

Multiple Price Equilibria in a Customer Market

Tadashi Minagawa* and Shin Kawai

Graduate School of Economics
Nagoya University
Furo-cho Chikusa-ku
Nagoya 464-8601 Japan
E-mail*: minagawa@soec.nagoya-u.ac.jp

Abstract

This paper considers the existence of multiple price equilibrium (price dispersion) in a customer market with perfect information and homogeneous agents. We introduce the congestion effect as an expected utility instead of waiting cost. There exists a continuum of asymmetric Nash equilibria, that is, any kind of price dispersion exists in equilibrium.

JEL classification: L11, L13

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1 Introduction

Numerous studies have been made regarding the existence of price dispersion (see Stiglitz 1989 and chapter 16 in Shy 1995 for discussions of various price dispersion theories)¹. Since the seminal paper of Stigler (1961), most of these models assume a lack of information concerning the prices offered by firms. Consumers learn the price of a firm either through search (e.g., Salop and Stiglitz 1977; Stiglitz 1979; Carlson and McAfee 1983; Burdett and Judd 1983) or through advertisements (e.g., Butters 1977; Bester and Petrakis 1995). Moreover, the assumption of heterogeneity of either consumers or firms is common in price dispersion models (e.g., Reinganum 1979; Wilde and Schwartz 1979; Rob 1985). In particular, the assumption of heterogeneous consumers is crucial in models that do not entail the lack of information (Luski 1976; Reitman 1991)². Chen and Kong (2004), how-

¹There are several price dispersion models of monetary economies through the random matching process, see Kamiya and Sato 2003 and the references therein.

²For a more detailed discussion regarding these assumptions, see Chen and Kong (2004).

ever, demonstrate that price dispersion is possible even in a world of perfect information and identical consumers and firms. The driving force in their model is service capacity cost and congestion cost. The congestion cost in their model can be interpreted as waiting cost (see Luski 1976; Reitman 1991).

The purpose of this paper is to provide another source of price dispersion. We adopt a model that is a variation of Chen and Kong (2004). Like Chen and Kong, both firms and consumers are *ex ante* identical and there are no search costs and no advertisement costs in our model, that is, all consumers know the exact prices charged by firms. Unlike their model, however, we introduce expected utility instead of waiting cost as a congestion effect. The uncertainty arises from the scarcity of the good sold at low prices. Imagine the bargain sales at some stores; if there are lots of consumers, some of them cannot purchase the good at a low price. It is natural to suppose that the consumer *ex ante* expects the probability of purchase at a low price as the relative quantity to the number of customers at the store. In other words, we assume that consumers take into account not only the prices but also the supply of goods.

In section 2, we present an oligopolistic market model in which firms face both price and quantity competition simultaneously. In section 3, the existence of multiple price equilibria is proved. In section 4, we show that the degree of price dispersion varies with the number of firms. In section 5, we modify the model by introducing a cost function. Fixed costs determine the number of firms and hence the degree of price dispersion. In section 6, we conclude the paper.

2 The Model

Consider an oligopolistic retail market in which there are two *ex ante* identical firms (or discount stores) and N identical consumers ($N > 0$). There is an indivisible good. Identical consumers have common preferences defined by

$$u(x, y - p) = ax + y - px$$

where $x \in \{0, 1\}$, $y > 0$, and $a > 0$ denote consumption of the good, income, and the reservation utility, respectively. The term $(y - p)$ represents the *residual income* when the consumer purchases the good. Each consumer purchases at most one unit of the good if the price is equal to or less than reservation utility $a > 0$. Each firm i ($i = 1, 2$) sells the good to his customers n_i at zero cost. The good is sold at high price p_h as a *list price* or a *regular price* which is a manufacturer's suggested retail price. For simplicity, suppose that the high price level equals the reservation utility. Suppose that each firm sells the good at a low price $p_{li} \in [0, p_h)$ with a limited quantity

$s_i \in (0, N]$. The low price can be interpreted as a *bargain price* or a *sale price* in order to obtain customers from its rival store. We assume that each firm restricts himself to the quantity s_i less than or equal to the number of customers at both firms (i.e., $s_i \leq n_i$, $i = 1, 2$) in order to avoid the Bertrand competition and the outcome with zero profit³.

ASSUMPTION 1 each firm takes p_h as given and $p_h = a > 0$.

