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Algebraic description of D-modules associated to 3×3 Matrices

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Abstract

In this paper we give a classification of regular holonomic $\mathcal{D}$-modules whose characteristic variety is contained in the union of the conormal bundles to the orbits of the group of invertible matrices of order 3. The main result is an equivalence between the category of these differential modules and the one of graded modules of finite type over the Weyl algebra of invariant differential operators under the action of the group of invertible matrices. We inferre that such objects can be understood in terms of finite diagrams of complex vector spaces of finite dimension related by linear maps.

1 Introduction

Let X be the complex vector space of square matrices of order 3. The product group of invertible matrices $GL_3(\mathbb{C}) \times GL_3(\mathbb{C})$ acts linearly on X by right and left multiplication: $((g,h), A) \longmapsto gAh^{-1}$. Denote by G the quotient group of $GL_3(\mathbb{C}) \times GL_3(\mathbb{C})$ by $\{(\lambda I_3, -\lambda I_3), \lambda \in \mathbb{C}^{\times}\}$ the kernel of this action. As usual $\mathcal{D}_X$ will refer to the sheaf of analytic differential operators on X. The action of G on X defines a morphism, $L : \mathcal{G} \longrightarrow \Theta_X, A \longmapsto L(A)$, from the Lie algebra $\mathcal{G}$ of G to the subalgebra Θ_X of $\mathcal{D}_X$ consisting of vector fields on X (infinitesimal generators of this action). The characteristic variety Λ of infinitesimal generators has four irreducible components which are the conormal bundles to the orbits of G:

$$\Lambda := \overline{T^*_{X_0}X} \bigcup \overline{T^*_{X_1}X} \bigcup \overline{T^*_{X_2}X} \bigcup \overline{T^*_{X_3}X} \tag{1}$$

where X_i is the set of matrices of rank exactly $i = 0, 1, 2, 3$. The aim of this paper is to give a combinatorial classification of regular holonomic $\mathcal{D}_X$-modules $\mathcal{M}$ whose characteristic variety char($\mathcal{M}$) is contained in Λ (see [15]). We will denote by $\mathrm{Mod}^{\mathrm{rh}}_{\Lambda}(\mathcal{D}_X)$ the category of these $\mathcal{D}_X$-modules. The main ingredient for obtaining a classification of these objects is the right-left action on X of G, and the extension of this action to an action of the univesal covering ($\widetilde{G} := SL_3(\mathbb{C}) \times SL_3(\mathbb{C}) \times \mathbb{C}$) of G on the differential systems in $\mathrm{Mod}^{\mathrm{rh}}_{\Lambda}(\mathcal{D}_X)$.

Our own findings indicate that such $\mathcal{D}_X$-modules have a finite presentation by generators and relations. More precisely, we will first introduce the algebra $\overline{\mathcal{B}}$, of algebraic operators on X which are invariant by $\widetilde{G}$ and give the description of this algebra by generators and relations. We will also describe the quotient algebra $\mathcal{B}$ of $\overline{\mathcal{B}}$ which acts freely on the ring of homogenous functions which are invariant by $SL_3(\mathbb{C}) \times SL_3(\mathbb{C})$. Then we inferre that there is an equivalence of categories between $\mathrm{Mod}_{\Lambda}^{\mathrm{rh}}(\mathcal{D}_X)$ and the category of graded $\mathcal{B}$-modules of finite type, the image by this equivalence of a differential system being its set of "global homogeneous sections" (i.e. global sections of finite type for the Euler vector field on X). From this result we will obtain a result of combinatorial classification. In other words we deduce that any object of this category can be understood in terms of finite diagrams of linear maps. Note that, before our study, many classical results of the same type have been obtained in various situations by other mathematicians, notably L. Boutet de Monvel [2] gave, very elegantly, a description of regular holonomic $\mathcal{D}_{\mathbb{C}}$-modules by using pairs of finite dimensional $\mathbb{C}$-vector spaces and certain linear maps. Galligo, Granger and Maisonobe [5] obtained using, the fundamental result of Kashiwara [9] and Mebkhout [14] i.e. the so called Riemann-Hilbert correspondence, a classification of regular holonomic $\mathcal{D}_{\mathbb{C}^n}$-modules with singularities along $x_1 \cdots x_n$ by 2^n-tuples of $\mathbb{C}$-vector spaces with a set of linear maps. R. MacPherson and K. Vilonen [13] treated the case with singularities along the curve $y^n = x^m$. M. Narvaez [18] treated the case $y^2 = x^p$ using the method of Beilinson and Verdier. Finally, the author [16], [17] constructed an explicit presentation in the case of the quadratic cone in $\mathbb{C}^n$(see also [6]) etc. This paper is organized as follows: In section 2, first we review the necessary results on homogeneous $\mathcal{D}_X$-modules (see [16], [17]): on the one hand, if $\mathcal{M}$ is a coherent $\mathcal{D}_X$-module with a good filtration stable under the action of the Euler vector field on X, denoted by θ, then $\mathcal{M}$ is generated by a finite number of global sections $(s_j)_{j=1,\cdots,p} \in \Gamma(X, \mathcal{M})$ such that $\dim_{\mathbb{C}} \mathbb{C}[\theta] s_j < +\infty$ (see Theorem 1.3 of [16]). On the other hand, the infinitesimal action of the group G lifts to an action of the universal covering $\widetilde{G} := SL_3(\mathbb{C}) \times SL_3(\mathbb{C}) \times \mathbb{C}$ of G on $\mathcal{M}$. Let us emphasize that the introduction of the universal covering is not necessary for the action on X (which of course goes down to G) but is required for the differential systems and then, that the hypothesis on the characteristic variety of $\mathcal{M}$ is essential. Next, we give a description of the $\mathbb{C}$-algebra $\overline{\mathcal{B}} := \Gamma(X, \mathcal{D}_X)^{\widetilde{G}}$ of $\widetilde{G}$-invariant differential operators with polynomial coefficients. We get the following result: let $x_1 = (x_{ij})$, $d_1 = {}^t(\frac{\partial}{\partial x_{ij}})$ be matrices with entries in $\mathcal{D}_X$. The group $\widetilde{G}$ acts on these matrices by $g \cdot (x_1, d_1) = (a x_1 b^{-1}, b d_1 a^{-1})$ for $\forall g = (a, b) \in \widetilde{G}$. Denote by $x_2 := \det(x_1)\, x_1^{-1}$ (resp. d_2) the adjoint matrice of x_1 (resp. d_1) and by Tr the trace map. We set $\delta := \frac{1}{3}\mathrm{Tr} x_1 x_2 = \det(x_{ij})$, $\Delta := \frac{1}{3}\mathrm{Tr} d_1 d_2 = \det\left(\frac{\partial}{\partial x_{ij}}\right)$, $\theta := \mathrm{Tr} x_1 d_1$ (the Euler vector field on X), $q := \mathrm{Tr} x_2 d_2$.

