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Price-based unit commitment problem

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Abstract The unit commitment problem is determining the schedules for power generating units and the generating level of each unit. The decisions concern which units to commit during each time period and at what level to generate power to meet the electricity demand. The problem is a typical scheduling problem in an electric power system. The electric power industry is undergoing restructuring and deregulation. In this paper we develop a stochastic programming model which incorporates power trading. We need to incorporate the uncertainties of electric power demand or electricity price into the unit commitment problem. It is assumed that demand and price uncertainty can be represented by a scenario tree. We propose a stochastic integer programming model in which the objective is to maximize expected profits. In this model, on/off decisions for each generator are made at the first stage. The approach to solving the problem is based on Lagrangian relaxation and dynamic programming.
Keywords: Electric Power; Unit Commitment; Stochastic Programming; Lagrangian Relaxation

1 Introduction

The economic operation and planning of electric power generation occupy an important position in the electric power industry. Wood and Wollenberg [17] offered a brief overview and applied many operations research methods to real electric power problems. Electric power utilities have to maintain sufficient capacity to meet electricity demand during peak load periods. The unit commitment problem is determining the schedules for power generating units and the generating level of each unit. The decisions concern which units to commit during each time period, and at what level to generate power to meet the electricity demand. The objective is to minimize the operational cost. This is the sum of the fuel and the start-up costs. The problem is a typical scheduling problem in an electric power system.

Many types of optimization technique have been applied to the unit commitment problem. Delson and Shahidehpour [6] illustrated how linear and integer programming were applied to power system engineering including generation scheduling, the allocation of reactive power supply and the planning of capital investment in generation equipment. The paper by Sheble and Fahd [9] is a survey in this field for the period from the late 1960's to the early 1990's. They classified the solution techniques into exhaustive enumeration, priority list, dynamic programming, integer and mixed-integer programming, branch-and-bound, linear programming, network flow programming, Lagrangian relaxation, and expert systems/artificial neural networks. Among these approaches, the Lagrangian relaxation technique seems to be the most promising because it decomposes the original problem into smaller subproblems. Muckstadt and Koenig [7] used this approach by relaxing the demand constraints. Bard [1] used the Lagrangian relaxation to disaggregate the problem into subproblems that were then solved by dynamic programming.

In these studies, the electricity demand at any period is known in advance. For many actual problems, however, such an assumption is often unjustified. These data contain uncertainty and are represented as random variables since the data represent information about the future. Takriti, Birge and Long [13] is the first paper that deals with the stochastic programming approach. Stochastic programming (Birge [3], Birge and Louveaux [4], Shiina [10]) is a method for an optimization problem under uncertainty. Takriti, Birge and Long [13] extended a technique used in the traditional deterministic unit commitment problem. The uncertainty in demand is modeled by introducing a set of scenarios. The problem is decomposed and solved using a Lagrangian relaxation type method, called a progressive hedging algorithm (Rockafellar and Wets [8]). For each scenario, the relaxed deterministic problem is then decomposed into a single generator subproblem using Lagrangian relaxation. Each subproblem can be solved efficiently by dynamic programming. Takriti and Birge [15] generalized this approach and showed that the duality gap of the relaxation is bounded by a certain constant. Shiina [11] developed a stochastic programming model in which on/off decisions for generators are made at the first stage. Takriti and Birge [14] developed a technique for refining the solution obtained by solving the Lagrangian relaxation problem. Their approach is to select a schedule among the feasible solutions sought up to the latest iteration, and to combine them so that the demand constraint can be met. The suggested model is a mixed-integer programming problem which is solved by branch-and-bound method. Though the numerical results indicated improvements, the approach is no more than heuristic. An efficient method for finding feasible schedules, and for refining them, is required.

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Shiina and Birge [12] propose a new algorithm, based on the Dantzig-Wolfe decomposition (Dantzig-Wolfe [5]) and the column generation approach (Barnhart et al. [2]), to solve the stochastic unit commitment problem.

The electric power industry is undergoing restructuring and deregulation. Takriti, Krasenbrink and Wu [16] presented a stochastic programming model that incorporates power trading and uncertainty in electricity demand and spot prices. They modeled the spot market as two generating units which represent buying and selling. In this model, the on/off decisions of generating units are defined as recourse variables. But some types of generators such as coal fired units involve time delay before the generators become available.

