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Payoff Improvement of Measurable α-Cores through Communications *

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Abstract

The purpose of this paper is to clarify the relationship between coarse α-core and fine α-core in strategic form games with differential information. We analyze the role of information transmission among members in a coalition. In this paper it is proved that players can be better off through communications.

Key Words: coarse α-core, fine α-core, measurability, differential information

JEL Classifications: C79, D82.

1 Measurable α-Cores

1.1 Basic Definitions

In this section we define some notions. Let $N = \{1, ..., n\}$ be the set of players. We denote by $\mathcal{N}$ the set of all nonempty subsets of N, which is called the set of coalitions. Let Ω be a finite set. The set Ω represents the states of the world, and the generic element ω is called

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a state. As we consider how players send their own information through communication, we introduce an additional structure to the strategic form games.

DEFINITION 1. A strategic form game with differential information is a specified list of data $\{X_i, u_i, \mathcal{P}_i, \mu_i\}_{i\in N}$, where

(1) X_i is the set of strategies for player i,

(2) $u_i : \Omega \times \prod_{i\in N} X_i \to \mathbb{R}$ is player i's payoff function,

(3) $\mathcal{P}_i$ is a partition of Ω, which means players i's information, and

(4) μ_i is a strictly positive probability measure on Ω that represents player i's prior.

1.2 Measurable Conditions

In this subsection we consider the measurability of strategies. Without the measurability conditions, all strategies can be taken by players, although they have no concern with communications. The strategy, which each player does not know, may be chosen. That sounds strange. All strategies cannot always be chosen by them. In this sense it is important to introduce the measurable conditions.

We would like to consider the measurability and its implications on the core concept that deals with pooling information. To this end, we define the set of measurable strategies. According to the degree of communications among players, some variations of measurability conditions can be defined.

For notational convenience, we denote X_i^{Ω} by Σ_i. As usual, we define $\Sigma_S = \prod_{i\in S} \Sigma_i$ as the set of joint strategies in the coalition S, $\Sigma = \Sigma_N$, and $\Sigma_{-S} = \prod_{i\in N\setminus S} \Sigma_i$ as the set of complemental coalition's strategies. These representative elements are σ_S, σ, and σ_{-S} respectively.

An information structure for $S \in \mathcal{N}$ is a collection $(\Pi_i)_{i\in S}$ for partition of Ω. For each partition Π of Ω, we denote by $\Pi(\omega)$ the element of Π which contains ω. If $\Pi(\omega)$ is included in $\Pi'(\omega)$ for all ω, It is called Π is finer than Π', or Π' is coarser than Π. The meet of an information structure for S is the finest partition of Ω that is coarser than each Π_i for all i

in S, which is denoted by $\wedge_{i\in S}\Pi_i$. This situation describes that each player in S obtains the self-evident information among S through communications. For this reason no information is pooled in the coalition S. Similarly, the join of partitions $(\Pi_i)_{i\in S}$ is the coarsest partition that is finer than each Π_i, which is denoted by $\vee_{i\in S}\Pi_i$. This situation describes that each player in S fully communicates and pools their information.

DEFINITION 2. For all $S \in \mathcal{N}$, the set of *coarse-measurable strategies for S* is denoted by

$$\Sigma_S^c := \{\sigma_S \in \Sigma_S | \bigwedge_{i\in S} \mathcal{P}_i - \text{measurable}\}.$$

For all $S \in \mathcal{N}$, the set of *fine-measurable strategies for S* is denoted by

$$\Sigma_S^f := \left\{ \sigma_S \in \Sigma_S \middle| \exists(\Pi_i)_{i\in S} : \begin{array}{c} 1.\Pi_i \text{ is finer than or equal to } \mathcal{P}_i, \\ 2.\Pi_i \text{ is coarser than or equal to } \bigvee_{i\in S}\mathcal{P}_i, \\ 3.\sigma_S \text{ is } \bigwedge_{i\in S}\Pi_i - \text{measurable} \end{array} \right\}.$$

For all $S \in \mathcal{N}$, the set of *private-measurable strategies for S* is denoted by

$$\Sigma_S^p := \{\sigma_S \in \Sigma_S | \sigma_i \text{ is } \mathcal{P}_i\text{-measurable for all } i \in S\}.$$

Coarse-measurable strategies are common knowledge among the members in a coalition. They can cooperate to coordinate the strategies, however, they choose only common knowledge strategies among them. In the fine-measurable strategy case, the members in a coalition can communicate and make use of the information freely. In the private-measurable strategy situation, every player in a coalition does not reveal his own information. They can coordinate the strategies, while they do not pool any information.