2.1 The firm i 's demand function

Each consumer chooses one of two stores for purchase of the good. Several *unlucky* consumers may purchase the good at a high price since the high-price equals the reservation utility level based on Assumption 1. Taking price and quantity vector $(p_h, p_{l1}, p_{l2}, s_1, s_2)$ as givens, the consumers rationally expect the number of the customers in each store. The consumer's expected utility function V from purchase of the good is defined by:

$$V(p_h, p_{li}, s_i) = \frac{s_i}{n_i}(a + y - p_{li}) + \left(1 - \frac{s_i}{n_i}\right)(a + y - p_h), \quad (1)$$

where $y > 0$ is income, and $a > 0$ is the reservation utility. The term $(y - p)$ represents the *residual income* when he purchases the good.

The number of customers at each store changes as long as there is the chance to obtain the larger surplus. Then n_i is determined at which the expected utility from each store is indifferent, that is,

$$V(p_h, p_{l1}, s_1) = V(p_h, p_{l2}, s_2); \quad (2)$$

hence, from (1), we obtain,

$$\frac{s_1}{n_1}(p_h - p_{l1}) = \frac{s_2}{n_2}(p_h - p_{l2}). \quad (3)$$

Using $n_1 + n_2 = N$, we can rewrite (3) as,

$$\begin{aligned} n_i(p_h, p_{li}, p_{lj}, s_i, s_j) &= N \left(\frac{(p_h - p_{li}) s_i}{(p_h - p_{li}) s_i + (p_h - p_{lj}) s_j} \right), \\ &= N \left(\frac{C_i}{C_1 + C_2} \right) \quad i, j = 1, 2, i \neq j, \end{aligned} \quad (4)$$

where C_i ($i = 1, 2$) represents the consumer surplus of firm i 's customers; that is,

$$C_i \equiv (p_h - p_{li}) s_i = ((a + y - p_{li}) - (a + y - p_h)) s_i. \quad (5)$$

³For a more detailed discussion, see Minagawa and Kawai (2004).

Equation (4) is the firm i 's demand function. The firm i can attract consumers by decreasing the low price or increasing the limited quantity; that is,

$$\frac{\partial n_i(\cdot)}{\partial p_{li}} < 0, \frac{\partial n_i(\cdot)}{\partial s_i} > 0, \text{ and } \frac{\partial n_i(\cdot)}{\partial p_{lj}} > 0, \frac{\partial n_i(\cdot)}{\partial s_j} < 0,$$

for every $p_{li} \in [0, p_h)$, $s_i \in (0, N]$. From (4), we must notice that the condition $s_i \leq n_i$ ($i=1,2$) is satisfied if and only if

$$(p_h - p_{li})(N - s_i) \geq C_j \quad i, j = 1, 2, i \neq j. \quad (6)$$

This condition can be rewritten as⁴

$$C_i + C_j \leq \min\{(p_h - p_{li})N, (p_h - p_{lj})N\}. \quad (7)$$

ASSUMPTION 2 Each firm takes action within the condition (7).

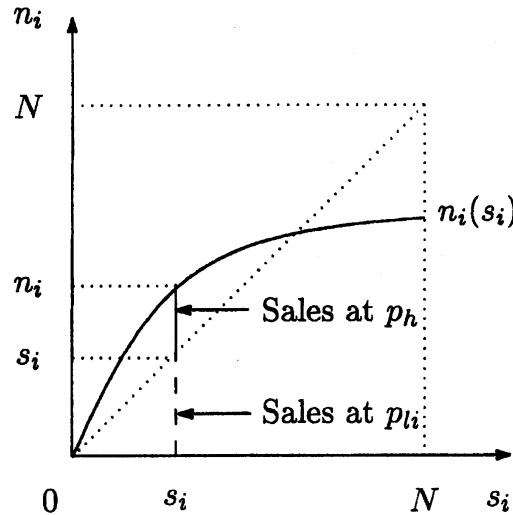


Figure 1: An Example of Strategy of s_i

2.2 Two-seller Game

Here we solve for an oligopoly equilibrium. We first have to define a *price and quantity competition* as a normal-form game. There are two firms as players of this game. Let each firm's actions be defined as choosing its low price and quantity levels taking high price p_h as given, and assume that both firms choose their actions simultaneously. Thus, each firm i chooses $p_{li} \in [0, p_h)$ and $s_i \in (0, N]$, $i = 1, 2$. The payoff function of each firm i can be defined by

$$\pi_i(p_h, p_{li}, p_{lj}, s_i, s_j) = p_{li}s_i + p_h(n_i(p_h, p_{li}, p_{lj}, s_i, s_j) - s_i).$$

⁴See Minagawa and Kawai (2004) for a more detailed discussion of this condition.