In this paper we develop a new stochastic programming model in which power trading is incorporated and switching decisions for generators are made at the first stage. We assume that there are hourly spot markets for electricity so that buying or selling do not affect spot price. We need to incorporate the uncertainty of electric power demand or electricity price into the unit commitment problem. It is assumed that demand and price uncertainty can be represented by a scenario tree. We propose a stochastic integer programming model in which the objective is to maximize expected profits. In this model, no artificial units are necessary to model the spot market. This problem is formulated as a multi-stage stochastic quadratic integer programming problem because the fuel cost function is assumed to be a convex quadratic function. The solution approach is based on Lagrangian relaxation method and dynamic programming. The problem is decomposed into subproblems of single units. The feasible schedule is obtained by solving dynamic programming on a scenario tree. To refine the solution obtained by dynamic programming, we solve an economic dispatch problem in which the equality demand constraint is relaxed to the inequality constraint with upper and lower limit. To solve this problem we develop an algorithm which combines the lambda iteration method and golden section.

2 Uncertainty in Electricity Demand and Price

We assume the duration of the planning horizon in T periods. Since the electricity demand or spot market price at any point in a period may be uncertain, we have to model the unit commitment problem as a stochastic programming problem.

To model uncertainty, we define the total demand for electricity and spot market price during period t as a random variable $\tilde{d}_t(\geq 0)$ and $\tilde{e}_t(\geq 0)$, respectively. It is assumed that $\tilde{d}_t$ and $\tilde{e}_t$ are defined on a known probability space and have a finite discrete distribution. Let d_t and e_t be realizations of random variable $\tilde{d}_t$ and $\tilde{e}_t$. A sequence of the realization of electricity demand and spot market price $(d_1,\ldots,d_T,e_1,\ldots,e_T)$ is called scenario. It is assumed that we have a set of S scenarios. We use a superscript s to denote an index of scenario s. We associate a probability p_s with each scenario $d^s=(d_1^s,\ldots,d_T^s), e^s=(e_1^s,\ldots,e_T^s), s=1,\ldots,S$. The scenarios are described using a scenario tree.

If two scenarios $(d^{s_1},e^{s_1}),(d^{s_2},e^{s_2}),$ $(s_1\neq s_2)$ satisfy the following conditions $(d_1^{s_1},\ldots,d_t^{s_1})=(d_1^{s_2},\ldots,d_t^{s_2})$, $(e_1^{s_1},\ldots,e_t^{s_1})=(e_1^{s_2},\ldots,e_t^{s_2})$ for a period t, they are indistinguishable up to period t. The decisions made for scenario (d^{s_1},e^{s_1}) up to period t must be the same as those made for (d^{s_2},e^{s_2}) up to period t. These two scenarios are said to be included in the same bundle at period t. The set of indices for the scenarios $\{1,\ldots,S\}$ at each period can be partitioned into disjoint subsets which represent scenario bundles. We define $B(s,t)$ to be the bundle in which scenario s is a member at period t.

If $B(s',t)=B(s,t)$ and $B(s',t+1)\neq B(s,t+1), s'<s$, the period $t+1$ is a point when scenario s splits from the other scenario s'. The scenario s' is called a predecessor of scenario s. If there are multiple predecessors for s, we define the scenario with the lowest index as the predecessor of s. The predecessor of scenario s is denoted by $Pred(s)$. The period $\tau(s)$ is defined as the first period in which a scenario s does not share a bundle with another scenario $s'<s$. For scenario 1, we define $\tau(1)=1$ and $Pred(1)=1$.

3 Stochastic Unit Commitment Problem

We assume that there are I generating units. The status of unit i at period t is represented by the 0-1 variable u_{it}^s. Unit i is on at time period t under scenario s, if $u_{it}^s=1$, and off if $u_{it}^s=0$. The power generating level of the unit i at period t under scenario s is represented by $x_{it}^s(\geq 0)$.

In this model, we assume that the amount of power which can be traded with the power pool is limited. We denote the amount of power which can be bought from power pool at period t by $\underline{d_t}$, the amount which can be sold to power pool at period t by $\overline{d_t}$.

$$\sum_{i=1}^{I} x_{it}^s \geq d_t^s - \underline{d_t}, t=1,\ldots,T, s=1,\ldots,S \tag{1}$$

If $\sum_{i=1}^{I} x_{it}^s < d_t^s$, the sum of the levels of generation is less than the demand. In this case, the demand shortage is satisfied by buying from the power pool.

$$\sum_{i=1}^{I} x_{it}^s \leq d_t^s + \overline{d_t}, t = 1, \ldots, T, s = 1, \ldots, S \tag{2}$$

If $\sum_{i=1}^{I} x_{it}^s > d_t^s$, the sum of the levels of generation is greater than the demand. In this case, the surplus of demand is sold to the power pool.