In noncooperative situations, players are not permitted to communicate. For this reason, they cannot take joint strategies. Which action do they choose, if they could communicate as we consider? Aumann and Peleg (1960) defined two naive behavior principles, which are called α-behavior and β-behavior principles. These principles have been considered to be useful to predict what they do in communicative situations. In this paper, we deal with α-behavior principle, which has been analyzed in many papers, for the benchmark to analyze cooperative behavior situations.

Before the definition of α-effectiveness, we consider the players' expected payoffs. For each $\sigma \in \Sigma$ and partition Π_i, we denote by $Eu_i(\sigma|\Pi_i)$ the conditional expected utility for player i, which is defined as

$$Eu_i(\sigma|\Pi_i)(\omega^*) := \sum_{\omega\in\Pi_i(\omega^*)} \frac{\mu_i(\omega)}{\mu_i(\Pi_i(\omega^*))} u_i(\omega, \sigma_1(\omega), ..., \sigma_n(\omega)) \quad \text{for all } \omega^*.$$

DEFINITION 3. For all $S \in \mathcal{N}$ and $\omega^* \in \Omega$, *coarse measurable α-effectiveness for Σ^c* is defined as

$$V_c(S, \omega^*; \Sigma^c) := \bigcup_{\sigma_S\in\Sigma_S^c} \bigcap_{\sigma_{-S}\in\Sigma_{-S}^c} \left\{ u \in \mathbb{R}^N \;\middle|\; \begin{array}{r} \forall i \in S,\ \forall \omega \in (\bigwedge_{i\in S} \mathcal{P}_i)(\omega^*), \\ Eu_i(\sigma_S, \sigma_{-S}|\mathcal{P}_i)(\omega) \geq u_i \end{array} \right\},$$

which we call an optimistic case. For all $S \in \mathcal{N}$ and $\omega^* \in \Omega$, *coarse measurable α-effectiveness for Σ^f* is defined as

$$V_c(S, \omega^*; \Sigma^f) := \bigcup_{\sigma_S\in\Sigma_S^c} \bigcap_{\sigma_{-S}\in\Sigma_{-S}^f} \left\{ u \in \mathbb{R}^N \;\middle|\; \begin{array}{r} \forall i \in S,\ \forall \omega \in (\bigwedge_{i\in S} \mathcal{P}_i)(\omega^*), \\ Eu_i(\sigma_S, \sigma_{-S}|\mathcal{P}_i)(\omega) \geq u_i \end{array} \right\},$$

which we call a pessimistic case. For all $S \in \mathcal{N}$ and $\omega^* \in \Omega$, *coarse measurable α-effectiveness for Σ* is defined as

$$V_c(S, \omega^*; \Sigma) := \bigcup_{\sigma_S\in\Sigma_S^c} \bigcap_{\sigma_{-S}\in\Sigma_{-S}} \left\{ u \in \mathbb{R}^N \;\middle|\; \begin{array}{r} \forall i \in S,\ \forall \omega \in (\bigwedge_{i\in S} \mathcal{P}_i)(\omega^*), \\ Eu_i(\sigma_S, \sigma_{-S}|\mathcal{P}_i)(\omega) \geq u_i \end{array} \right\},$$

which we call a non-measurability case.

In the optimistic case, the member of a complemental coalition use only common knowledge information and choose coarse measurable strategies. In the pessimistic case, they can share the information and choose fine-measurable strategies. In the non-measurability case, a complemental coalition can choose any strategy, including strategies which are not measurable for any player.

We define the coarse measurable α-core concepts.