The first term of RHS is the revenue from bargain sales and the second term is the revenue from regular sales. Assume, for simplicity, that the costs of production are zero.

Firm i takes (p_{lj}, s_j) as given and chooses (p_{li}, s_i) to

$$\begin{aligned} \max_{p_{li}, s_i} \pi_i(\cdot) &= p_{li}s_i + p_h(n_i(\cdot) - s_i) \\ &= p_{li}s_i + p_h \left(\frac{NC_i}{C_i + C_j} - s_i \right) \end{aligned} \quad (8)$$

$i, j = 1, 2, \quad i \neq j.$

The first-order conditions are given by

$$\frac{\partial \pi_i}{\partial p_{li}} = s_i - p_h \left(\frac{NC_j s_i}{(C_i + C_j)^2} \right) = 0, \quad (9)$$

and

$$\frac{\partial \pi_i}{\partial s_i} = p_{li} + p_h \left(\frac{N(p_h - p_{li})C_j}{(C_i + C_j)^2} - 1 \right) = 0. \quad (10)$$

The second-order conditions are satisfied since

$$\frac{\partial^2 \pi_i}{\partial p_{li}^2} = -p_h(\cdot) < 0, \quad \text{and} \quad \frac{\partial^2 \pi_i}{\partial s_i^2} = -p_h(\cdot) < 0.$$

for every $p_{li} \in [0, p_h)$ and $s_i \in (0, N]$ ($i = 1, 2$).

From (9) and (10), we obtain, respectively,

$$p_{li} = p_h - \frac{\sqrt{Np_h(p_h - p_{lj})s_j} - (p_h - p_{lj})s_j}{s_i}, \quad (11)$$

and

$$s_i = \frac{\sqrt{Np_h(p_h - p_{lj})s_j} - (p_h - p_{lj})s_j}{p_h - p_{li}}. \quad (12)$$

Substituting (12) into (11), we find that the solution to this problem is *indeterminate*⁵. We can, however, derive the condition of symmetric Nash equilibrium by substituting $s_i = s_j = s^*$ and $p_{li} = p_{lj} = p_i^*$ for (11) and (12). In this process, we obtain (see Figure 2):

$$p_{li} = p_h + (p_h - p_{lj}) - \sqrt{\frac{p_h N (p_h - p_j)}{s^*}}, \quad (13)$$