Rapid changes of temperature are not allowed for thermal units. When unit i is switched on, it must continue to run for at least a certain period L_i. These minimum up-time constraints are described in (3).

$$u_{it} - u_{i,t-1} \leq u_{i\tau}, \tau = t+1, \ldots, \min\{t + L_i - 1, T\}, t = 2, \ldots, T \tag{3}$$

Similarly, when unit i is switched off, it must continue to be off at least l_i periods. These constraints are called minimum down-time constraints (4).

$$u_{i,t-1} - u_{it} \leq 1 - u_{i\tau}, \tau = t+1, \ldots, \min\{t + l_i - 1, T\}, t = 2, \ldots, T \tag{4}$$

Let $[q_i, Q_i]$ be an operating range of the generating unit i. That is, x_{it}^s has to satisfy the following constraints (5).

$$q_i u_{it} \leq x_{it}^s \leq Q_i u_{it}, i = 1, \ldots, I, t = 1, \ldots, T, s = 1, \ldots, S \tag{5}$$

For two scenarios which are members of the same bundle, the decision variables must be the same.

$$\begin{aligned} x_{it}^{s_1} = x_{it}^{s_2}, \quad & i = 1, \ldots, I, t = 1, \ldots, T, \\ & \forall s_1, s_2 \in \{1, \ldots, S\}, s_1 \neq s_2, B(s_1, t) = B(s_2, t) \end{aligned} \tag{6}$$

This type of constraint is called a nonanticipativity constraint, or a bundle constraint.

The fuel cost function $f_i(x_{it}^s)$ is given by a convex quadratic function of x_{it}^s. This function relates to the output of power generated by unit i and depends on the consumption of fuel. The fuel cost function is regarded as convex quadratic, since the incremental fuel cost is a linear increasing function of x_{it}^s. The start up cost function $g_i(u_{i,t-1}, u_{it})$ satisfies the condition $g_i(0,1) > 0, g_i(0,0) = 0, g_i(1,0) = 0, g_i(1,1) = 0$. The mathematical formulation of the stochastic unit commitment problem is described as follows.

(Stochastic Unit Commitment Problem)

$$\begin{aligned} \min \quad & \sum_{s=1}^{S} p_s \sum_{t=1}^{T} \sum_{i=1}^{I} \{f_i(x_{it}^s) u_{it}^s + g_i(u_{i,t-1}, u_{i,t})\} - \sum_{s=1}^{S} p_s \sum_{t=1}^{T} e_t^s (\sum_{i=1}^{I} x_{it}^s - d_t^s) \\ \text{subject to} \quad & \sum_{i=1}^{I} x_{it}^s \geq d_t^s - \underline{d_t}, t = 1, \ldots, T, s = 1, \ldots, S \\ & \sum_{i=1}^{I} x_{it}^s \leq d_t^s + \overline{d_t}, t = 1, \ldots, T, s = 1, \ldots, S \\ & u_{it} - u_{i,t-1} \leq u_{i\tau}, \tau = t+1, \ldots, \min\{t + L_i - 1, T\}, \\ & \quad i = 1, \ldots, I, t = 2, \ldots, T \\ & u_{i,t-1} - u_{it} \leq 1 - u_{i\tau}, \tau = t+1, \ldots, \min\{t + l_i - 1, T\}, \\ & \quad i = 1, \ldots, I, t = 2, \ldots, T \\ & u_{it} \in \{0, 1\}, i = 1, \ldots, I, t = 1, \ldots, T \\ & q_i u_{it} \leq x_{it}^s \leq Q_i u_{it}, i = 1, \ldots, I, t = 1, \ldots, T, s = 1, \ldots, S \\ & x_{it}^{s_1} = x_{it}^{s_2}, i = 1, \ldots, I, t = 1, \ldots, T, \\ & \quad \forall s_1, s_2 \in \{1, \ldots, S\}, s_1 \neq s_2, B(s_1, t) = B(s_2, t) \end{aligned}$$

The problem results in a large scale mixed integer nonlinear programming problem that combines S deterministic unit commitment problems. The objective is to maximize the expected operational profit over all possible scenarios.

4 Solution Algorithm

4.1 Lagrangian Relaxation

First, we consider solving the stochastic unit commitment problem using a Lagrangian relaxation approach. Instead of solving the problem directly, we solve the Lagrangian relaxation which results from relaxing the demand constraints. Let $\lambda_t^s(\geq 0), \mu_t^s(\geq 0)$ be Lagrange multipliers associated with constraints (1), (2), respectively. The Lagrangian relaxation problem is shown as follows.