DEFINITION 4. For $\Sigma^k = \Sigma^c,\ \Sigma^f$, and Σ,
the coarse measurable α-core at ω is $C(Vc, \omega; \Sigma^k) := V_c(N, \omega, \Sigma^k) \setminus \bigcup_{S\in\mathcal{N}} \mathrm{int}V_c(S, \omega, \Sigma^k)$, and the coarse measurable α-core is $C(Vc; \Sigma^k) := (V_c(N, \omega, \Sigma^k) \setminus \bigcup_{S\in\mathcal{N}} \mathrm{int}V_c(S, \omega, \Sigma^k))_{\omega\in\Omega}$.

Let σ' satisfy $Eu_i(\sigma'|\mathcal{P}_i)(\omega) = u_i$ for all $i \in S$ and $\omega \in \wedge\mathcal{P}_i(\omega^*)$. For each $\Sigma^k = \Sigma^c,\ \Sigma^f$ and Σ, an element u in $\mathrm{int}V_c(S, \omega^*, \Sigma^k)$ states that σ' is improved by S for some strategy bundle. As players can choose joint strategy in Σ^c_S, it is considered that they can cooperate. However, they use only common information while they do not pool their information.

Especially, we call $C(V_c; \Sigma^c)$ the coarse measurable α-core. We can prove that the inclusion relation can hold among these core concepts.

REMARK 1.
$C(V_c; \Sigma^c) \subset C(V_c; \Sigma^f) \subset C(V_c; \Sigma)$.

When communications are available, we can also define various α-effectivenesses in the same way.

DEFINITION 5. For all $S \in \mathcal{N}$ and $\omega^* \in \Omega$, *fine measurable α-effectiveness for* Σ^c is defined as

$$V_f(S, \omega^*; \Sigma^c) := \left\{ u \in \mathbb{R}^N \;\middle|\; \begin{array}{l} \exists(\Pi_i)_{i\in S} \ : \\ 1.\Pi_i \text{ is finer than or equal to } \mathcal{P}_i, \\ 2.\Pi_i \text{ is coarser than or equal to } \bigvee_{i\in S} \mathcal{P}_i, \\ 3.\exists\sigma \in \Sigma^f_S : \wedge_{i\in S}\Pi_i\text{-measurable and} \\ \qquad \forall i \in S,\ \forall\sigma_{-S} \in \Sigma^c_{-S},\ \forall\omega \in (\bigwedge_{i\in S}\Pi_i)(\omega^*), \\ \qquad\quad Eu_i(\sigma_S, \sigma_{-S}|\Pi_i)(\omega) \geq u_i \end{array} \right\},$$

which we call an optimistic case. For all $S \in \mathcal{N}$ and $\omega^* \in \Omega$, *fine measurable α-effectiveness* *for* Σ^f is defined as

$$V_f(S,\omega^*;\Sigma^f) := \left\{ u \in \mathbb{R}^N \;\middle|\; \begin{array}{l} \exists(\Pi_i)_{i\in S} \;: \\ 1.\Pi_i \text{ is finer than or equal to } \mathcal{P}_i, \\ 2.\Pi_i \text{ is coarser than or equal to } \bigvee_{i\in S}\mathcal{P}_i, \\ 3.\exists\sigma \in \Sigma^f_S : \wedge_{i\in S}\Pi_i\text{-measurable and} \\ \qquad \forall i \in S,\ \forall\sigma_{-S} \in \Sigma^f_{-S},\ \forall\omega \in (\bigwedge_{i\in S}\Pi_i)(\omega^*), \\ \qquad\quad Eu_i(\sigma_S,\sigma_{-S}|\Pi_i)(\omega) \geq u_i \end{array} \right\},$$

which we call a pessimistic case. For all $S \in \mathcal{N}$ and $\omega^* \in \Omega$, *fine measurable α-effectiveness for* Σ is defined as

$$V_f(S,\omega^*;\Sigma) := \left\{ u \in \mathbb{R}^N \;\middle|\; \begin{array}{l} \exists(\Pi_i)_{i\in S} \;: \\ 1.\Pi_i \text{ is finer than or equal to } \mathcal{P}_i, \\ 2.\Pi_i \text{ is coarser than or equal to } \bigvee_{i\in S}\mathcal{P}_i, \\ 3.\exists\sigma \in \Sigma^f_S : \wedge_{i\in S}\Pi_i\text{-measurable and} \\ \qquad \forall i \in S,\ \forall\sigma_{-S} \in \Sigma^c_{-S},\ \forall\omega \in (\bigwedge_{i\in S}\Pi_i)(\omega^*), \\ \qquad\quad Eu_i(\sigma_S,\sigma_{-S}|\Pi_i)(\omega) \geq u_i \end{array} \right\},$$

which we call a non measurability case.