and

$$s_i = -s_j + \sqrt{\frac{Np_h s_j}{p_h - p_i^*}}. \quad (14)$$

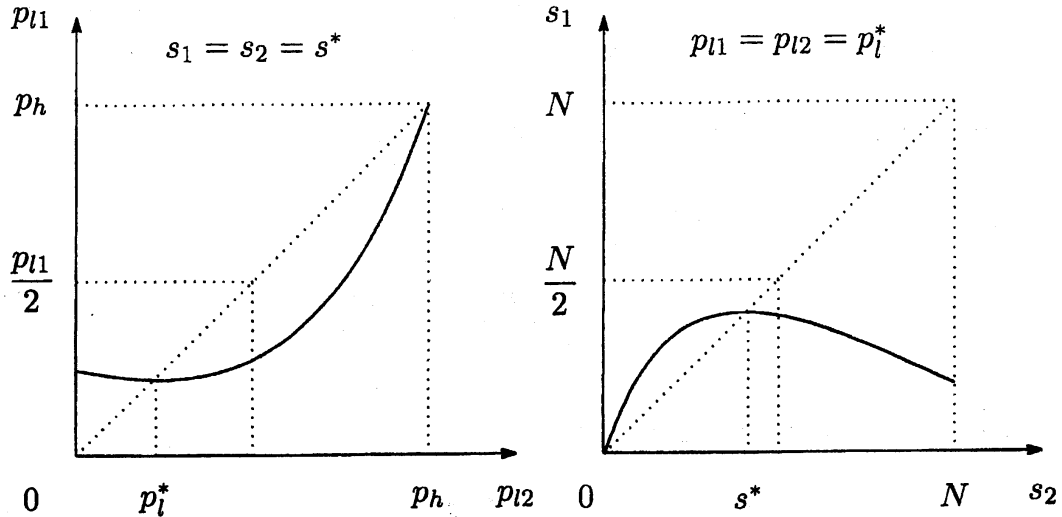


Figure 2: An Example of Symmetric Nash Equilibrium

Therefore, a set of a price and a quantity levels that satisfies (11) or (12) is

$$p_{li} = p_{lj} = p_i^* = \left(1 - \frac{N}{4s^*}\right) p_h \quad \text{and} \quad s_i = s_j = s^* = \frac{p_h N}{4(p_h - p_i^*)}. \quad (15)$$

Notice that, in equilibrium, the number of customers in each firm become $n_i^* = n^* = N/2$. From Assumption 1, s^* must satisfy $s^* \leq N/2$ and hence, from (15), p_i^* must satisfy $p_i^* \leq p_h/2$. The low price level, on the other hand, should be nonnegative (i.e., $p_i^* \geq 0$), thus the quantity is bounded below (i.e., $s^* \geq N/4$).

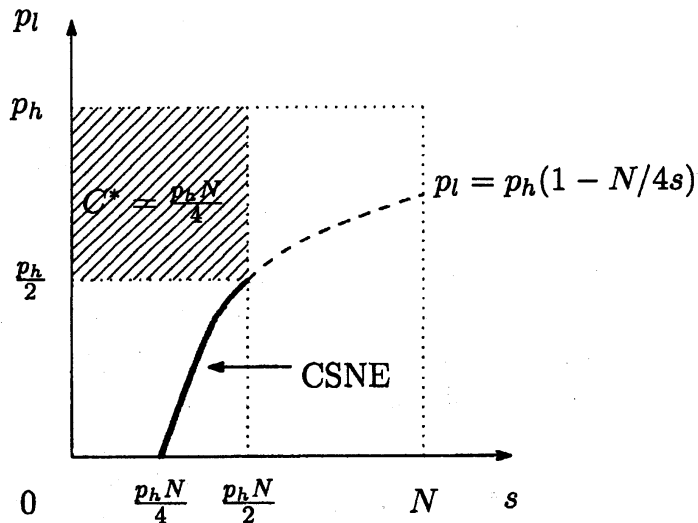


Figure 3: A Continuum of Symmetric Nash Equilibria (CSNE)

From the above discussions, we can establish the following proposition.

⁵We will explain the reason for this in the next section.

PROPOSITION 2.1 *There exists a continuum of symmetric Nash equilibria in which the good is sold at high price p_h and low price p_l^* . Any set of $p_{li} = p_l^* \in [0, p_h/2]$ and $s_i = s^* \in [N/4, N/2]$ for $i = 1, 2$ which satisfies*

$$(p_h - p_l^*)s^* = \frac{p_h N}{4}$$

is an equilibrium. The number of customers and the profit of firm i are $N/2$ and $p_h N/4$, respectively, in all equilibria.

Figure 3 illustrates a continuum of symmetric Nash equilibria in Proposition 2.1.

3 Existence of Multiple Price Equilibria

3.1 Two-seller Game and Two-price Equilibria

Proposition 2.1 shows that there exists a continuum of *symmetric* Nash equilibria. In this section, we will show that a continuum of *asymmetric* Nash equilibria do exist in which there are price dispersions among low price levels. From (8), the profit maximization problem of firm i can be rewritten as

$$\begin{aligned} \max_{C_i} \pi_i(C_i, C_j) &= p_h \left(\frac{NC_i}{C_i + C_j} \right) - (p_h - p_{li})s_i \\ &= p_h N \left(\frac{C_i}{C_i + C_j} \right) - C_i, \quad i, j = 1, 2, i \neq j, \end{aligned} \quad (16)$$

where C_i is the consumers' surplus at firm i , which is defined by (5). This payoff function implies that the firm i gives away the surplus to consumers in order to obtain his customer from the rival store. This is the reason for the indeterminacy in (11) and (12). The set of strategies (p_{li}, s_i) is reduced to the unique strategy variable C_i ($\in (0, p_h N]$).