(Lagrangian Relaxation Problem)

$$\begin{aligned}
L(\lambda,\mu) = \min \quad & \sum_{s=1}^{S} p_s \sum_{t=1}^{T}\sum_{i=1}^{I}\{f_i(x_{it}^s)u_{it}^s + g_i(u_{i,t-1},u_{i,t})\} - \sum_{s=1}^{S} p_s \sum_{t=1}^{T} e_t^s(\sum_{i=1}^{I} x_{it}^s - d_t^s) \\
& -\sum_{s=1}^{S}\sum_{t=1}^{T}\lambda_t^s(\sum_{i=1}^{I} x_{it}^s - d_t^s + \underline{d_t}) - \sum_{s=1}^{S}\sum_{t=1}^{T}\mu_t^s(-\sum_{i=1}^{I} x_{it}^s + d_t^s + \overline{d_t}) \\
\text{subject to} \quad & u_{it} - u_{i,t-1} \leq u_{i\tau}, \tau = t+1,\dots,\min\{t+L_i-1,T\}, \\
& i=1,\dots,I, t=2,\dots,T \\
& u_{i,t-1} - u_{it} \leq 1 - u_{i\tau}, \tau = t+1,\dots,\min\{t+l_i-1,T\}, \\
& i=1,\dots,I, t=2,\dots,T \\
& u_{it} \in \{0,1\}, i=1,\dots,I, t=1,\dots,T \\
& q_i u_{it} \leq x_{it}^s \leq Q_i u_{it}, i=1,\dots,I, t=1,\dots,T, s=1,\dots,S \\
& x_{it}^{s_1} = x_{it}^{s_2}, i=1,\dots,I, t=1,\dots,T, \\
& \forall s_1,s_2 \in \{1,\dots,S\}, s_1 \neq s_2, B(s_1,t) = B(s_2,t)
\end{aligned}$$

This relaxation decomposes the problem into smaller single-generator subproblems. The objective function of Lagrangian relaxation problem $L(\lambda,\mu)$ can be rewritten as follows.

$$\begin{aligned}
L(\lambda,\mu) = \min \quad & \sum_{i=1}^{I}\sum_{t=1}^{T}\sum_{s=1}^{S}[p_s\{f_i(x_{it}^s)u_{it}^s - e_t^s x_{it}^s\} - \lambda_t^s x_{it}^s + \mu_t^s x_{it}^s] + \sum_{i=1}^{I}\sum_{t=1}^{T} g_i(u_{i,t-1},u_{i,t}) \\
& + \sum_{s=1}^{S}\sum_{t=1}^{T}\{p_s e_t^s d_t^s - \lambda_t^s(-d_t^s + \underline{d_t}) - \mu_t^s(d_t^s + \overline{d_t})\}
\end{aligned} \tag{7}$$

The last term of objective function (7) is a constant. So the function (7) is separable in each unit.

The Lagrangian relaxation problem can be solved by calculating dynamic programming on the scenario tree. First, we solve the following generation level decision problem to obtain optimal $\hat{x}_{it}^s, t=\tau(s),\dots,T, s=1,\dots,S$. The problem is a convex quadratic programming problem which can be solved easily.

(Generation Level Decision Problem for Unit i at Period t under Scenario s)

$$\begin{aligned}
\min \quad & \sum_{s''\in B(s,t)} p_{s''}\{f_i(x_{it}^s) - e_t^s x_{it}^s\} - (\lambda_t^s - \mu_t^s)x_{it}^s \\
\text{subject to} \quad & q_i \leq x_{it}^s \leq Q_i
\end{aligned}$$

Let $\hat{x}_{it}^s, t=\tau(s),\dots,T, s=1,\dots,S$ be solutions of the generation level decision problem. By setting $x_{it}^{s''} = \hat{x}_{it}^s, t=1,\dots,T, \forall s'' \in B(s,t)$, the scenario bundle constraints can be satisfied.

Then the binary variables $u_{it}^s, t=1,\dots,T, s=1,\dots,S$ are determined. The calculation of dynamic programming is done by the following recursive equations. A unit i must be in one of $L_i + l_i$ states. The first L_i states mean that the unit i is on, and the last l_i states mean that the unit i is off. Let $C_i(t,k)$ be the optimal cost of unit i under the scenario s from stage t to the end of the horizon, if unit i is in state k at stage t. The recursive equations are defined as shown in (8).

$$C_i(t,k) = \begin{cases}
C_i(t+1,k+1) + \sum_{s\in\{s':\tau(s')\leq t\}} (\sum_{s''\in B(s,t)} p_{s''})\{f_i(\hat{x}_{it}^s) - e_t^s\hat{x}_{it}^s\} - (\lambda_t^s - \mu_t^s)\hat{x}_{it}^s & \text{if } 1 \leq k < L_i \\
\min[C_i(t+1,k) + \sum_{s\in\{s':\tau(s')\leq t\}} (\sum_{s''\in B(s,t)} p_{s''})\{f_i(\hat{x}_{it}^s) - e_t^s\hat{x}_{it}^s\} - (\lambda_t^s - \mu_t^s)\hat{x}_{it}^s & \\
C_i(t+1,k+1) + \sum_{s\in\{s':\tau(s')\leq t\}} (\sum_{s''\in B(s,t)} p_{s''})\{f_i(\hat{x}_{it}^s) - e_t^s\hat{x}_{it}^s\} - (\lambda_t^s - \mu_t^s)\hat{x}_{it}^s] & \text{if } k = L_i \\
C_i(t+1,k+1) & \text{if } L_i < k < L_i + l_i \\
\min\{C_i(t+1,k), C_i(t+1,1) + g_i(0,1)\} & \text{if } k = L_i + l_i
\end{cases} \tag{8}$$