For each $\Sigma^k = \Sigma^c, \Sigma^f$, and Σ, u in $\text{int}V_f(S,\omega^*;\Sigma^k)$ means that there exist σ' and an information structure $(\Pi_i)_{i\in S}$ such that $Eu_i(\sigma'|\Pi_i)(\omega) = u_i$ for all i in S and ω in $(\wedge_{i\in S}\Pi_i)(\omega^*)$. The intuition of this definition is that the strategy bundle σ' is improved upon by S for some strategy bundle and information structures. The players in the coalition S can choose joint strategies through communications, and pool their information.

We also define the fine measurable α-core concepts in the same way.

DEFINITION 6. For $\Sigma^k = \Sigma^c, \Sigma^f$, and Σ,
the fine measurable α-core at ω is $C(V_f,\omega;\Sigma^k) := V_f(N,\omega,\Sigma^k) \setminus \bigcup_{S\in\mathcal{N}} \text{int}V_f(S,\omega,\Sigma^k)$, and the fine measurable α-core is $C(V_f,\omega;\Sigma^k) := (V_f(N,\omega,\Sigma^k) \setminus \bigcup_{S\in\mathcal{N}} \text{int}V_f(S,\omega,\Sigma^k))_{\omega\in\Omega}$.

Particularly, we call $C(V_f;\Sigma^f)$ the fine measurable α-core.

Remark 2.
$C(V_f;\Sigma^c) \subset C(V_f;\Sigma^f) \subset C(V_f;\Sigma)$.

Remark 3.
If $(\bigvee_{i\in S}\mathcal{P}_i)(\omega) = \{\omega\}$ for all $\omega \in \Omega$, then $\Sigma^f_S = \Sigma_S$ for all S.

As we focus on the strategy that players choose, we define the set of α-core strategies corresponding to V_c and V_f.

Definition 7. For $\Sigma^k = \Sigma^c, \Sigma^f$, and Σ, we define

$$X(V_c;\Sigma^k) := \left\{\sigma \in \Sigma \,\middle|\, \left((Eu_i(\sigma|\mathcal{P}_i)(\omega))_{i\in N}\right)_{\omega\in\Omega} \in C(V_c;\Sigma^k)\right\},$$

which is called the set of *coarse α-core strategies*, and

$$X(V_f;\Sigma^k) := \left\{\sigma \in \Sigma \,\middle|\, \exists(\Pi_i)_{i\in N} \;:\; \begin{array}{l} 1.\Pi_i \text{ is not coarser than } \mathcal{P}_i, \\ 2.\Pi_i \text{ is not finer than } \bigvee_{i\in N}\mathcal{P}_i, \\ 3. \left((Eu_i(\sigma|\mathcal{P}_i)(\omega))_{i\in N}\right)_{\omega\in\Omega} \in C(V_f;\Sigma^k)\} \end{array}\right\},$$

which is called the set of *fine α-core strategies.*

2 Basic Relations and Examples

First, we apply the various α-cores to some famous games. Second, we examine the basic relations between coarse measurable α-core and fine measurable α-core.

Example 1. Presoners' Dilemma

This example suggests that the coarse measurable α-core concept does not include the fine measurable α-core concept. The set of players is $\{1,2\}$. The state set is given by $\{\omega_1,\omega_2\}$. Let the information structure of player 1 $\mathcal{P}_1$ be $\{\{\omega_1\},\{\omega_2\}\}$ and player 2's $\mathcal{P}_2$ be $\{\{\omega_1,\omega_2\}\}$. Player 2 places equal probability on the each state. The row player and colum player denote player 1 and 2 respectively.

Let $X_i = \{C, D\}$ for $i = 1, 2$. The set of the coarse measurable strategy for $\{1, 2\}$ is $\Sigma^c = \{((C,C),(C,C)),((C,C),(D,D)),((D,D),(C,C)),((D,D),(D,D))\}$. The first component of the bundle $((C,C),(C,C))$ means player 1's action is C at both states. The set of the fine measurable strategy for $\{1,2\}$ is $\Sigma^f = \Sigma$. The payoff matrices are given by the following table, which represent the prisoners' dilemma with two states.