Proposition 1 *The algebra $\overline{\mathcal{B}}$ is generated over $\mathbb{C}$ by δ, Δ, θ, q such that*

$$\begin{array}{ll}
(r_1) & [\theta,\ \delta] = +3\delta \\
(r_2) & [\theta,\ \Delta] = -3\Delta \\
(r_3) & [\theta,\ q] = 0 \\
(r_4) & [q,\ \delta] = 2\theta\delta \\
(r_5) & [q,\ \Delta] = -2\Delta\theta \\
(r_6) & [\Delta,\ \delta] = \frac{q}{3} + 2(\frac{\theta}{3}+1)(\frac{\theta}{3}+3).
\end{array}$$

In section 3, we study regular holonomic $\mathcal{D}_X$-modules with support on $\overline{X_2} = X_0 \cup X_1 \cup X_2$. We provide a very concrete characterization of such $\mathcal{D}_X$-modules. This study is fundamental to Section 4 which is devoted to the proof of the following result:

Theorem 2 *Let $\mathcal{M}$ be an object in $\mathrm{Mod}^{\mathrm{rh}}_{\Lambda}(\mathcal{D}_X)$. $\mathcal{M}$ is generated by its $\widetilde{G}$-invariant global sections $(s_j)_{j=1,\cdots,p} \in \Gamma(X, \mathcal{M})$ such that $\dim_{\mathbb{C}} \mathbb{C}[\theta]\, s_j < \infty$.*

This theorem is at the heart of the proof of our main theorem which is stated in section 5 as follows: let $\mathcal{W}$ be the Weyl algebra on X. Denote by $\mathcal{B}$ the quotient algebra of $\overline{\mathcal{B}}$ by the two sided ideal generated by $A := \delta\Delta - \frac{\theta}{3}(\frac{\theta}{3}+1)(\frac{\theta}{3}+2)$ and $B := \frac{q}{3} - \frac{\theta}{3}(\frac{\theta}{3}+1)$. Namely, we set $\mathcal{B} := \overline{\mathcal{B}}/\overline{\mathcal{B}}(A,B)\overline{\mathcal{B}}$. We will denote by $\mathrm{Mod}^{\mathrm{gr}}(\mathcal{B})$ the category of graded $\mathcal{B}$-modules T of finite type such that $\dim_{\mathbb{C}} \mathbb{C}[\theta]\, u < \infty$ for $^{\forall} u \in T$.
If $\mathcal{M}$ is an object in the category $\mathrm{Mod}^{\mathrm{rh}}_{\Lambda}(\mathcal{D}_X)$, denote by $\Psi(\mathcal{M})$ the submodule of $\Gamma(X,\mathcal{M})$ consisting of $\widetilde{G}$-invariant global sections u in $\mathcal{M}$ such that $\dim_{\mathbb{C}} \mathbb{C}[\theta]\, u < \infty$. Then $\Psi(\mathcal{M})$ is an object in the category $\mathrm{Mod}^{\mathrm{gr}}(\mathcal{B})$. Conversely, if T is an object in the category $\mathrm{Mod}^{\mathrm{gr}}(\mathcal{B})$, one associates to it the $\mathcal{D}_X$-module $\Phi(T) = \mathcal{M}_0 \bigotimes_{\mathcal{B}} T$, where $\mathcal{M}_0 = \mathcal{W}/\mathcal{I}$ with $\mathcal{I}$ the left ideal generated by infinitesimal generators of G. Then $\Phi(T)$ is an object in the category $\mathrm{Mod}^{\mathrm{rh}}_{\Lambda}(\mathcal{D}_X)$. Thus, we have defined two functors

$$\begin{cases} \Psi : \mathrm{Mod}^{\mathrm{rh}}_{\Lambda}(\mathcal{D}_X) \longrightarrow \mathrm{Mod}^{\mathrm{gr}}(\mathcal{B}) \\ \\ \Phi : \mathrm{Mod}^{\mathrm{gr}}(\mathcal{B}) \longrightarrow \mathrm{Mod}^{\mathrm{rh}}_{\Lambda}(\mathcal{D}_X) \end{cases} \tag{2}$$

We get the following result:

Theorem 3 *The functors Φ and Ψ induce equivalence of categories*

$$\mathrm{Mod}^{\mathrm{rh}}_{\Lambda}(\mathcal{D}_X) \xrightarrow{\sim} \mathrm{Mod}^{\mathrm{gr}}(\mathcal{B}). \tag{3}$$

Finally, we close this study by describing the objects in the category $\mathrm{Mod}^{\mathrm{gr}}(\mathcal{B})$ in terms of finite diagram of linear maps.

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2 Homogeneous modules and invariant operators

In this paper, we will use the theory of analytic $\mathcal{D}$-modules developed in [7], [8], [9], [10], [11], [12]. The first part of this section consists in the review of necessary results on homogeneous $\mathcal{D}$-modules (see [16], [17]). In the second part, we describe the algebra of invariant differential operators under the action of invertible matrices.

