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Remark on divergent multizeta series

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There are several manners of regularizing divergent zeta series. It is useful to compare these, since they are used to describe relations among convergent zeta series. Here we compare the two main ones, related to the two shuffle products. The remark in this paper (§1) was described in a lecture at the CIRM, april 2000, and used e.g. by G. Racinet [13]. §3 gives a more formal version; in §2 I have briefly recalled the basic notions about polylogarithms and zeta-numbers used in §3.

1 Logarithmically divergent series

1.1 Series

Let $\mathcal{K} \subset \mathbb{C}[[z]]$ denote the set of formal series $f = \sum f_n z^n$ with complex coefficients in which the coefficient f_n has an asymptotic expansion of the form

$$f_n \sim \sum_1^\infty p_k(\operatorname{Log} n)\, n^{-k} \tag{1}$$

for $n \to \infty$, where for each k, p_k is a polynomial. Polylogarithm series $f = Li_a$ belong to $\mathcal{K}$ and zeta values are special cases of convergent series $f(1)$, cf. below.

We set $S_n(f) = S_n = \sum_0^n f_k$. Condition (1) is equivalent to the fact that S_n has a similar asymptotic expansion:

$$S_n \sim \sum_0^\infty P_k(\operatorname{Log} n)\, n^{-k}. \tag{2}$$

This implies that the series is convergent for $|z| < 1$ and its sum, still denoted $f(z)$ admits for $z \to 1-0$ an asymptotic expansion:

$$f(z) \sim \sum_{k=0}^\infty (1-z)^k Q_k(\operatorname{Log} \frac{1}{1-z}) \tag{3}$$

where the Q_k are also polynomials (the converse is not true).

We will denote the leading polynomials

$$S_f = P_0, \qquad M_f = Q_0 \tag{4}$$

The following result is immediate:

Lemma 1 *The following statements are equivalent:*
1) The numerical series $\sum f_n$ is convergent.
2) The numerical series $\sum f_n$ is absolutely convergent.
3) The polynomial $S_f = P_0$ is constant.
4) The polynomial $M_f = Q_0$ is constant.
S_f and M_f are then equal constants: $S_f = M_f = \sum f_n$.

1.2 Products.

Let $k \in \mathbb{C}$ be a field. On the set of formal series $k[[T]]$ we have two associative and commutative products defined by:

$$f \times g = \sum f_p g_q \, z^{p+q}\,, \qquad f * g = \sum f_p g_q \, z^{\sup(p,q)} \tag{5}$$

the first is the usual product (corresponding to the shuffle product ш below), the second is the unique product such that if $S_n(f) = \sum_1^n f_k$ is the n-th partial sum, we have $S_n(f * g) = S_n(f)S_n(g)$ for all n (corresponding to the $*$ shuffle product below).

It is immediate that $\mathcal{K}$ is a sub-algebra for both products, i.e. the coefficients of $f \times g$ and $f * g$ have a logarithmic asymptotic expansion as above if f and g do.

The map $f \to M_f(X)$, resp. $f \to S_f(X)$ a homomorphism of the algebra $(\mathcal{K}, \times)$, resp. $(\mathcal{K}, *)$, to the polynomial algebra $\mathbb{C}[T]$ (surjective if $k = \mathbb{C}$).

Corollary 1 *Let $I \subset \mathcal{K}$ be the subset of convergent series with vanishing sum: I is an ideal of $\mathcal{K}$ for both multiplication laws $\times, *$. It contains all elements of the form $f * g - f \times g$ for $f, g \in \mathcal{K}$, f convergent.*

Indeed M_f is multiplicative for $\times$ (resp. S_f for $*$) and $M_f = 0 \Leftrightarrow S_f = 0$. The second assertion follows from the fact that, if f is convergent, setting $a = f(1) = \sum f_n$, we have $f * g - f \times g = (f-a) * g - (f-a) \times g$; since $f - a$ is convergent with vanishing sum, the same is true for both terms: $M_{(f-a)\times g} = 0$, $S_{(f-a)*g} = 0$.

A typical regularized zeta value is obtained by applying the character $f \mapsto M_f(\theta)$ (resp. or $f \mapsto S_f(\theta)$) to a divergent polygarithm series, for some number θ.