The first-order condition of this problem is

$$\frac{\partial \pi_i}{\partial C_i} = p_h N \left(\frac{C_j}{(C_i + C_j)^2} \right) - 1 = 0.$$

The second-order condition is satisfied since

$$\frac{\partial^2 \pi_i}{\partial C_i^2} = -p_h \left(\frac{C_j}{(C_i + C_j)^3} \right) < 0$$

for every $C_i \in (0, p_h N]$. Hence, the best-response function of firm i as a function of the consumer surplus level of firm j is given by

$$C_i = R_i(C_j) = \sqrt{p_h N C_j} - C_j. \quad (17)$$

The solution of this game is⁶

$$C^* = \frac{p_h N}{4}. \quad (19)$$

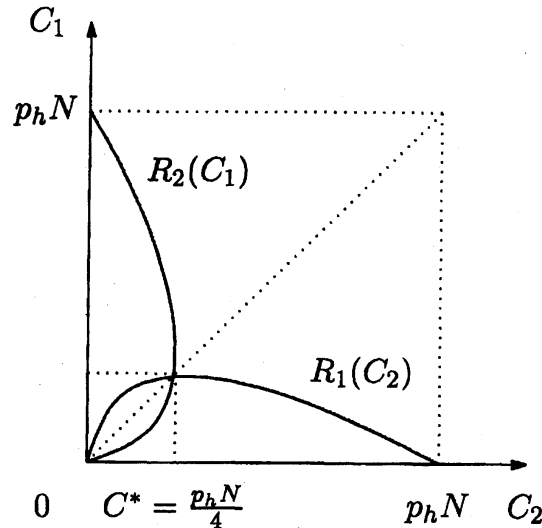


Figure 4: The Best-Response Functions

The original game's strategy is the set of p_{li} and s_i . We find that any set of p_{li} and s_i which satisfy (19) is a Nash equilibrium. In other words, there is a continuum of asymmetric Nash equilibria in the original game.

From the above discussions, we have established the following proposition.

PROPOSITION 3.1 *There exists a continuum of asymmetric Nash equilibria in which the good is sold at one high price p_h and two low prices (p_{l1}^*, p_{l2}^*) . Any set of $p_{li} = p_{li}^* \in [0, p_h/2]$ and $s_i = s_i^* \in [N/4, N/2]$, $i = 1, 2$, which satisfies*

$$C^* = (p_h - p_{li}^*)s_i^* = \frac{p_h N}{4}, \quad i = 1, 2.$$

is an equilibrium. The number of customers and the profit of firm i are $N/2$ and $p_h N/4$, respectively, in all equilibria.

Notice that the symmetric Nash equilibria in Proposition 2.1 is included in the equilibria in Proposition 3.1. Figure 5 illustrates an example of the best response correspondence of firm 1 when firm 2 adapts a set of equilibrium strategies (p_{l2}^*, s_2^*) .

⁶Note that $C^* = 0$ is another solution of (17). However, it cannot be a Nash equilibrium