2.1 Homogeneous modules

Definition 4 *Let $\mathcal{M}$ be a $\mathcal{D}_X$-module. We say that $\mathcal{M}$ is homogeneous if there is a good filtration stable under the action of the Euler vector field θ on X. We say that a section s in $\mathcal{M}$ is homogeneous if $\dim_{\mathbb{C}} \mathbb{C}[\theta]\, s < \infty$. The section s is said to be homogeneous of degree $\lambda \in \mathbb{C}$, if there exists $j \in \mathbb{N}$ such that $(\theta - \lambda)^j s = 0$.*

Theorem 5 *([16, Theorem 1.3.]) Let $\mathcal{M}$ be a coherent homogeneous $\mathcal{D}_X$-module with a good filtration $(F_k\mathcal{M})_{k\in\mathbb{Z}}$ stable by θ. Then*
i) $\mathcal{M}$ is generated by a finite number of global sections $(s_j)_{j=1,\cdots,q} \in \Gamma(X, \mathcal{M})$ such that $\dim_{\mathbb{C}} \mathbb{C}[\theta]\, s_j < \infty$,
ii) For any $k \in \mathbb{N}$, $\lambda \in \mathbb{C}$, the vector space $\Gamma(X, F_k\mathcal{M}) \bigcap \left[\bigcup_{p\in\mathbb{N}} \ker(\theta-\lambda)^p\right]$ of homogeneous global sections of $F_k\mathcal{M}$ of degree λ is finite dimensional.

Note that a similar result was proved in the case of regular holonomic $\mathcal{D}$-modules by J. L. Brylinski, B. Malgrange, J. L. Verdier (see [4]).

Remark 6 *The action of the group G (preserving the good filtration) on a $\mathcal{D}_X$-module $\mathcal{M}$ is given by an isomorphism $u : p_1^+(\mathcal{M}) \xrightarrow{\sim} p_2^+(\mathcal{M})$ where $p_1 : G \times X \longrightarrow X$ is the projection on X, and $p_2 : G \times X \longrightarrow X$, $(g, x) \longmapsto g \cdot x$ defines the action of G on X (satisfying the associativity conditions). In fact u is an isomorphism above the isomorphism of algebras $\widetilde{u} : p_1^+(\mathcal{D}_X) \xrightarrow{\sim} p_2^+(\mathcal{D}_X)$.*

As mentioned in the introduction, $\mathrm{Mod}_\Lambda^{\mathrm{rh}}(\mathcal{D}_X)$ stands for the category of regular holonomic $\mathcal{D}$-modules whose characteristic variety is contained in Λ and $\widetilde{G} := SL_3(\mathbb{C}) \times SL_3(\mathbb{C}) \times \mathbb{C}$ denote the universal covering of G. Let $\mathcal{M}$ be an object in the category $\mathrm{Mod}_\Lambda^{\mathrm{rh}}(\mathcal{D}_X)$. By virtue of Theorem 5 we get the following proposition:

Proposition 7 *([16, Proposition 1.6.])The infinitesimal action of G on $\mathcal{M}$ lifts to an action of $\widetilde{G}$ on $\mathcal{M}$, compatible with the one of G on X and $\mathcal{D}_X$.*

2.2 Invariant operators

Let us recall that $\mathcal{W}$ indicates the Weyl algebra on X. We describe the subalgebra of $\mathcal{W}$ of $\widetilde{G}$-invariant differential operators. We denote it by $\overline{\mathcal{B}}$. Let $x_1 = (x_{ij})$, $d_1 = {}^t(\frac{\partial}{\partial x_{ij}})$ be matrices with entries in $\mathcal{D}_X$. The group $\widetilde{G}$ (resp. G) acts on these matrices by right and left multiplication: for any $g = (a, b) \in \widetilde{G}$, we have $g \cdot (x_1, d_1) = (ax_1b^{-1},\ bd_1a^{-1})$. Let $x_2 := \det(x_1)\, x_1^{-1}$ (resp. d_2) be the adjoint matrice of x_1(resp. d_1). Denote by Tr the trace map. We set $\delta := \frac{1}{3}\mathrm{Tr} x_1 x_2 = \det(x_{ij})$, $\Delta = \frac{1}{3}\mathrm{Tr} d_1 d_2 = \det\left(\frac{\partial}{\partial x_{ij}}\right)$, $\theta = \mathrm{Tr} x_1 d_1$ (the Euler vector field on X), $q := \mathrm{Tr} x_2 d_2$. We have the following proposition:

Proposition 8 *The algebra $\overline{\mathcal{B}}$ is generated over $\mathbb{C}$ by δ, Δ, θ, q such that*

$$\begin{array}{rclr}
[\theta,\ \delta] & = & +3\delta, & (r_1) \\
[\theta,\ \Delta] & = & -3\Delta, & (r_2) \\
[\theta,\ q] & = & 0, & (r_3) \\
[q,\ \delta] & = & 2\theta\delta, & (r_4) \\
[q,\ \Delta] & = & -2\Delta\theta, & (r_5) \\
[\Delta,\ \delta] & = & \frac{q}{3} + 2(\frac{\theta}{3}+1)(\frac{\theta}{3}+3). & (r_6)
\end{array}$$

Let G_0 be the image of the group $SL_3(\mathbb{C}) \times SL_3(\mathbb{C})$ in G. The G_0-invariant homogeneous functions are the elements in $\mathbb{C}[\delta]$. Clearly, the algebra $\overline{\mathcal{B}} \subset \mathcal{W}$ acts on $\mathbb{C}[\delta]$. Denote by $\mathcal{J}$ the kernel of this action. Then $\mathcal{J}$ contains the following homogeneous operators $\delta\Delta - \frac{\theta}{3}(\frac{\theta}{3}+1)(\frac{\theta}{3}+2)$ and $\frac{q}{3} - \frac{\theta}{3}(\frac{\theta}{3}+1)$. Denote by $\mathcal{I}$ the left ideal generated by infinitesimal generators of G. Then $\mathcal{J}$ is the two sided ideal of $\widetilde{G}$-invariant differential operators with polynomial coefficients, $P \in \overline{\mathcal{B}}$, which are also contained in the ideal $\mathcal{I}$ that is $\mathcal{J} := \overline{\mathcal{B}} \cap \mathcal{I}$. This last can be described concisely as follows:

Lemma 9 *The ideal $\mathcal{J}$ is generated by*

$$\delta\Delta - \frac{\theta}{3}(\frac{\theta}{3}+1)(\frac{\theta}{3}+2) \text{ and } \frac{q}{3} - \frac{\theta}{3}(\frac{\theta}{3}+1). \tag{4}$$

We close this section by the following corollary which is an immediate consequence of Proposition 8. Denote by $\mathcal{B}$ the quotient algebra of $\overline{\mathcal{B}}$ by the two sided ideal $\mathcal{J}$ that is $\mathcal{B} := \overline{\mathcal{B}}/\mathcal{J}$. The algebra $\mathcal{B}$ acts faithfully on the set of G_0-invariant homogeneous functions that is on $\mathbb{C}[\delta]$.

Corollary 10 *The algebra $\mathcal{B} := \overline{\mathcal{B}}/\mathcal{J}$ is generated over $\mathbb{C}$ by δ, Δ, θ such that*

$$\begin{array}{ll} (r_1) & [\theta,\ \delta] = +3\delta \\ (r_2) & [\theta,\ \Delta] = -3\Delta \\ (r_6) & [\Delta,\ \delta] = \ 3(\frac{\theta}{3}+1)(\frac{\theta}{3}+2) \end{array}$$

3 $\mathcal{D}$-modules with support on the set of matrices of rank ≤ 2

Let $\overline{X_i}$ be the set of matrices of rank i or less ($i = 0, 1, 2, 3$). We still denote by δ, the determinant map $\delta : X \longrightarrow \mathbb{C}, x \longmapsto \det(x)$. This section is concerned with the description of regular holonomic $\mathcal{D}_X$-modules with support on the hypersurface $\overline{X_2} := \{x \in X,\ \delta(x) = 0\}$. Such a description is done with the help of the characterization of the inverse image by δ of the $\mathcal{D}_{\mathbb{C}}$-module $\mathcal{O}_{\mathbb{C}}\left(\frac{1}{t}\right)$ where t is a coordinate of $\mathbb{C}$. Without going in further detail, it is important to point out that this study is fundamental for the next section.

3.1 Inverse image

For a $\mathcal{D}_{\mathbb{C}}$-module $\mathcal{N}$, we denote by $\delta^+\mathcal{N}$ its inverse image by the determinant map. Let t be a coordinate of $\mathbb{C}$ and put $\partial_t = \frac{\partial}{\partial t}$. We have the following elementary lemmas:

Lemma 11 *The Transfer module $\mathcal{D}_{X \overset{\delta}{\to} \mathbb{C}}$ is generated over $\mathcal{D}_{X\times\mathbb{C}}$ by an element K subject to the relations*

$$\delta K = Kt, \qquad d_1 K \ = \ x_2 K \partial_t. \tag{5}$$

$\mathcal{D}_{X \overset{\delta}{\to} \mathbb{C}}$ is flat over $\delta^{-1}(\mathcal{D}_{\mathbb{C}})$ and the relations ([?]) imply the following equalities

$$\begin{aligned} x_1 d_1 K &= I_3 K t \partial_t && (6)\\ \theta K &= 3Kt\partial_t && (7)\\ d_2 K &= x_1 K \partial_t\,(t\partial_t + 1) && (8)\\ x_2 d_2 K &= I_3 K t \partial_t\,(t\partial_t + 1) && (9)\\ qK &= 3Kt\partial_t\,(t\partial_t+1) && (10)\\ \Delta K &= K\partial_t\,(t\partial_t+1)\,(t\partial_t+2) && (11) \end{aligned}$$

Therefore, if $\mathcal{N}$ is a $\mathcal{D}_{\mathbb{C}}$-module, the inverse image functor, $\mathcal{N} \longrightarrow \delta^+\mathcal{N}$ is reduced to its first term that is the module $\mathcal{D}_{X \overset{\delta}{\to} \mathbb{C}} \bigotimes_{\delta^{-1}(\mathcal{D}_{\mathbb{C}})} \delta^{-1}\mathcal{N}$ and it is an exact functor. Consequently, if $\mathcal{N}$ is a regular holonomic $\mathcal{D}_{\mathbb{C}}$-module with singularity at $t=0$, then its inverse image $\mathcal{M} := \delta^{+}\mathcal{N}$ decomposes at least as $\mathcal{N}$. Moreover, if the operator of multiplication by t is invertible on $\mathcal{N}$ then the operator of multiplication by δ is also invertible on the inverse image $\mathcal{M} := \delta^{+}\mathcal{N}$. In particular, in this case, any meromorphic section (of $\delta^{+}\mathcal{N}$) defined in $X \backslash \overline{X_2}$ extends to the whole X. To put it more precisely, let j be the embedding $X \backslash \overline{X_2} \hookrightarrow X$. Denote by j_*(resp. j^*) the "meromorphic" algebraic direct (resp. inverse) image (see [1]). If $\mathcal{M}$ is a $\mathcal{D}_X$-module, we set $\overline{\mathcal{M}} := j_* j^* (\mathcal{M})$ the algebraic module of meromorphic sections of $\mathcal{M}$ with pole in $\overline{X_2}$. We have a canonical homomorphism $\mathcal{M} \longrightarrow \overline{\mathcal{M}}$ and it defines an exact functor $\mathcal{M} \longrightarrow \overline{\mathcal{M}}$ as j_*. We have the following proposition:

Proposition 12 *Let $\mathcal{N}$ be a regular holonomic $\mathcal{D}_{\mathbb{C}}$-module with singularity at $t=0$. Assume that the operator of multiplication by t is invertible on $\mathcal{N}$ then*

i) the operator of multiplication by δ is invertible on the inverse image $\delta^{+}\mathcal{N}$,

in particular

ii) the canonical homomorphism

$$\delta^{+}\mathcal{N} \xrightarrow{\sim} \overline{\delta^{+}\mathcal{N}} \tag{12}$$

is an isomorphism that is the meromorphic sections defined in $X \backslash \overline{X_2}$ extend to the whole X.