In the recursive equation of dynamic programming (8) at period t, the probability of scenario s ($\tau(s) \leq t$) is replaced by $\sum_{s''\in B(s,t)} p_{s''}$. This procedure makes possible an exact calculation of dynamic programming. We define the optimal $u_{it}, i=1,\dots,I, t=1,\dots,T$ obtained from (8) by $\hat{u}_{it}, i=1,\dots,I, t=1,\dots,T$.

Then we maximize the Lagrangian dual function because the function is concave.

(Lagrangian Dual Problem)

$$\max L(\lambda,\mu) = \sum_{s=1}^{S} p_s \sum_{t=1}^{T}\sum_{i=1}^{I}\{f_i(\hat{x}_{it}^s)\hat{u}_{it}^s + g_i(\hat{u}_{i,t-1},\hat{u}_{i,t})\} - \sum_{s=1}^{S} p_s \sum_{t=1}^{T} e_t^s(\sum_{i=1}^{I}\hat{x}_{it}^s - d_t^s)$$
$$-\sum_{s=1}^{S}\sum_{t=1}^{T}\lambda_t^s(\sum_{i=1}^{I}\hat{x}_{it}^s - d_t^s + \underline{d_t}) - \sum_{s=1}^{S}\sum_{t=1}^{T}\mu_t^s(-\sum_{i=1}^{I}\hat{x}_{it}^s + d_t^s + \overline{d_t})$$

subject to $\lambda \geq 0, \mu \geq 0$

The optimal objective value of the Lagrangian relaxation problem provides the lower bound of the original objective value if the solution satisfies the constraints (1) and (2). Then we wish to obtain the largest possible lower bound. However, the function is not differentiable, so we use the subgradient optimization technique. Let λ^0, μ^0 be any initial multiplier. The Lagrangian multipliers are refined for $l = 1, 2, \dots,$ as shown in (9), where α_l denotes a step size at iteration l.

$$\lambda^{l+1} = \lambda^l + \alpha_l \underline{\xi^l}, \ \mu^{l+1} = \mu^l + \alpha_l \overline{\xi^l} \tag{9}$$

A vector $(\underline{\xi^l}, \overline{\xi^l})$ is called subgradient of $L(\lambda,\mu)$ at (λ^l,μ^l) if inequality (10) holds.

$$L(\lambda,\mu) \leq L(\lambda^l,\mu^l) + (\lambda^\top - \lambda_l^\top)\underline{\xi^l} + (\mu^\top - \mu_l^\top)\overline{\xi^l} \tag{10}$$

In the Lagrangian dual problem, $(\underline{\xi^l}, \overline{\xi^l})$ is given as follows.

$$\underline{\xi^l} = \sum_{i=1}^{I}\hat{x}_{it}^s + d_t^s - \underline{d_t}, \overline{\xi^l} = \sum_{i=1}^{I}\hat{x}_{it}^s - d_t^s - \overline{d_t} \tag{11}$$

The nessesary and sufficient condition for the convergence of the sequence (λ^l, μ^l) to obtain the optimal solution is shown as follows.

$$\alpha_l \|\xi^l\| \to 0 \text{and} \sum_l \alpha_l \|(\underline{\xi^l}, \overline{\xi^l})\| \to 0 \tag{12}$$

A geometric convergence rate can be achieved if we set α_l as (13), where L^* denotes an optimal value of $L(\lambda,\mu)$.

$$\alpha_l = \frac{L^* - L(\lambda^l)}{\|\xi^l\|^2} \tag{13}$$

But we cannot know the value of L^* in advance. Instead, we adopt a heuristic for selecting the step length.

$$\alpha_l = \frac{\theta^l(UB - L(\lambda^l))}{\|\xi^l\|^2} \tag{14}$$

In this expression, UB denotes an upper bound of $L(\lambda,\mu)$ and θ^l is chosen between 0 and 2.

This procedure terminates if the difference between the upper bound and lower bound is relatively small. For further details, the reader should refer to the paper by Takriti, Krasenbrink and Wu [16].

4.2 Economic dispatch problem with power trading

The optimal solution obtained from solving the Lagrangian relaxation problem may not give a primal feasible solution. After solving the Lagrangian relaxation problem by dynamic programming, the set of solutions $\hat{x}_{it}^s, \hat{u}_{it}, i = 1,\dots,I, s = 1,\dots,S, t = \tau(s),\dots,T$ is obtained. To modify the level of power generation $\hat{x}_{it}^s$, we solve the following economic dispatch problem with power trading for $s = 1,\dots,S, t = \tau(s),\dots,T$.

