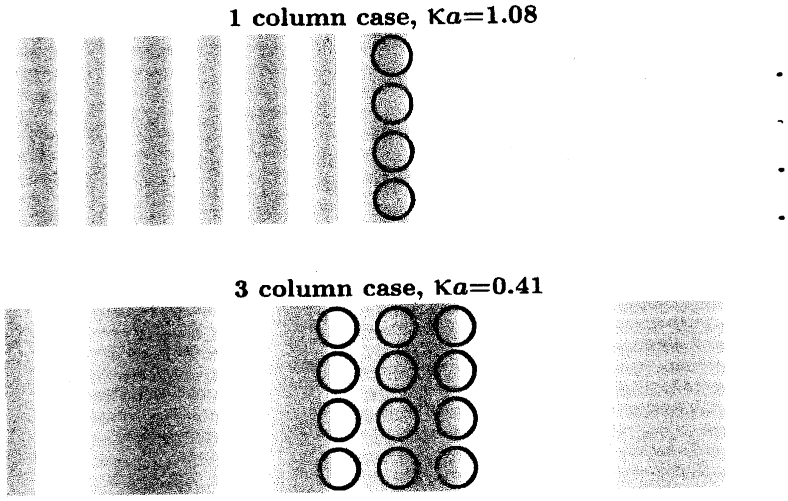

Fig. 3 Scattering from columns of cylinders.

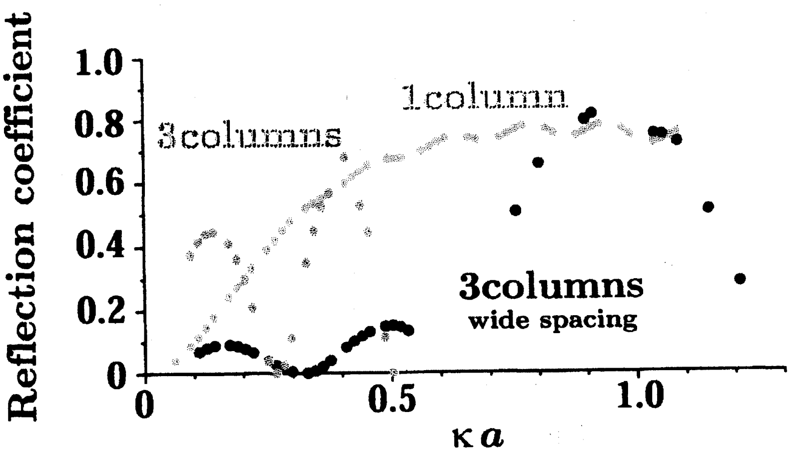

Fig. 4 Reflection coefficient.

References

[1] A. Prosperetti and H.N. Oğuz. PHYSALIS: A new $o(N)$ method for the numerical simulation of disperse flows of spheres. Part I: Potential flow. *J. Comput. Phys.* **167**: 196-216, 2001.