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A ZETA FUNCTION ASSOCIATED WITH THE BERGMAN KERNEL

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On the unit ball $\Omega_0 = \{z \in \mathbb{C}^n : |z|^2 < 1\}$, let us consider the Hilbert space $H_s(\Omega_0)$ of L^2 holomorphic functions with respect to the measure $(1-|z|^2)^{-1-s}/\Gamma(-s)|dz|^2$. If $s < 0$, then $H_s(\Omega_0)$ is non-trivial and admits a reproducing kernel

$$K_s(z) = \pi^{-n}\Gamma(n-s)\,(1-|z|^2)^{s-n},$$

which we call the weighted Bergman kernel. From this formula it is clear that K_s can be analytically continued to $s \in \mathbb{C}$. Note that K_s has single poles at $s = n, n+1, n+2, \dots$ but then $(1-|z|^2)^{s-n}$ is real analytic on $\mathbb{C}^n$. Thus, as a microfunction, K_s is holomorphic in $s \in \mathbb{C}$.

In this talk, I show that this argument can be generalized to strictly pseudoconvex domains in $\mathbb{C}^n$. Then K_s has more poles and some of the residues give CR invariants of the boundary. We call K_s a zeta function associated with the Bergman kernel (I'll explain the reason in the talk). The main tool of the proof is Kashiwara's microlocal analysis of the Bergman kernel [2]. The computation of the residues are done by using the simple holonomic system for the weighted Bergman kernels. More details can be found in [1].

References

[1] K. Hirachi, A link between the asymptotic expansions of the Bergman kernel and the Szegö kernel, to appear in "Complex Analysis in Several Variables," Advanced Studies in Pure Mathematics, Math. Soc. Japan, Tokyo. Available from `http://www.ms.u-tokyo.ac.jp/ hirachi/papers.html`

[2] M. Kashiwara, *Analyse micro-locale du noyau de Bergman* Sém. Goulaouic-Schwartz, École Polytech., Exposé n° VIII, 1976–77.

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