(Economic Dispatch Problem with Power Trading at Period t under Scenario s)

$$\min \ \sum_{i=1}^{I} f_i(x_{it}^s)\hat{u}_{it}^s - e_t^s(\sum_{i=1}^{I} x_{it}^s - d_t^s)$$

subject to

$$\sum_{i=1}^{I} x_{it}^s \geq d_t^s - \underline{d_t}, t = 1,\dots,T, s = 1,\dots,S$$
$$\sum_{i=1}^{I} x_{it}^s \leq d_t^s + \overline{d_t}, t = 1,\dots,T, s = 1,\dots,S$$
$$q_i\hat{u}_{it} \leq x_{it}^s \leq Q_i\hat{u}_{it}, i = 1,\dots,I, t = 1,\dots,T, s = 1,\dots,S$$

Lagrangian relaxation

- **Step 0.** Given Lagrangian multipliers $\lambda^0, \mu^0 (> 0)$.
- **Step 1.** Decompose Lagrangian dual into smaller single-generator subproblems by relaxing constraints (1) and (2).
- **Step 2.** Solve generation level decision problem to obtain $\hat{x}^s_{it}, s = 1, \ldots, S, t = \tau(s), \ldots, T$. Then solve dynamic programming problem by recursive equations (8) to obtain $\hat{u}_{it}, i = 1, \ldots, I, t = \tau(s), \ldots, T$.
- **Step 3.** Update Lagrange multiplier by subgradient optimization (9). Go to step 1.

Figure 1: Algorithm of Lagrangian relaxation method

If the value of $\sum_{i=1}^I x^s_{it}$ is fixed and the constraints $\sum_{i=1}^I x^s_{it} \geq d^s_t - \underline{d_t}$, $\sum_{i=1}^I x^s_{it} \leq d^s_t + \overline{d_t}$ do not exist, the problem can be solved by the lambda iteration method (Wood and Wollenberg [17]). The lambda iteration method, which is based on the method of indeterminate coefficents, seeks the optimal value of undetermined multipliers using binary search. We develop a solution method which combines the lambda iteration method and the golden section. In our approach, the economic power dispatch problem with power trading is formulated as a parametric optimization problem. Then the golden section is applied to select an optimal parameter. The lambda iteration method is also modiified in order to treat the constraints $\sum_{i=1}^I x^s_{it} \geq d^s_t - \underline{d_t}$, $\sum_{i=1}^I x^s_{it} \leq d^s_t + \overline{d_t}$.

(Parametric Optimization Problem: P(α))

$$\begin{aligned} z(\alpha) = \min \quad & \sum_{i=1}^I f_i(x^s_{it})\hat{u}^s_{it} - e^s_t(\sum_{i=1}^I x^s_{it} - d^s_t) \\ \text{subject to} \quad & \sum_{i=1}^I x^s_{it} = \alpha \\ & q_i\hat{u}_{it} \leq x^s_{it} \leq Q_i\hat{u}_{it}, i = 1, \ldots, I \end{aligned}$$

We solve the parametric optimization problem in the following range of parameter α.

$$\max\{\sum_{i=1}^I q_i\hat{u}_{it}, d^s_t - \underline{d_t}\} \leq \alpha \leq \min\{\sum_{i=1}^I Q_i\hat{u}_{it}, d^s_t + \overline{d_t}\}$$

The algorithm to solve economic dispatch problem with power trading is shown in Figure 2.

Algorithm to solve Economic Dispatch Problem with Power Trading by Golden Section

- **Step 0.** Set $\underline{\alpha} = \max\{\sum_{i=1}^I q_i\hat{u}_{it}, d^s_t - \underline{d_t}\}$, $\overline{\alpha} = \min\{\sum_{i=1}^I Q_i\hat{u}_{it}, d^s_t + \overline{d_t}\}$, $\epsilon > 0$ and $k = 0$.
- **Step 1.** Set $\alpha_1 = \underline{\alpha} + F_1(\overline{\alpha} - \underline{\alpha}), \alpha_2 = \underline{\alpha} + F_2(\overline{\alpha} - \underline{\alpha})$, where $F_1 = \frac{3-\sqrt{5}}{2}, F_2 = \frac{\sqrt{5}-1}{2}$.
- **Step 2.** If $\overline{\alpha} - \underline{\alpha} < \epsilon$, then stop.
- **Step 3.** If $z(\alpha_1) < z(\alpha_2)$, then set $\overline{\alpha} = \alpha_2$, $\alpha_2 = \alpha_1$ and $\alpha_1 = \underline{\alpha} + F_1(\overline{\alpha} - \underline{\alpha})$. If $z(\alpha_1) \geq z(\alpha_2)$, then set $\underline{\alpha} = \alpha_1$, $\alpha_1 = \alpha_2$ and $\alpha_2 = \underline{\alpha} + F_2(\overline{\alpha} - \underline{\alpha})$. Go to step 2.