3.2 Characterization of $\delta^{+}\left(\mathcal{O}_{\mathbb{C}}\left(\frac{1}{t}\right)\right)$

Let us give an explicit description of the inverse image $\delta^{+}\left(\mathcal{O}_{\mathbb{C}}\left(\frac{1}{t}\right)\right)$ where $t \in \mathbb{C}$. In particular, we describe all the submodules of $\delta^{+}\left(\mathcal{O}_{\mathbb{C}}\left(\frac{1}{t}\right)\right)$by way of its irreducible (simple) submodules. This study is carried out with a view to using such modules in order to prove that any regular holonomic $\mathcal{D}_X$-module in the category $\mathrm{Mod}_{\Lambda}^{\mathrm{rh}}(\mathcal{D}_X)$ is generated by its $\widetilde{G}$-invariant sections.
Let $P = \delta^{+}\left(\mathcal{O}_{\mathbb{C}}\left(\frac{1}{t}\right)\right) = \mathcal{O}_X\left(\frac{1}{\delta}\right)$. The $\mathcal{D}_X$-module P is generated by its $\widetilde{G}$-invariant homogeneous sections. Namely P is generated by the combinations of the $e_k := \delta^k$ where $k \leq 0$. Note that if we want to emphasize the tensor structure in the inverse image, $e_k := \left(K \cdot t^k\right) \otimes 1 = K \otimes t^k$ where $k \leq 0$ and K is the generator of Transfer module. We get

$$\delta e_k = e_{k+1}, \qquad d_1 e_k \quad = \quad k x_2 e_{k-1}. \tag{13}$$

These relations imply the following

$$d_2 e_k = k\,(k+1)\,x_1 e_{k-1}, \tag{14}$$

$$\Delta e_k = k\,(k+1)\,(k+2)\,e_{k-1}. \tag{15}$$

The $\mathcal{D}_X$-module P has 4 submodules denoted by P_j, generated respectively by e_j $(j = 0, -1, -2, -3)$. Denote by P^j the 4 subquotients associated to P_j: $P^0 = P_0$; $P^j = P_j/P_{j+1}$ if $j = -1, -2, -3$. The quotient $P^j$is an irreducible holonomic $\mathcal{D}_X$-module of multiplicity 1, whose microsupport is $\Lambda_{3+j} := \overline{T^*_{X_{3+j}} X}$ $(j = 0, -1, -2, -3)$. Indeed we have $P^0 = P_0 = \mathcal{O}_X$ and the following description

Irreducible $\mathcal{D}$-modules	Associated generators and relations	
P^{-1} with $\mathrm{char}\left(P^{-1}\right) = \overline{T^*_{X_2} X}$	generator $\widetilde{e}_{-1}$, $\left\{\begin{array}{c} \delta\widetilde{e}_{-1} = 0 \\ d_2\widetilde{e}_{-1} = 0 \\ x_1 d_1 \widetilde{e}_{-1} = -I_3 \widetilde{e}_{-1} \\ \theta\widetilde{e}_{-1} = -3\widetilde{e}_{-1} \end{array}\right.$	(16)
P^{-2} with $\mathrm{char}\left(P^{-2}\right) = \overline{T^*_{X_1} X}$	generator $\widetilde{e}_{-2}$, $\left\{\begin{array}{c} x_2\widetilde{e}_{-2} = 0 \\ \Delta\widetilde{e}_{-2} = 0 \\ x_1 d_1 \widetilde{e}_{-2} = -2I_3 \widetilde{e}_{-2} \\ \theta\widetilde{e}_{-2} = -6\widetilde{e}_{-2} \end{array}\right.$	(17)
P^{-3} with $\mathrm{char}\left(P^{-3}\right) = \overline{T^*_{X_0} X}$	generator $\widetilde{e}_{-3}$, $\left\{\begin{array}{c} x\widetilde{e}_{-3} = 0 \\ \theta\widetilde{e}_{-3} = -9\widetilde{e}_{-3} \end{array}\right.$	(18)

Then, with the aid of the relations (14), (15), and the basic fact that the P^j are irreducible modules, we can see that any submodule $\mathcal{M}$ of P which is not contained in P_j contains P_{j+1}, this means that the P^j are the only submodules of P. Thus we have the following Lemma:

Lemma 13 *P_0, P_{-1}, P_{-2}, P_{-3} are the only submodules of P.*

The following remark will be used in the proof of the next proposition:

Remark 14 *The hypersuface $\overline{X_2}$ is smooth out of $\overline{X_1}$ and is a "normal" variety along X_1 (smooth). Indeed along X_1, the variety $\overline{X_2}$ is locally isomorphic to the product of X_1 (smooth) and a quadratic cone.*

We get the following proposition:

Proposition 15 *Any section $s \in \Gamma\left(X\backslash\overline{X_1}, P_{-2}\right)$ (resp. $\Gamma\left(X\backslash X_0, P_{-1}\right)$) of P_{-2} (resp. P_{-1}) defined on the complementary of $\overline{X_1}$ (resp. $X_0 = \{0\}$) extends to the whole X.*

Proof. The $\mathcal{D}_X$-module P_j is the union of the modules $\mathcal{O}_X e_k$ $(j \leq k \leq 0)$ so that the associated graded module $\mathrm{gr}(P_j)$ is the sum of modules $\mathcal{O}_{X_{3+j}} \widetilde{e}_k$ $(j = -1, -2, -3)$. Since the hypersurface $\overline{X_2}$ is a normal variety along $\overline{X_1}$ (see Remark 14) and $\overline{X_1}$ is normal, then the "property of extension" is true here for the functions. ∎