1\2	C	D
C	5,5	1,6
D	6,1	3,3

state ω_1

1\2	C	D
C	3,3	0,4
D	4,0	2,2

state ω_2

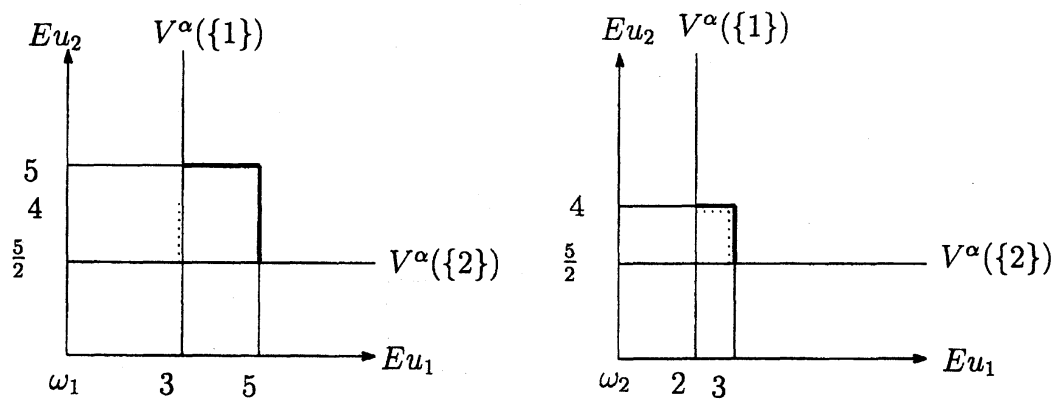

Figure 1: Prisonars' dilemma

The bold line of figure 1 is the fine measurable α-core. The dot line of this graph is the coarse measurable α-core. Two graphs suggest that the coarse measurable α-core concept does not include the fine measurable α-core concept. Moreover, $((C,C),(C,C))$ is the fine α-core strategy and also the coarse α-core strategy.

REMARK 4.

In fine measurable α-core, player 2's information at ω_1 is $\{\{\omega_1, \omega_2\}\}$, and at ω_2 is $\{\{\omega_1, \omega_2\}\}$.

At state ω_1, player 1 and 2 pool their information through communication. Player 1 transmit his own information to player 2. At state ω_2, the information transmission does not occur through communication. ◇

Example 2. Coordination game

The settings are similar to Example 1, except for the payoff matrices. The payoff matrices are defined as below.