Figure 2: Algorithm to solve Economic Dispatch Problem with Power Trading

In step 3 of the algorithm shown in Figure 2, parametric optimization problem P(α) is solved. To solve problem P(α), the lambda iteration method is modified. We briefly summarize the outline of our method. We consider the following economic dispatch problem in which the constraints $q_i\hat{u}_{it} \leq x^s_{it} \leq Q_i\hat{u}_{it}, i = 1, \ldots, I$ are relaxed.

(Relaxed economic Dispatch Problem)

$$\min \quad \sum_{i=1}^{I} f_i(x_{it}^s)\hat{u}_{it}^s - e_t^s(\sum_{i=1}^{I} x_{it}^s - d_t^s)$$

$$\text{subject to} \quad \phi = \sum_{i=1}^{I} x_{it}^s - \alpha = 0$$

To solve the relaxed economic dispatch problem, the indeterminate coefficents method is applied. The constraint function ϕ is multiplied by an undetermined multiplier π and added to the objective function. This function is known as the Lagrange function denoted by L.

$$L = \sum_{i=1}^{I} f_i(x_{it}^s)\hat{u}_{it}^s - e_t^s(\sum_{i=1}^{I} x_{it}^s - d_t^s) - \pi(\sum_{i=1}^{I} x_{it}^s - \alpha)$$

The nessesary condition for an extreme value of the objective function is shown as follows.

$$\frac{\partial L}{\partial x_{it}^s} = f_i^{'}(x_{it}^s)\hat{u}_{it} - e_t^s - \pi = 0, i = 1, \ldots, I \quad (15)$$

$$\frac{\partial L}{\partial \pi} = \phi = \sum_{i=1}^{I} x_{it}^s - \alpha = 0 \quad (16)$$

The lambda iteration method seeks an optimal solution by obtaining the optimal π using the binary search. In our algorithm, the lambda iteration method is modified so as to satisfy the constraints $q_i\hat{u}_{it} \leq x_{it}^s \leq Q_i\hat{u}_{it}, i = 1, \ldots, I$. The algorithm is shown in Figure 3.

Modified Algorithm of Lambda Iteration

- **Step 0.** Suppose $\sum_{i=1}^{I} q_i\hat{u}_{it} - \alpha < 0$ and $\sum_{i=1}^{I} Q_i\hat{u}_{it} - \alpha > 0$. Solve $f_i^{'}(q_i)\hat{u}_{it} - e_t^s - \underline{\pi} = 0$ and $f_i^{'}(Q_i)\hat{u}_{it} - e_t^s - \overline{\pi} = 0$ to obtain $\underline{\pi}$ and $\overline{\pi}$.
- **Step 1.** Let $\hat{x}_{it}^s$ be the solution of the equation $f_i^{'}(x_{it}^s)\hat{u}_{it} - e_t^s - \frac{\underline{\pi}+\overline{\pi}}{2} = 0$ If $\hat{x}_{it}^s < q_i$, then $\hat{x}_{it}^s = q_i$. Else if $\hat{x}_{it}^s > Q_i$, then $\hat{x}_{it}^s = Q_i$.
- **Step 2.** If $\sum_{i=1}^{I} \hat{x}_{it}^s - \alpha < 0$, then $\overline{\pi} = \underline{\pi} + \frac{\underline{\pi}+\overline{\pi}}{2}$. Otherwise, if $\sum_{i=1}^{I} \hat{x}_{it}^s - \alpha > 0$, then $\underline{\pi} = \underline{\pi} + \frac{\underline{\pi}+\overline{\pi}}{2}$. Go to step 1.

Figure 3: Modified Algorithm of Lambda Iteration

5 Numerical Experiments

This section demonstrates how our algorithm works. The column generation method for the stochastic unit commitment problem was implemented using C language on SUN Enterprise 420R (450MHz UltraSPARCII). The whole framework of the algorithm was coded in C.

We applied our solution method to the problems based on an electric power utility in Japan. Some of the data was modified for the sake of confidentiality. The test problems considered in this section consist of $I = 10, 20, 30$ units, $T = 168$ periods and $S = 16$ scenarios. We have predicted demand for electric power $\overline{d}_t$ for $t = 1, \ldots, T$. The demand data $\overline{d}_t$ was also multiplied by $I/10$. The scenarios are generated from the predicted power demand by increasing or reducing predicted demand $\overline{d}_t$, as shown in Table 1.