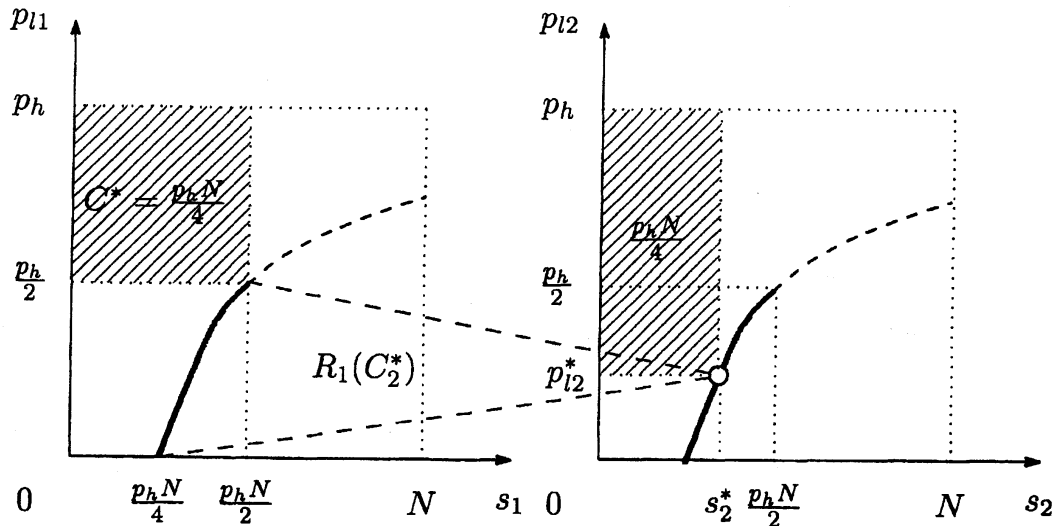


Figure 5: A Continuum of Asymmetric Nash Equilibria

3.2 M -seller Game and M -price Equilibria

Suppose now that the market consists of M (≥ 1) identical firms. We found that, in the two-seller game, the firm's strategy is represented by choosing the consumer surplus level, instead of choosing price and quantity levels independently. In the M -seller game, we need to deduce the firm i 's demand function as a function of the consumer surplus levels of all firms. Then (3) can be modified by

$$\frac{C_i}{n_i} = \frac{C_j}{n_j}, \quad i, j = 1, 2, \dots, M, \quad i \neq j. \quad (20)$$

This condition means that the average consumer surplus per capita at the store must be equal among the stores in equilibrium. Although there are M equations in (20), one of them is not independent. Hence, there are $M - 1$ independent equations and $\sum_{i=1}^M n_i = N$. Solving these $(M - 1) + 1$ equations with M unknowns, the firm i 's demand function can be calculated as

$$n_i(C_i, C_{-i}) = N \left(\frac{C_i}{C_i + C_{-i}} \right), \quad \text{where } C_{-i} = \sum_{j \neq i}^{M-1} C_j. \quad (21)$$

The condition (6) can be rewritten as

$$(p_h - p_{li})N - C_i \geq C_{-i}, \quad \text{for } i = 1, 2, \dots, M,$$

because the profit of firm i is not continuous at $C_i = 0$ for $C_j = 0$. That is if $C_j = 0$,

$$\pi_i(C_i, 0) = \begin{cases} \frac{p_h N}{2}, & C_i = 0, \\ p_h N - C_i, & C_i > 0. \end{cases} \quad (18)$$

Then, the firm i can obtain larger profit by increasing C_i (> 0).

and hence,

$$C_i + C_{-i} \leq \min\{(p_h - p_{l1})N, (p_h - p_{l2})N, \dots, (p_h - p_{lM})N\}. \quad (22)$$

ASSUMPTION 3 Each firm takes action within the condition (22).

Using this demand function (21), firm i chooses C_i to

$$\max_{C_i} \pi_i(C_i, C_{-i}) = p_h N \left(\frac{C_i}{C_i + C_{-i}} \right) - C_i.$$

The first order condition is given by

$$\frac{\partial \pi_i}{\partial C_i} = p_h N \left(\frac{C_{-i}}{(C_i + C_{-i})^2} \right) - 1 = 0.$$

Hence, the best-response function of firm i as a function of the consumer surplus levels of firm $-i$ is given by

$$C_i = R_i(C_{-i}) = \sqrt{p_h N C_{-i}} - C_{-i}. \quad (23)$$

Since all firms are identical regarding cost structure, we can find that the solution where $C_i = C^*$ for all $i = 1, \dots, M$. Substituting the common C^* into the already derived best-response functions. We have it that

$$C^* = \sqrt{p_h N (M - 1) C^*} - (M - 1) C^*.$$

Hence, there are two solutions:

$$C^* = \left(1 - \frac{1}{M} \right) \frac{p_h N}{M}. \quad (24)$$

Similar to the discussion of the two-seller game, $C^* = 0$ could not be a Nash equilibrium since if $C_{-i} = 0$, from (18), the firm i has an incentive to deviate from that state. We now can establish the following proposition.

PROPOSITION 3.2 *There exists a continuum of asymmetric Nash equilibria in which the good is sold at one high price and M low prices $(p_{l1}^*, \dots, p_{lM}^*)$. Any set of*

$$0 \leq p_{li}^* \leq \frac{p_h}{M}, \quad \text{and} \quad \left(1 - \frac{1}{M} \right) \frac{N}{M} \leq s_i^* \leq \frac{N}{M} \quad (25)$$

which satisfies

$$C^* = (p_h - p_{li}^*) s_i^* = \left(1 - \frac{1}{M} \right) \frac{p_h N}{M}, \quad i = 1, 2, \dots, M.$$

is an equilibrium. The number of customers of firm i is the same in each equilibrium, which is $n_i^* = n^* = N/M$. The profit of the firm i is also the same as

$$\pi_i^* = \pi^* = \frac{p_h N}{M^2}, \quad i = 1, 2, \dots, M.$$

in each equilibrium.