1\2	A	B
A	2,2	0,0
B	0,0	1,1

state ω_1

1\2	A	B
A	1,1	0,0
B	0,0	2,2

state ω_2

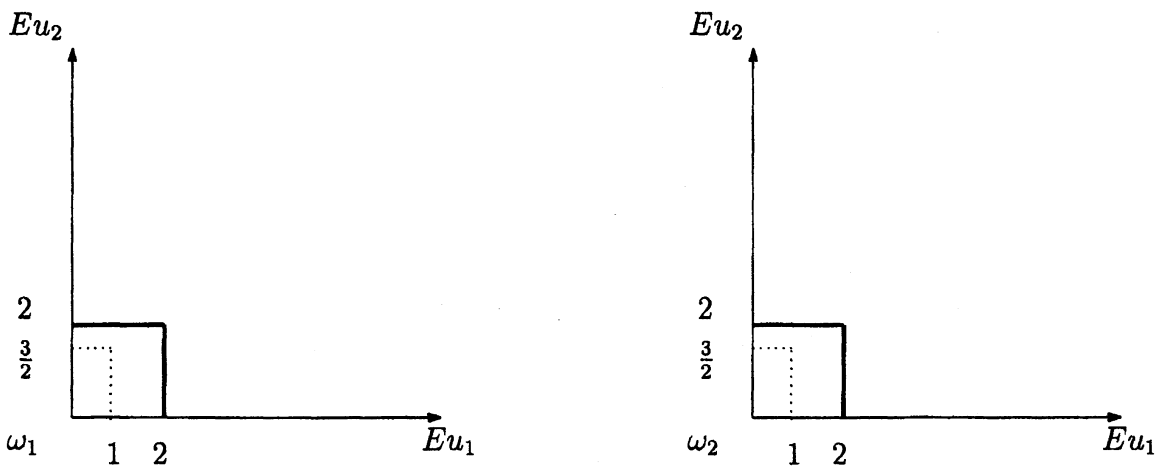

Figure 2: Coordination game

First, the fine measurable α-core is denoted by the bold lines. The coarse measurable α-core is denoted by the dot lines of the figure. In the fine measurable α-core, both players make use of the information and achieve the efficient payoffs at both states. In the coarse measurable α-core, players cannot make use of sophisticated information. Consequently, *the fine measurable α-core and the coarse measurable α-core have no relation.*

Second, to realize the fine measurable α-core, each player choose the strategy "A at ω_1 and B at ω_2 ". The fine measurable α-core strategy is $((A,B),(A,B))$. On the other hand, the coarse measurable α-core strategies are $((A,A),(A,A))$ and $((B,B),(B,B))$. Hence, we can obtain that *the fine measurable α-core strategies and the coarse measurable α-core strategies have no relation.*

REMARK 5.

In fine measurable α-core, player 2's information is $\{\{\omega_1\}, \{\omega_2\}\}$ at each state.

For each state, the information transmission occurs. $\diamondsuit$

Example 1 and Example 2 suggest that the inforamtion transmission does not always occur through communication.

Example 3. n-persons social dilemma game

Let $N = \{1, ..., n\}$ be the set of players. The set of strategies is defined as $X_i = \{C, D\}$, where we interpret C as cooperation to produce some public goods, and D as non-cooperation. The notation r is the minimum number to implement public productions. Namely, if the number of players, who choose C, is greater than or equal to r, then the public goods are produced. Otherwise, the public goods are not produced, and the players, who choose C, pay the cost K.

The set $\Omega = \{\omega_1, \omega_2\}$ denotes the states of the world. The information structure $\mathcal{P}_1 = \{\{\omega_1\}, \{\omega_2\}\}$ and $\mathcal{P}_j = \{\{\omega_1, \omega_2\}\}$ for all $j \neq 1$. For notational convenience, the number of players who choose C is denoted by c. Player i's payoff function is given by

$$u_i(\omega, \sigma_1(\omega), ..., \sigma_n(\omega)) = \begin{cases} p(\omega) & \text{if } c \geq r,\ \sigma_i = D \\ p(\omega) - K & \text{if } c \geq r,\ \sigma_i = C \\ 0 & \text{if } c < r,\ \sigma_i = D \\ -K & \text{if } c < r,\ \sigma_i = C. \end{cases}$$

$p(\omega)$ is the utility from public goods and K is the cost to pay for taking part in the production of public goods. We assume $p(\omega_1) > p(\omega_2) > K > 0$. That is, the utility from public goods at state ω_1 is higher than at state ω_2.

This setting is summarized by the following graph.

At point A, $n - r$ players choose D who obtain $p(\omega)$, and r players choose C, who obtain $p(\omega) - K$. At point B, all players choose C and obtain $p(\omega) - K$. At point C, all

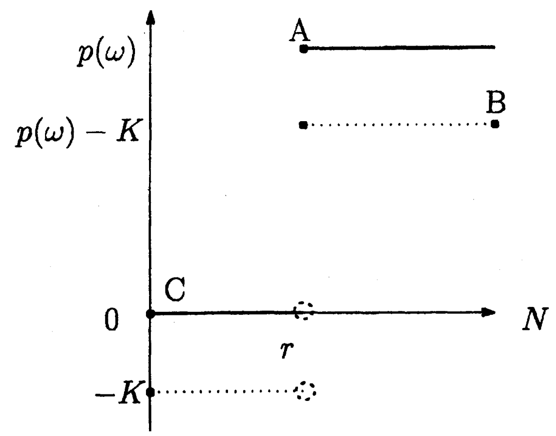

Figure 3: n-persons social dilemma game

players choose D and so on. In the coarse measurable α-core, r players choose C at ω_1, and they choose C at ω_2. $n-r$ players choose D for each state. On the other hand, in the fine measurable α-core, r players choose C and $n-r$ players choose D at each state.