We assume that hourly spot price e_t^s follows the next equation (17).

$$e_t^s = \sum_{i=1}^{I} f^{'}(\frac{q_i + Q_i}{2})/I + \frac{d_t^s - \overline{d}_t}{1000I} \quad (17)$$

Table 1: Demand increases and decreases for different scenarios

Scenario	Probability	Period t						
		Sun.	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.
s	p_s	1-24	25-48	49-72	73-96	97-120	121-144	145-168
1	0.02	0	+4%	+8%	+12%	+16%	+16%	+16%
2	0.03	0	+4%	+8%	+12%	+12%	+12%	+12%
3	0.04	0	+4%	+8%	+8%	+10%	+10%	+10%
4	0.05	0	+4%	+8%	+8%	+8%	+8%	+8%
5	0.06	0	+4%	+4%	+6%	+8%	+8%	+8%
6	0.08	0	+4%	+4%	+6%	+6%	+6%	+6%
7	0.10	0	+4%	+4%	+4%	+6%	+6%	+6%
8	0.12	0	+4%	+4%	+4%	+4%	+4%	+4%
9	0.12	0	-4%	-4%	-4%	-4%	-4%	-4%
10	0.10	0	-4%	-4%	-4%	-6%	-6%	-6%
11	0.08	0	-4%	-4%	-6%	-6%	-6%	-6%
12	0.06	0	-4%	-4%	-6%	-8%	-8%	-8%
13	0.05	0	-4%	-8%	-8%	-8%	-8%	-8%
14	0.04	0	-4%	-8%	-8%	-10%	-10%	-10%
15	0.03	0	-4%	-8%	-12%	-12%	-12%	-12%
16	0.02	0	-4%	-8%	-12%	-16%	-16%	-16%

Table 2 presents the results of our test problems. The first column provides the number of generators. For example, $I = 10 \times 3$ indicates that there are 10 sets of generators each of which contains 3 identical generating units. The columns labeled "Primal" provide the objective values which are modified from the solution of Lagrangian relaxation by the economic dispatch algorithm shown in Figure 2. The columns labeled "Dual" provides the value of Lagrangian. The number of iteration of Lagrangian relaxation is set to 1000, and the number of iteration of golden section and lambda iteration (Figure 3) is limited to 20 and 30, respectively.

We compare the conventional unit commitment model, in which we minimize the total cost, with our model which we developed by incorporating power trading. In the conventional model, power trading is not taken into consideration. The column labeled "Gain" shows the profit generated from considering power trading.

Table 2: Computational Results

I = Sets of generators $\times$ identical units	Conventional unit commitment			Unit commitment with power trading			Gain	Gain
	Primal	Dual	Gap(%)	Primal	Dual	Gap(%)		(%)
$10 = 10 \times 1$	3486224	3398633	2.51	3402920	3366875	1.05	83304	2.39
$20 = 10 \times 2$	6973285	6802345	2.45	6803181	6752377	0.75	170104	2.44
$30 = 10 \times 3$	10458953	10208589	2.39	10186560	10142858	0.43	272393	2.60

Then, we consider the value of the stochastic solution. Let $\overline{x}_{it}^{s}, \overline{u}_{it}$ be the solution of the unit commitment problem which is obtained by replacing all random variables with their expected values. This solution is referred to expected-value solution. We appliy this expected-value solution to all available scenarios and the expected value of the objective function at $(\overline{x}, \overline{u})$ can be computed. The difference between the expected objective value of the expected solution and the optimal objective value of the original stochastic programming problem is called the value of the stochastic solution, and is denoted as VSS. The value of the stochastic solution is shown in Table 3.

Table 3: Value of stochastic solution

$I =$	Expected objective value using expected-value solution	Expected objective value using stochastic solution	VSS	VSS (%)
$10 = 10\times 1$	3442207	3402920	39287	1.14%
$20 = 10\times 2$	6884416	6803181	81235	1.18%
$30 = 10\times 3$	10326624	10186560	140064	1.36%

6 Concluding Remarks

We proposed a stochastic integer programming model in which the objective is to maximize expected profits. This problem is formulated as a multi-stage stochastic quadratic integer programming problem because the fuel cost function is assumed to be a convex quadratic function. The approach to solving the problem is based on Lagrangian relaxation method and dynamic programming. The feasible schedule is obtained by solving dynamic programming on a scenario tree. To refine the solution obtained by dynamic programming, we solve an economic dispatch problem in which the equality demand constraint is relaxed to the inequality constraint with upper and lower limit. To solve this problem we develop an algorithm which combines the lambda iteration method and golden section. More research is necessary to make a scenario set which reflects real demand or spot price. As for application to real power systems, the coordination of the operation of hydroelectric generation plants is left as a future problem.

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