1.3 Relation between M_f and Q_f

Corollary 1 shows that the polynomials M_f et S_f determine each other for $f \in \mathcal{K}$. We will explicit the formula which links them. We use the generating series $F = F(T, z)$, with T is a formal parameter:

$$F = \sum \frac{T^n}{n!} \left(\log \frac{1}{1-z}\right)^n = (1-z)^{-T} \tag{6}$$

so that $M_F(X) = e^{TX}$. Then for F we get

$$\sum S_n z^n = \frac{F}{1-z} = (1-z)^{-T-1} = \sum (1+T)\dots(1+\frac{T}{n})\, z^n$$

i.e.

$$S_n = (1+T)\dots(1+\frac{T}{n}) \sim \frac{e^{T\log n}}{T!}.$$

Taking limits (the limit holds in any reasonable sense, in particular the coefficients of the T^k converge) we get

$$S_F(X) = \frac{1}{T!}\, e^{TX} = \frac{1}{\frac{d}{dX}!}\, e^{TX}$$

Theorem 1 *For any $f \in \mathcal{K}$ we have :* $S_f(X) = \frac{1}{\frac{d}{dX}!}\, Q_f(X)$

If we explicit the Taylor-series expansion of $\frac{1}{T!}$, this can be rewritten

$$S_f(X) = \exp\,[\sum_1^\infty \frac{(-1)^{j-1}}{j}\ \zeta(j)(\frac{d}{dX})^j]\ Q_f(X)$$

with here the convention $\zeta(1) = \gamma$, the Euler constant.

2 Polylogarithms.

In this section we recall basic facts about polylogarithms. Let k be a field of characteristic 0 ($k = \mathbb{Q}$, $\mathbb{R}$ or $\mathbb{C}$).

2.1 Polylogarithm functions.

• Let $\mathcal{L} = k\langle X_0, X_1\rangle$ denotes the free algebra with two generators X_0, X_1 ($\mathcal{L}_k$ if there is a risk of confusion). It has a canonical basis W consisting of words (monomials): a word of degree N is $w = X_{i_0}\dots X_{i_N}$, $i_k = 0$ or 1; $w = 1$ if $N = 0$ (empty word).

• $\widehat{\mathcal{L}}$ denotes the completion of $\mathcal{L}$: its elements are formal series $f = \sum f_w w$. We identify $\mathcal{L}$ with the graded dual of $\widehat{\mathcal{L}}$ (it might be better to use another notation; the free (concatenation) product lives on $\widehat{\mathcal{L}}$; the shuffle products below live on the dual.

• Duality endows $\mathcal{L}$ with a structure of left or right $\widehat{\mathcal{L}}^0$-module, via inner products, e.g. $\langle a \lrcorner g, g\rangle = \langle a, gf\rangle$. Since 1 is a generator of the left or right free module $\mathcal{L}$, it is a cogenerator in the dual, i.e. if $a \in \mathcal{L}, a \neq 0$ there exists $f \in \widehat{\mathcal{L}}$ such that $a \lrcorner f = 1$ (or $f \llcorner a = 1$).

• To each $a \in \mathcal{L}$ we associate linearly a polylogarithm function $Li_a(z)$ so that the $\widehat{\mathcal{L}}$-valued the formal series $L = L(z) = \sum_{w\in W} Li_w(z) w$ satisfies the differential equation

$$dL(z) = (\frac{dz}{z}X_0 + \frac{dz}{1-z}X_1)L, \quad L^{reg}(0) = 1 \tag{7}$$

The coefficients Li_w are holomorphic functions on $]0,1[$, and extend as ramified functions to the whole complex plane. They have logarithmic singularities at $0,1,\infty$ i.e. of the form $\phi = \sum_1^K \phi_k(u)(\log u)^k$ if u is a local parameter ($u = z, 1-z$ or $\frac{1}{z}$) and ϕ_k is holomorphic in a neighborhood of the singular point. The regularized value at $z=0$ is defined by $\phi^{reg}(0) = \phi_0(0)$.

If $w = X_{i_1} \dots X_{i_N}$, we have $Li_w = P_{i_1} \dots P_{i_N}(1)$ where P_0, P_1 are the operators $P_0 f(z) = (\int_0^z f(t)\frac{dt}{t})^{reg}$, $P_1 f(z) = \int_0^z f(t)\frac{dt}{1-t}$. In particular for $0 < x < 1$

$$(8) \quad Li_w(x) = \int \dots \int_{x>t_1>\dots t_n>0} \omega_{i_1}(t_1)\dots\omega_{i_N}(t_N), \quad \text{with} \quad \omega_0 = \frac{dt}{t},\ \omega_1 = \frac{dt}{1-t}$$

• Let Γ denote the fundamental group of $\mathbb{C} \setminus \{0;1\}$; we choose as base point $1+0$, and in what follows we denote c_1c_0 the path c_0 followed by c_1 (the opposite of the usual order). Γ is a free group generated by γ_0, γ_1, with γ_0, a small loop around 0, and γ_1 a small loop around 1 preceded and followed by the real path $[+0, 1-0]$.