In this game, we can obtain a core strategy inclision property.

REMARK 6.

(1) $X(V_c;\Sigma^c) \subset X(V_f;\Sigma^f)$

(2) For each $\omega \in \Omega$ and $u \in C(V_c,\omega;\Sigma^c))$, there exists $v \in C(V_f,\omega;\Sigma^f))$ such that $v \geq u$.

If player 1 informs the other players, all players can be better off. In other words, more efficient point is obtained through communications. ◇

We can prove the similar property to Remark 6-(2) in general model.

PROPOSITION 1 (COMMUNICATION PROPERTY)

1. Assume that for all $v \in V_f(N,\omega;\Sigma^f)$, ω, and S, $\{u_S|(u_S,v_{-S}) \in V_f(S,\omega;\Sigma^f)\} \subset \{u_S|(u_S,v_{-S}) \in V_f(N,\omega;\Sigma^f)\}$. For all $\omega \in \Omega$ and $u \in C(V_c,\omega;\Sigma^f)$, there exists $v \in C(V_f,\omega;\Sigma^f)$ such that $v \geq u$.

2. Assume that for all $v \in V_f(N,\omega;\Sigma^c)$, ω, and S, $\{u_S|(u_S,v_{-S}) \in V_f(S,\omega;\Sigma^c)\} \subset \{u_S|(u_S,v_{-S}) \in V_f(N,\omega;\Sigma^c)\}$. For all $\omega \in \Omega$ and $u \in C(V_c,\omega;\Sigma^c)$ there exists

$v \in C(V_f, \omega; \Sigma^c)$ such that $v \geq u$.

Proof. **(1)** Fix $\omega \in \Omega$. From $u \in C(V_c, \omega; \Sigma^f)$, there exist $\sigma \in \Sigma^c$, for all $\omega' \in (\bigwedge_{i \in N} \mathcal{P}_i)(\omega)$,

$$Eu_i(\sigma|\mathcal{P}_i)(\omega') \geq u_i.$$

Consider $\Pi_i := \mathcal{P}_i$, then $u \in V_f(N, \omega; \Sigma^f)$ since $\Sigma^f \supset \Sigma^c$.

By the definition of $V_f(N, \omega; \Sigma^f)$, for some $\sigma \in \Sigma^f$, for all $i \in N$, there exist Π_i which is finer than or equal to $\mathcal{P}_i$ and coarser than or equal to $\bigvee_{i \in S} \mathcal{P}_i$,

$$Eu_i(\sigma|\Pi_i)(\omega') \geq u_i$$

for all $\omega' \in (\bigwedge_{i \in N} \Pi_i)(\omega)$.

Now define

$$MV_f(N, \omega; \Sigma^f) := \left\{ u \in V_f(N, \omega; \Sigma^f) \,\middle|\, \neg \exists v \in V_f(N, \omega; \Sigma^f) : \begin{array}{ll} 1.v_i \geq u_i & \forall i \in S \\ 2.v_j \geq u_j & \exists j \in S \end{array} \right\}.$$

Then we can choose $v \in \{v \in MV_f(N, \omega; \Sigma^f) | v \geq u\}$.

It is sufficient to show for all $\alpha \in MV_f(N, \omega; \Sigma^f)$, $S \in \mathcal{N}$, and $\beta \in V_f(S, \omega; \Sigma^f)$, there exist $i \in S$ such that $\beta_i > \alpha_i$.

Suppose there exist $\alpha \in MV_f(N, \omega; \Sigma^f)$, $S \in \mathcal{N}$, and $\beta \in V_f(S, \omega; \Sigma^f)$ such that for all $i \in S$, $\beta_i \leq \alpha_i$. We obtain $\alpha \in MV_f(S, \omega; \Sigma^f)$ and $(\beta_S, \alpha_{-S}) \in V_f(N, \omega; \Sigma^f)$. From the assumption, $(\beta_S, \alpha_{-S}) \in V_f(N, \omega; \Sigma^f)$. This is a contradiction to $\alpha \in MV_f(N, \omega; \Sigma^f)$.

(2) The proof is similar to (1). As we substitute Σ^c for Σ^f, the proof is completed. □

We cannot suggest the communication property between the coarse measurable α-core and the fine measurable α-core generally, as we have to concern coalitional deviations. Using $\Sigma^c_{\{i\}} = \Sigma^f_{\{i\}}$ for all i, we can apply proposition 4 to the case of two persons' games.