Note that each firm does not necessarily set different prices. Thus, there exists any kind of price distribution in equilibrium.

4 Multiple Price Equilibria and Welfare

4.1 Varying the Number of Sellers

We now investigate the changes in the degree of price dispersion among low price levels as we change the number of firms in the industry. First, note that substituting $M = 1$ into (24) yields $C^* = 0$, that is, the firm does not adopt the discount strategy hence the equilibrium price becomes monopoly price p_h . Second, substituting $M = 2$ yields the duopoly solution described in Proposition 3.1.

Now we let the number of firms grow with no bounds. Then, we have it that, from (25),

$$\lim_{M \rightarrow \infty} p_{li}^* = 0 \quad \text{and} \quad \lim_{M \rightarrow \infty} s_i^* = 0.$$

The former equation $\lim p_{li}^* = 0$ implies that price dispersions disappear in the limit. The latter equation $\lim s_i^* = 0$ should be regarded as the firm i selling at a low price within the limit as the number of sales itself converges to zero; i.e., $\lim n^* = 0$. In fact, from (25), the range of the total supply of the good at a low price is

$$\left(1 - \frac{1}{M}\right) N \leq S^* < N, \quad \text{where } S^* = \sum s_i^*.$$

Hence, the limit of S^* is

$$\lim_{M \rightarrow \infty} S^* = N.$$

These equations imply that the multiple price equilibria converge to the unique competitive price (i.e., $p_l = 0$) equilibrium.

PROPOSITION 4.1 *As the number of firms increases,*

1. *The multiple price equilibria converge to the unique competitive equilibrium,*
2. *The variance of price dispersion decreases.*

4.2 Welfare Analysis

We have assumed that the utility function has a special form

$$u(1, y - p) = a + y - p,$$

where $u(1, y - p)$ denotes the utility function when one unit of the good is purchased (hence the first factor of this function is 1) at a price p (hence the second factor of this function is the residual income $y - p$). We can,

however, generalize it to the *risk-neutral* class with respect to income. Using the expression $u(1, y - p)$, each consumer's expected utility function from the store i is rewritten as

$$V(p_h, p_{li}, s_i) = \left(\frac{s_i}{n_i}\right) u(1, y - p_{li}) + \left(1 - \frac{s_i}{n_i}\right) u(1, y - p_h). \quad (26)$$

From (26), the equal-expected-utility condition $V(p_h, p_{li}, s_i) = V(p_h, p_{lj}, s_j)$ can be written as

$$\frac{s_i}{n_i}(u(1, y - p_{li}) - u(1, y - p_h)) = \frac{s_j}{n_j}(u(1, y - p_{lj}) - u(1, y - p_h)). \quad (27)$$

Since we assume here that the utility function u is risk neutral (i.e., a linear function with respect to residual income $y - p$), and p_h equals the reservation utility⁷, the consumer surplus from the purchase of the good at a low price can be written as

$$\begin{aligned} u(1, y - p_{li}) - u(1, y - p_h) &= \gamma(y - p_{li}) - \gamma(y - p_h) \\ &= \gamma(p_h - p_{li}), \quad \gamma \geq 1, \end{aligned} \quad (28)$$

where γ is the marginal utility of income when the good is purchased. Then, (27) becomes

$$\frac{s_i}{n_i}(p_h - p_{li}) = \frac{s_j}{n_j}(p_h - p_{lj}).$$

and hence

$$\frac{C_i}{n_i} = \frac{C_j}{n_j}$$

Therefore, there is no need to modify the discussions of the previous sections even if $\gamma > 1$.

From (28), the consumer surplus from each firm is γC^* . Thus, the consumer surplus in this market is

$$CS^*(M) = \gamma M \cdot C^* = \gamma \left(1 - \frac{1}{M}\right) p_h N.$$

The producer surplus is aggregate profit,

$$PS^*(M) = M\pi^*(M) = \frac{p_h N}{M}.$$

Thus, the social welfare of M-firm equilibrium is

$$\begin{aligned} W^*(M) &= CS^*(M) + PS^*(M) \\ &= \gamma \left(1 - \frac{1}{M}\right) p_h N + \frac{p_h N}{M} \\ &= \left(\gamma - (\gamma - 1) \frac{1}{M}\right) p_h N \end{aligned}$$

From the above discussions, we can establish the following proposition.