4 Invariant sections

In this section, we intend to show that any regular holonomic $\mathcal{D}_X$-module $\mathcal{M}$ in the category $\mathrm{Mod}_{\Lambda}^{\mathrm{rh}}(\mathcal{D}_X)$ is generated by its $\widetilde{G}$-invariant homogeneous global sections. This fact is at the heart of the proof of our main theorem. In an attempt to do it, first we restrict the $\mathcal{D}_X$-module $\mathcal{M}$ to a section of the projection defined by δ the deteminant map. This allows us to consider $\mathcal{M}$ as an inverse image by δ of a $\mathcal{D}_{\mathbb{C}}$-module $\mathcal{N}$ outside of $X_1 \cup X_0 =: \overline{X_1}$ (the singular part of the hypersurface $\overline{X_2} := \{x \in X,\ \delta(x) = 0\}$). Namely

$$\mathcal{M}_{|X\setminus(X_1\cup X_0)} \simeq \delta^+\mathcal{N}_{|X\setminus(X_1\cup X_0)} \tag{19}$$

Next, using the fundamental results of the previous section we will get the desired theorem.
To begin with, let us recall that the determinant map $\delta : X \longrightarrow \mathbb{C}, x \longmapsto \det(x)$ is submersive out of $X_1 \cup X_0 =: \overline{X_1}$. Denote by $i : \mathbb{C} \longrightarrow X, t \longmapsto \begin{pmatrix} t & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ a section of δ ($\delta \circ i = Id_{\mathbb{C}}$). Denote by $D := i(\mathbb{C})$ its image. Let $\mathcal{M}$ be an object in the category $\mathrm{Mod}_{\Lambda}^{\mathrm{rh}}(\mathcal{D}_X)$, we get the following lemma:

Lemma 16 *D is non characteristic for $\mathcal{M}$ i.e. $\overline{T_D^*X} \bigcap char(\mathcal{M}) \subset T_X^*X$.*

Since the line D is non characteristic for the $\mathcal{D}_X$-module $\mathcal{M}$ (see Lemma 16), then $\mathcal{M}$ is canonically isomorphic to $\delta^+ i^+(\mathcal{M})$ in the neighborhood of D i.e.

$$\mathcal{M}_{|_D} \simeq \delta^+ i^+ \mathcal{M}_{|_D}. \tag{20}$$

We know from Kashiwara [7] that the sheaf $\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \delta^+ i^+ \mathcal{M})$ is constructible. Also $\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \delta^+ i^+ \mathcal{M})$ is a locally constant sheaf on the fibers $\delta^{-1}(t)$, $t \in \mathbb{C}$. As the group $\widetilde{G}$ acts on the $\mathcal{D}_X$-modules $\mathcal{M}$ and $\delta^+ i^+ \mathcal{M}$, it acts also on the sheaf $\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \delta^+ i^+ \mathcal{M})$ and because of the action of $\widetilde{G}$ the stratas are the orbits of $\widetilde{G}$ that is X_0, X_1, X_2, X_3 (see [12]). The sheaf $\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \delta^+ i^+ \mathcal{M})$ has a canonical section u defined in the neighborhood of the line D (corresponding with the isomorphism $\mathcal{M} \xrightarrow{\sim} \delta^+ i^+(\mathcal{M})$ which induces the identity on D). Since the fibers $\delta^{-1}(t)$, $t \in \mathbb{C}$ are simply connected, we have the following proposition:

Proposition 17 *The canonical isomorphism $u : \mathcal{M} \xrightarrow{\sim} \delta^+ i^+(\mathcal{M})$ defined in the neighborhood of D such that $i^+.u = Id_{|D}$, extends to $X \setminus (X_1 \cup X_0)$.*

From now on, let us denote by $\mathcal{N} := i^+\mathcal{M}$ the restriction of the $\mathcal{D}_X$-module $\mathcal{M}$ to the transversal line D. We know from Proposition 17 that the $\mathcal{D}_X$-module $\mathcal{M}$ is isomorphic to $\delta^+\mathcal{N}$ on $X \setminus \overline{X_1}$:

$$\mathcal{M}_{|_{X\setminus\overline{X_1}}} \simeq \delta^+\mathcal{N}_{|_{X\setminus\overline{X_1}}}. \tag{21}$$

In particular this isomorphism is true out of the hypersurface $\overline{X_2}$.

$$\mathcal{M}_{|X\setminus\overline{X_2}} \simeq \delta^+\mathcal{N}_{|X\setminus\overline{X_2}}. \tag{22}$$

Recall that $\overline{\mathcal{M}}$ (see section 3.1) indicates the $\mathcal{D}_X$-module of meromorphic sections of $\mathcal{M}$ defined on $X\setminus\overline{X_2}$. According to an argument of Kashiwara, since $\mathcal{M}$ and $\delta^+\mathcal{N}$ are regular holonomic and isomorphic out of $\overline{X_2}$, then their corresponding "meromorphic" modules are also isomorphic that is

$$\overline{\mathcal{M}} \simeq \overline{\delta^+\mathcal{N}}\,. \tag{23}$$

Now consider the left exact functor (see section 3.1)

$$\mathcal{M} \longrightarrow \overline{\mathcal{M}}\left(\simeq \overline{\delta^+\mathcal{N}}\right). \tag{24}$$

By using the basic fact that $\overline{\delta^+\mathcal{N}} \simeq \delta^+\mathcal{N}$ (see relation (12) of Proposition 12) and the morphism (24), it follows that there exists a morphism

$$v : \mathcal{M} \longrightarrow \delta^+\mathcal{N} \tag{25}$$

which is an isomorphism out of the hypersurface $\overline{X_2}$.
Now we can prove the following theorem:

Theorem 18 *$\mathcal{M}$ is generated by its $\widetilde{G}$-invariant homogeneous global sections.*