$L(z)$ is a ramified function; z should be thought of as a homotopy class of paths avoiding $0,1,\infty$, with origin $1+0$.

Since the polar part of $dL\,L^{-1}$ at $z=0$, resp. 1, is $\frac{X_0}{z}$, resp. $-\frac{X_1}{z-1}$, we get, for $\gamma \in \Gamma$

$$L(z\gamma = L(z)\phi(\gamma)$$

where $\phi : \Gamma \to \widehat{\mathcal{L}}^\times$ is the group homomorphism such that

$$\phi(\gamma_0) = e^{2i\pi X_0}, \qquad \phi(\gamma_1) = L^{reg}(1-0)^{-1}e^{-2i\pi X_1}L^{reg}(1-0).$$

Equivalently

$$(9) \qquad Li_a(z\gamma) = Li_{w\lrcorner\phi\gamma}(z) \qquad \text{for any} \quad \gamma \in \Gamma.$$

It follows in particular that the image $\phi(\mathbb{Z}(\Gamma)$ is dense in $\widehat{\mathcal{L}}$, and the algebras generated by the fundamental group, or by X_0, X_1 acting by right interior product, are the same. In particular for any $a \in \mathcal{L}, f \in \widehat{\mathcal{L}}$, $Li_{a\llcorner f}$ is a finite linear combination of branches of Li_a.

• The map $a \in \mathcal{L}_\mathbb{C} \mapsto Li_a$ is injective, i.e. the $Li_w, w \in W$ are linearly independent over $\mathbb{C}$ (in fact over the field of rational functions $\mathbb{C}(z)$); this is a simple special case of the Riemann-Hilbert theorem: if $a \in \mathcal{L}_\mathbb{C}, a \neq 0$ there exists $f \in \mathcal{L}_\mathbb{C}$ such that $a\lrcorner f = 1$, so $Li_{a\lrcorner f} = 1$ is a linear combination of branches of Li_a, and Li_a does not vanish]. The range of this map is exactly the set of functions which are ramified at $0,1,\infty$, of finite type and unipotent monodromy, and any branch of which at 0, 1 or ∞ has a logarithmic singularity as above.

2.2 Polyzeta numbers

• If $a \in \mathcal{L}$, the function Li_a (or rather its first branch) is holomorphic at the origin iff a is orthogonal to $\mathcal{L}X_0$ ($a\lrcorner X_0 = 0$). The Taylor series of Li_a then belongs to the algebra $\mathcal{K}$ of §1. We denote $\mathcal{A} \subset \mathcal{L}$ the sub-algebra $\mathcal{A} = k + \mathcal{L}X_1$, orthogonal of $\mathcal{L}X_0$.

$\mathcal{A}$ is a free algebra with generators the $y_j = X_0^{j-1}X_1$. It has a basis consisting of monomials $y_s = y_{s_1} \dots y_{s_r}$, indexed by indexed by sequences $s = (s_1, \dots, s_r)$ of integers ≥ 1 (the concatenation product, in this order, $y_\emptyset = 1$).

The polylogarithm series are the Taylor series at $z = 0$ of the $Li_a, a \in \mathcal{A}$: we have

$$Li_{y_s}(z) = \sum_{n_1 > \dots n_s \geq 1} \frac{z^{n_1}}{n_1^{s_1} \dots n_r^{s_r}} \tag{10}$$

• We denote $\mathcal{A}^0 \subset \mathcal{A}$ the sub-algebra $k + X_0\mathcal{L}X_1$ (orthogonal to $X_0\mathcal{L} + \mathcal{L}X_1$). If $s = s_1, \dots s_r$ $(r > 0)$ we have $y_s \in \mathcal{A}^0$ iff $s_1 > 1$. Then the polygarithm series $Li_s(1)$ is obviously convergent ($Li_s(z) = \int_0^z \frac{dx}{x}..$ has a limit for $z \to 1 - 0$). The corresponding zeta number is

$$\zeta_s = Li_s(1) = \sum_{n_1 > \dots n_s \geq 1} \frac{1}{n_1^{s_1} \dots n_r^{s_r}} \tag{11}$$

2.3 Products.

Let us recall that two shuffle products, ш and $*$, are defined on $\mathcal{L}$ resp. $\mathcal{A}$.