⁷The reservation price p_h is now defined by $p_h = \{p | u(1, y - p) = U(0, y)\}$

PROPOSITION 4.2 *The consumer surplus increases and the producer surplus decreases with respect to M . The social welfare increases if $\gamma > 1$. That is,*

If $\gamma > 1$,

$$\lim_{M \rightarrow \infty} CS^*(M) = \gamma p_h N, \quad \lim_{M \rightarrow \infty} PS^*(M) = 0$$

and

$$\lim_{M \rightarrow \infty} W^*(M) = \left(1 - \left(1 - \frac{1}{\gamma}\right) \frac{1}{M}\right) \gamma p_h N = \gamma p_h N.$$

Notice that if all firms charge the high price, each firm's profit is $p_h N/M$. Therefore, this market has the prisoner's dilemma characteristic as in usual imperfect competition models.

5 Introducing a Cost Function

In multiple price equilibria, the supremum low price level is at most $p_h/2$. This is not realistic because we observe that, for example, the good is sold at 75% of its regular price, etc. We can, however, explain this by introducing a cost function. The cost function is defined by

$$K(n_i) = kn_i + A,$$

where $k \in [0, p_h)$ and $A > 0$ are the marginal costs and fixed costs, respectively. As in Varian (1980), this function is based on the casual observation that retail stores are characterized by fixed costs of rent and sales force, plus constant variable costs (the wholesale cost) of the good being sold. Since the marginal cost is k , it seems natural that the lower bound of the low price is k (and hence $p_{li} \in [k, p_h)$ and $C_i \in (0, (p_h - k)N]$). Formally, the profit of firm i is

$$\begin{aligned} \pi_i &= p_h n_i - C_i - K(n_i) \\ &= p_h N \left(\frac{C_i}{C_i + C_{-i}} \right) - C_i - k \left(N \left(\frac{C_i}{C_i + C_{-i}} \right) \right) - A. \end{aligned}$$

Substituting the cost function into profit, the firm i chooses $C_i \in (0, (p_h - k)N]$ to

$$\max_{C_i} \pi_i(C_i, C_{-i}) = (p_h - k)N \left(\frac{C_i}{C_i + C_{-i}} \right) - C_i - A.$$

Since the marginal cost and the fixed cost are constant, the same arguments can be applied to this problem.

The first order condition is

$$(p_h - k)N \left(\frac{C_{-i}}{(C_i + C_{-i})^2} \right) = 1.$$

The best response function is

$$C_i = R(C_{-i}) \equiv \sqrt{(p_h - k)NC_{-i}} - C_{-i}$$

Therefore, we can establish the following result.

PROPOSITION 5.1 *There exists a continuum of asymmetric Nash equilibria in which the good is sold at one high price and M low prices $(p_{i1}^*, \dots, p_{iM}^*)$. Any set of*

$$k \leq p_{li}^* \leq \frac{p_h}{M} + \left(1 - \frac{1}{M}\right)k, \quad \text{and} \quad \left(1 - \frac{1}{M}\right)\frac{N}{M} \leq s_i^* \leq \frac{N}{M}. \quad (29)$$

which satisfies

$$C^* = (p_h - p_{li}^*)s_i^* = \left(1 - \frac{1}{M}\right)\frac{(p_h - k)N}{M}, \quad i = 1, 2, \dots, M.$$

is an equilibrium. The number of customers of firm i is the same in each equilibrium, which is $n_i^* = n^* = N/M$. The profit of the firm i is also the same as

$$\pi_i^* = \pi^* = \frac{(p_h - k)N}{M^2} - A, \quad i = 1, 2, \dots, M.$$

in each equilibrium. Furthermore, the number of firms is determined by $\pi^* = 0$; i.e.,

$$M^* = \sqrt{\frac{(p_h - k)N}{A}}. \quad (30)$$

Note that from (30), fixed cost A determines the number of firms and hence the degree of price dispersion.

6 Concluding Remarks

We found that price dispersion occurs in an oligopolistic retail market with perfect information, homogeneous agents, and no cost functions. The key role of price dispersion is that each firm can choose both price and quantity levels. This generates consumers' expectations of congestion. As a result, the number of customers is determined endogenously in this model. It is worth noticing that, in a multiple price equilibrium, each different price is paired with each different quantity. In this sense, the good is discriminated.

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