Proof. To begin with, recall that we have denoted by $P := \delta^+(\mathcal{O}_{\mathbb{C}}\left(\frac{1}{t}\right)) = \mathcal{O}_X\left(\frac{1}{\delta}\right)$ (see section 3.2). We know that the $\mathcal{D}_X$-module P is generated by its $\widetilde{G}$-invariant homogeneous sections $e_k = K \cdot t^k \otimes 1 = K \otimes t^k = \delta^k$ where $k \leq 0$ and K is the generator of the Transfer module $\mathcal{D}_{X \xrightarrow{\delta} \mathbb{C}}$ subject to the relations (13), (14), (15). In particular, P has 4 sub $\mathcal{D}$-modules which we have denoted by P_j, generated by e_j $(j = 0, -1, -2, -3)$ (see Lemma 13).
Let $\mathcal{M}^G \subset \mathcal{M}$ be the submodule generated by $\widetilde{G}$-invariant homogeneous global sections. We are going to show successively that the quotient $\mathcal{M}/\mathcal{M}^G$is a $\mathcal{D}_X$-module with support on $\overline{X_i}$, $i = 0, 1, 2$.
• $\mathcal{M}/\mathcal{M}^G$ is with support on $\overline{X_2}$: indeed, we know from Proposition 17 that $\mathcal{M}$ is isomorphic in $X\setminus\overline{X_2}$ to a module $\delta^+\mathcal{N}$. One may assume that the operator of multiplication by t is inversible on $\mathcal{N}$ such that there is an homomorphism $v : \mathcal{M} \longrightarrow \delta^+\mathcal{N}$ (see (25)) which is an isomorphism out of $\overline{X_2}$. The image $v(\mathcal{M})$ is a submodule of $\delta^+\mathcal{N}$ thus it is generated by its invariant homogeneous sections. Let s be an invariant global section of a quotient of $\mathcal{M}$, then the section s lifts to an invariant section $\widetilde{s}$ of $\mathcal{M}$ $(\widetilde{s} \in \Gamma(X, \mathcal{M})^G)$. Therefore $\mathcal{M}/\mathcal{M}^G$ is with support on $\overline{X_2}$.
• If $\mathcal{M}$ is with support on $\overline{X_2}$, it is isomorphic out of $\overline{X_1}$ to a direct sum of copies of P_{-3}/P_0 (the Dirac $\mathcal{D}_X$-module with support on $\overline{X_2}$). Then there is a morphism $\mathcal{M} \longrightarrow (P_{-3}/P_0)^N$ whose sections extend to the whole X, such that $\mathcal{M}/\mathcal{M}^G$ is with support on $\overline{X_1}$ because the submodules of P_{-3}/P_0 are

also generated by their invariant sections.

• In the same way, if $\mathcal{M}$ is with support on $\overline{X_1}$, then there is a morphism $\mathcal{M} \longrightarrow (P_{-3}/P_{-1})^N$, which is an isomorphism out of $\overline{X_0} = \{0\}$, such that $\mathcal{M}/\mathcal{M}^G$ is with support on $\{0\}$ because the submodules of P_{-3}/P_{-1} are also generated by their invariant sections.

• Finally, if $\mathcal{M}$ is with support on $\overline{X_0} = \{0\}$(the Dirac $\mathcal{D}_X$-module with support on $\{0\}$), the result is obvious. ■

5 Main result

Let us recall that $\mathcal{W}$ indicates the Weyl algebra on X and $\overline{\mathcal{B}} := \Gamma(X, \mathcal{D}_X)^{\widetilde{G}} \subset \mathcal{W}$ the subalgebra of $\widetilde{G}$-invariant differential operators. Then $\overline{\mathcal{B}}$ is generated over $\mathbb{C}$ by four operators δ, Δ, θ, q satisfying the relations (r_i) of Proposition 8. As in section ??, $\mathcal{I}$ stands for the ideal generated by infinitesimal generators of G and $\mathcal{J}:= \overline{\mathcal{B}} \cap \mathcal{I}$ is the two sided ideal generated by $A = \delta\Delta - \frac{\theta}{3}\left(\frac{\theta}{3}+1\right)\left(\frac{\theta}{3}+2\right)$ and $B = \frac{q}{3} - \frac{\theta}{3}\left(\frac{\theta}{3}+1\right)$ (see Lemma 9). We denoted by $\mathcal{B}$ the quotient algebra of $\overline{\mathcal{B}}$ by the ideal $\mathcal{J}$ i.e. $\mathcal{B} := \overline{\mathcal{B}}/\mathcal{J}$. The algebra $\mathcal{B}$ is generated over $\mathbb{C}$ by δ, Δ, θ such that

$$\begin{array}{ll} (r_1) & [\theta,\, \delta] = +3\delta \\ (r_2) & [\theta,\, \Delta] = -3\Delta \\ (r_6) & [\Delta,\, \delta] = 3(\frac{\theta}{3}+1)(\frac{\theta}{3}+2) \end{array}$$

(see corollary 10). It is a graded algebra by the action of homotheties $\mathbb{C}^\times$ and it acts naturally on $\widetilde{G}$-invariant sections.

We will denote by $\mathrm{Mod}^{\mathrm{gr}}(\mathcal{B})$ the category of graded $\mathcal{B}$-modules T of finite type such that $\dim_{\mathbb{C}} \mathbb{C}[\theta]\, u < \infty$ for $^\forall u \in T$. In other words, $T = \underset{\lambda\in\mathbb{C}}{\oplus} T_\lambda$ is a direct sum of $\mathbb{C}$-vector spaces ($T_\lambda = \bigcup\limits_{p\in\mathbb{N}} \ker(\theta-\lambda)^p$ is finite dimensional) equipped with three endomorphisms δ, Δ, θ of degree 3, -3, 0 respectively satisfying the relations $(r_i)_{i=1,2,6}$, with $(\theta-\lambda)$ being a nilpotent operator on each T_λ.

Let us recall that $\mathrm{Mod}^{\mathrm{rh}}_\Lambda(\mathcal{D}_X)$ stands for the category of regular holonomic $\mathcal{D}_X$-modules whose characteristic variety is contained in Λ.