Remark 7. (commucation property)

In the case of two persons' games, we obtain that for all $\omega \in \Omega$ and $u \in C(V_c(\cdot, \cdot; \Sigma^c))$, there exists $v \in C(V_f(\cdot, \cdot; \Sigma^f))$ such that $v \geq u$.

More information brings a player more payoff whether we impose the measurability condition or not.

PROPOSITION 2 (INFORMATION PROPERTY)

Let $\mathcal{P}'_i$ is finer than $\mathcal{P}_i$ for some i. If for some ω^* and all σ, $Eu_i(\sigma|\mathcal{P}_i)(\omega^*) < Eu_i(\sigma|\mathcal{P}'_i)(\omega^*)$ then player i's utility level in the coarse measurable α-core at ω^* increases when player i's information partition changes from $\mathcal{P}_i$ to $\mathcal{P}'_i$.

Proof. Define the coarse measurable α-core at ω with respect to $\mathcal{P}_i$ and the coarse measurable α-core at ω with respect to $\mathcal{P}'_i$ as $C(V_{c,}, \omega; \mathcal{P}_i)$ and $C(V_c, \omega; \mathcal{P}'_i)$.

Let $u \in C(V_c, \omega^*; \mathcal{P}_i)$. That is, there exists $\sigma \in \Sigma^c$ such that for all $j \in N$ and $\omega \in (\bigwedge_{j\in N} \mathcal{P}_j)(\omega^*)$,

$$Eu_j(\sigma|\mathcal{P}_j)(\omega) \geq u_j.$$

Then for some i,

$$Eu_i(\sigma|\mathcal{P}'_i)(\omega) > Eu_i(\sigma|\mathcal{P}_i)(\omega) \geq u_i.$$

Let $v_i := Eu_i(\sigma|\mathcal{P}'_i)(\omega)$ and $v_j := Eu_j(\sigma|\mathcal{P}_j)(\omega)$. Then $v \geq u$ holds.

Suppose v does not belong to $C(V_c, \omega^*; \mathcal{P}'_i)$, i.e., there exist $\omega \in (\mathcal{P}'_i \wedge \bigwedge_{j\in S\setminus i} \mathcal{P}_j)(\omega^*)$, S and $\sigma'_S \in \Sigma^c_{-S}$, for all $\sigma_{-S} \in \Sigma^c_{-S}$ and $j \in S$,

$$Eu_j(\sigma'_S, \sigma_{-S}|\mathcal{P}_j)(\omega) > v_j \geq u_j$$

If S does not include i, this inequality holds for all $\omega \in (\bigwedge_{j\in S} \mathcal{P}_j)(\omega^*)$. This contradicts $u \in C(V_c, \omega^*; \mathcal{P}_i)$. Hence we have to consider that S includes i and the above inequality holds for all $\omega \in (\mathcal{P}'_i \wedge \bigwedge_{j\in S\setminus i} \mathcal{P}_j)(\omega^*)$. In this case, define

$$\sigma''_S(\omega) := \begin{cases} \sigma'_S & \text{if } \omega \in (\mathcal{P}'_i \wedge \bigwedge_{j\in S\setminus i} \mathcal{P}_j)(\omega^*) \\ \sigma_S(\omega) & \text{otherwise} \end{cases}$$

Then we can obtain for all $\omega \in (\mathcal{P}'_i \wedge \bigwedge_{j\in S\setminus i} \mathcal{P}_j)(\omega^*)$ and $j \in S$,

$$Eu_j(\sigma''_S, \sigma_{-S}|\mathcal{P}_j)(\omega) > Eu_j(\sigma_S, \sigma_{-S})(\omega) = v_j > u_j,$$

since $(\mathcal{P}_i' \wedge \bigwedge_{j\in S\setminus i} \mathcal{P}_j)(\omega^*) \subset (\bigwedge_{j\in S} \mathcal{P}_i)(\omega^*)$. This contradicts $u \in C(V_c, \omega^*; \mathcal{P}_i)$. From this proof, we can obtain only player i's expected utility changes while the others' do not change. □

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