• ш is the unique product compatible with the usual product of holomorphic functions, i.e. $Li_{a ш b} = Li_a Li_b$.

$L \otimes L$ satisfies the differential equation

$$d(L \otimes L) = \left[\frac{dz}{z}(X_0 \otimes 1 + 1 \otimes X_0) + \frac{dz}{1-z}(X_1 \otimes 1 + 1 \otimes X_1)\right](L \otimes L),$$

$$(L \otimes L)^{reg}(0) = 1$$

so the dual coproduct is the algebra homomorphism $\Delta_ш : \mathcal{L} \to \mathcal{L} \otimes \mathcal{L}$ such that $\Delta_ш(X_i) = X_i \otimes 1 + 1 \otimes X_i$ for $i = 0, 1$ (the completion takes $\widehat{\mathcal{L}}$ to $\mathcal{L}\widehat{\otimes}\mathcal{L}$ - which is bigger than $\widehat{\mathcal{L}} \otimes \widehat{\mathcal{L}}$).

For the ш product law, L is of group type, i.e. $\Delta_ш L(z) = L(z) \otimes L(z)$; so are the monodromy elements $\phi(\gamma), \gamma \in \Gamma$.

• $*$ is only defined on $\mathcal{A}$. The dual coproduct Δ_* is characterized by the requirement that the generating series

$$u(T) = 1 + \sum_1^\infty y_j T^j \tag{12}$$

is of group type, i.e. for the generators we have

$$\Delta_*(y_j) = \sum_{p+q=j} y_p \otimes y_q \quad \text{(with the convention } y_0 = 1) \tag{13}$$

To the word $y_s \in \mathcal{A}$ one associates the elementary "quasi-symmetric" function (formal series) $P_s(t_1, \ldots, t_n, \ldots)$:

(14) $$P_s = \sum_{n_1 > \cdots > n_r} t_{n_1}^{s_1} \ldots t_{n_r}^{s_r}$$

Let us recall that the P_s form a basis of the algebra $\mathcal{P} \subset k[[t_1, ...t_n, ...]]$ of quasi-symmetric functions, which is a sub-algebra of the algebra all formal series in $t_1, \ldots$.

Since $u(T)$ of group type in $\mathcal{A}[[T]]$, so is the formal product

(15) $$U = \ldots u(t_n) \ldots u(t_1) = \sum P_s y_s$$

(infinite concatenation product, in the indicated order). So the linear map $\Psi : \mathcal{A} \to \mathcal{P}$ such that $\Psi(y_s) = P_s$ is an algebra isomorphism: $(\mathcal{A}, *) \to \mathcal{P}$.

For $f = Li_{y_s}(z)$, (10), (11) show that series $\zeta_f = \sum_{n_1 > \ldots n_r} n_1^{-s_1} \ldots n_r^{-s_r}$ is obtained by substituting $t_k = \frac{1}{k}$ in $U(f)$, and the N-th sum $S_N(f)$ is obtained by substituting $t_k = \frac{1}{k}$ if $k \leq N$, $t_k = 0$ otherwise; so we have $S_N(f * g) = S_N(f) S_N(g)$.

3 Relations between polylogarithm series.

3.1 Main identity - second version.

Any monomial $a \in \mathcal{A}$ is uniquely factorized as $y = y_1^k \cdot b$ for some integer k and some monomial $b \in \mathcal{A}^0$. An easy induction then shows that for either law $\mathcal{A}$ is a polynomial algebra: $\mathcal{A}_{ш} = \mathcal{A}^0_{ш}[y_1]$, $\mathcal{A}_* = \mathcal{A}^0_*[y_1]$. The successive powers $y_1^{*k}, y_1^{шk}$ are not equal, and the relation between them is given by the next result:

Theorem 2 *We have the following identity of formal series with coefficients in $\mathcal{A}$:*

(16) $$\exp^{ш} y_1 T = \exp^{*} \sum_1^{\infty} \frac{(-1)^{k-1}}{k} y_k T^k$$

where $\exp^{ш}$, *resp.* $\exp^*$ *denote the exponential series in* $\mathcal{A}_{ш}[[T]]$, *resp.* $\mathcal{A}_*[[T]]$.