If $\mathcal{M}$ is an object in the category $\mathrm{Mod}^{\mathrm{rh}}_\Lambda(\mathcal{D}_X)$, denote by $\Psi(\mathcal{M})$ the submodule of $\Gamma(X, \mathcal{M})$ consisting of $\widetilde{G}$-invariant homogeneous global sections u of $\mathcal{M}$ such that $\dim_{\mathbb{C}} \mathbb{C}[\theta]\, u < \infty$. Recall that (Theorem 5) $\Psi(\mathcal{M})_\lambda := [\Psi(\mathcal{M})] \cap \left[\bigcup\limits_{p\in\mathbb{N}} \ker\theta-\lambda)^p\right]$ is the $\mathbb{C}$-vector space of homogeneous global sections of degree λ of $\Psi(\mathcal{M})$ and $\Psi(\mathcal{M}) = \bigoplus\limits_{\lambda\in\mathbb{C}} \Psi(\mathcal{M})_\lambda$. Then $\Psi(\mathcal{M})$ is an object in the category $\mathrm{Mod}^{\mathrm{gr}}(\mathcal{B})$.

Conversely, if T is an object in the category $\mathrm{Mod}^{\mathrm{gr}}(\mathcal{B})$, one associates to it the

$\mathcal{D}_X$-module

$$\Phi(T) = \mathcal{M}_0 \bigotimes_{\mathcal{B}} T \tag{26}$$

where $\mathcal{M}_0 := \mathcal{W}/\mathcal{I}$ is a $(\mathcal{W}, \mathcal{B})$-module. Then $\Phi(T)$ is an object in the category $\mathrm{Mod}_\Lambda^{\mathrm{rh}}(\mathcal{D}_X)$.
Thus, we have defined two functors

$$\begin{cases} \Psi : \mathrm{Mod}_\Lambda^{\mathrm{rh}}(\mathcal{D}_X) \longrightarrow \mathrm{Mod}^{\mathrm{gr}}(\mathcal{B}) \\ \Phi : \mathrm{Mod}^{\mathrm{gr}}(\mathcal{B}) \longrightarrow \mathrm{Mod}_\Lambda^{\mathrm{rh}}(\mathcal{D}_X). \end{cases} \tag{27}$$

We get the two following lemmas:

Lemma 19 *The canonical morphism*

$$T \longrightarrow \Psi(\Phi(T)),\ t \longmapsto 1 \otimes t \tag{28}$$

is an isomorphism, and defines an isomorphism of functors $\mathrm{Id}_{\mathrm{Mod}^{\mathrm{gr}}(\mathcal{B})} \longrightarrow \Psi \circ \Phi$.
.

Lemma 20 *The canonical morphism*

$$w : \Phi(\Psi(\mathcal{M})) \longrightarrow \mathcal{M} \tag{29}$$

is an isomorphism and defines an isomorphism of functors $\Phi \circ \Psi \longrightarrow \mathrm{Id}_{\mathrm{Mod}_\Lambda^{\mathrm{rh}}(\mathcal{D}_X)}$.

Finally, our main result is an immediate consequence of the previous lemmas:

Theorem 21 *The functors* Φ *and* Ψ *induce equivalence of categories*

$$\mathrm{Mod}_\Lambda^{\mathrm{rh}}(\mathcal{D}_X) \xrightarrow{\sim} \mathrm{Mod}^{\mathrm{gr}}(\mathcal{B}). \tag{30}$$

5.1 Diagram associated to a $\mathcal{D}$-module

Now, using the previous result, we are going to obtain a result of combinatorial classification. Let us mention that the objects in the category $\mathrm{Mod}^{\mathrm{gr}}(\mathcal{B})$ can be understood in terms of finite diagrams of linear maps. This section consists in the classification of such diagrams. To put it more precisely, a graded $\mathcal{B}$-module T in the category $\mathrm{Mod}^{\mathrm{gr}}(\mathcal{B})$ defines an infinite diagram consisting of finite dimensional vector spaces T_λ (with $(\theta - \lambda)$ being a nilpotent operator on each T_λ, $\lambda \in \mathbb{C}$) and linear maps between them deduced from the multiplication by δ, Δ:

$$\cdots \rightleftarrows T_\lambda \underset{\Delta}{\overset{\delta}{\rightleftarrows}} T_{\lambda+3} \rightleftarrows \cdots \tag{31}$$

satisfying the relations $(r_i)_{i=1,\cdots,6}$ of Proposition 8 and the following one

$$\delta\Delta = \frac{\theta}{3}\left(\frac{\theta}{3}+1\right)\left(\frac{\theta}{3}+2\right), \; \Delta\delta = (\frac{\theta}{3}+1)\left(\frac{\theta}{3}+2\right)\left(\frac{\theta}{3}+3\right).$$

Such a diagram is completely determined by a finite subset of objects and arrows. Indeed

a) For $\sigma \in \mathbb{C}/3\mathbb{Z}$, denote by $T^\sigma \subset T$ the submodule $T^\sigma = \bigoplus_{\lambda=\sigma \bmod 3\mathbb{Z}} T_\lambda$. Then T is generated by the finite direct sum of T^σ's

$$T = \bigoplus_{\sigma\,\in\,\mathbb{C}/3\mathbb{Z}} T^\sigma = \bigoplus_{\sigma\,\in\,\mathbb{C}/3\mathbb{Z}} \left(\bigoplus_{\lambda=\sigma \bmod 3\mathbb{Z}} T_\lambda \right) \tag{32}$$

b) If $\sigma \neq 0 \bmod 3\mathbb{Z}$ $(\lambda = \sigma \bmod 3\mathbb{Z})$, then the linear maps δ and Δ are bijective. Therefore T^σ is completely determined by one element T_λ.

c) If $\sigma = 0 \bmod 3\mathbb{Z}$ $(\lambda = \sigma \bmod 3\mathbb{Z})$, then T^σ is completely determined by a diagram of four elements

$$T_{-9} \underset{\Delta}{\overset{\delta}{\rightleftarrows}} T_{-6} \underset{\Delta}{\overset{\delta}{\rightleftarrows}} T_{-3} \underset{\Delta}{\overset{\delta}{\rightleftarrows}} T_0. \tag{33}$$

In the other degrees δ or Δ are bijective.

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