This follows immediately from the equality $y_1^{шk} = k! y_{1\ldots1} = k! X_1^k$ so we have

$$U(y_1^{шk}) = k! \sum_{n_1 > \cdots > n_k} t_{n_1} \ldots t_{n_k} = \sum_{n_1 \neq \cdots \neq n_k} t_{n_1} \ldots t_{n_k}$$

and

$$U(\exp^{ш} y_1 T) = \prod (1 + t_j T) = \exp \sum_{k \geq 1, j} \frac{(-1)^{k-1}}{k} t_j^k T^k .$$

Since $\sum t_j^k = U(y_k)$ this implies (16).

One can reformulate this identity as follows: let D be the "left interior" product operator by X_1 in $\mathcal{A}$. This is a derivation, for both laws $*$ and ш since $X_1 = y_1$ is primitive for both coproducts $\Delta_{ш}, \Delta_*$. We have $Dy_{s_1} \dots y_{s_n} = 0$ if $s_1 > 1$, $y_{s_2} \dots y_{s_r}$ if $s_1 = 1$ and the fixed sub-algebra of D is the subring $\mathcal{A}^0 = k + X_0 \mathcal{L} X_1$ above, corresponding to the "obviously" convergent series. Theorem 1 can be expressed as follows:

Corollary 2 *Let $\Phi(D)$ be the (infinite order) differential operator on $(\mathcal{A}, *)$:*

$$\Phi(D) = \exp^*\left(\sum_{k \geq 2} \frac{(-1)^{k-1}}{k} y_k\, D^k\right). \tag{17}$$

Then for $f_k \in \mathcal{A}^0$ we have

$$\sum f_k * y_1^{ш k} = \Phi(D) \sum f_k * y_1^{*k} \tag{18}$$

3.2 Relations

Regularized divergent ζ values are obtained by extending the character $a \mapsto \zeta_a$ from $\mathcal{A}^0$ to $\mathcal{A}$ for one of the laws $*$ or ш; for this one only needs to define the zeta-value $\zeta_{y_1} = \theta$, so the extended character is $f \mapsto S_f(\theta)$, resp. $M_f(\theta)$ with the notation of §1; the most natural choices are $\theta = 0$ or $\theta = \gamma$, the Euler constant.

The two extensions are not equal. However both extensions coincide on on the elements $a \in \mathcal{A}$ such that the corresponding series $f = Li_a = \sum f_n z^n$ is convergent for $z = 1$ (i.e. $S_f = M_f =$ constant, as in §1); for such an element the zeta number $\zeta_a = Li_a(1)$ is still unambiguously defined. These elements form a sub-algebra, for both laws ш and $*$, which contains $\mathcal{A}^0 = k + X_0 \mathcal{L} X_1$.

By §1 the null-ideal J is the same for both extended characters: it is the set of all $a \in \mathcal{A}$ such that $Li_a(1)$ is convergent, and the sum is 0. J is not defined over $\mathbb{Q}$ (the coefficients of the extended character are not rational, and J is not of codimention 1 if k is smaller than $\mathbb{C}$). However J certainly contains all elements of the form $a * b - a ш b$ with $b \in \mathcal{A}^0$. E.g. we have $y_1 * y_2 - y_1 ш y_2 = y_3 - y_{12}$ hence $\zeta_3 - \zeta_{12} = 0$ (Euler).

Let V be the k-vector space generated by the $a * b - a ш b$ ($b \in \mathcal{A}^0$). V is stable by D ($D(a * b - a ш b) = Da * b - Da ш b$). In fact it follows easily from theorem 2 that the two $*$ or ш ideals $A * V$ and $A ш V$ are equal. Then $I = A * V = A ш V$ is stable by D so it is generated by $I \cap \mathcal{A}^0$, i.e. by the coefficients $f_j \in \mathcal{A}^0$ of $f = \sum f_k * y_1^{*k}$ (or $f = \sum f_k ш y_1^{ш k}$) for all $f = a * b - a ш b$ as above.

The ideal I thus introduced is rational (everything is well defined if $k = \mathbb{Q}$). The now standard conjecture is that for $k = \mathbb{Q}$ we have $I = J$, the null ideal of ζ, i.e. the only algebraic relations between zeta numbers are those that can be deduced formally from the comparison of the two product laws. This seems rather out of reach right now and present work deals with proving that all known relations can be reduced to those above, and making explicit the ring structure of the quotient ring $\mathcal{A}_0$ mod. these relations.

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