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群 VON NEUMANN 環の分類についての覚書

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1. CONVENTIONS

We focus on the classification problem for group von Neumann algebras. Let us first fix conventions. Group which are written as Γ, Λ, etc. are assumed to be discrete and countable. The left and right regular representations of Γ on $\ell_2\Gamma$ are denoted by λ and ρ respectively; $\lambda(s)$ and $\rho(s)$ are unitary operators on $\ell_2\Gamma$ such that

$$\lambda(s)\delta_t = \delta_{st} \quad \text{and} \quad \rho(s)\delta_t = \delta_{ts^{-1}}$$

for $s, t \in \Gamma$. The reduced group C*-algebra $C^*_\lambda\Gamma$ is the C*-algebra generated by λ, likewise for $C^*_\rho\Gamma$;

$$C^*_\lambda\Gamma = \overline{\lambda(\mathbb{C}\Gamma)}^{\|\ \|} \subset \mathbb{B}(\ell_2\Gamma) \quad \text{and} \quad C^*_\rho\Gamma = \overline{\rho(\mathbb{C}\Gamma)}^{\|\ \|} \subset \mathbb{B}(\ell_2\Gamma).$$

The group von Neumann algebra $\mathcal{L}\Gamma$ is the von Neumann algebra generated by λ;

$$\mathcal{L}\Gamma = \lambda(\mathbb{C}\Gamma)'' = \rho(\mathbb{C}\Gamma)'.$$

We sometime use $\mathcal{M}$ instead of $\mathcal{L}\Gamma$, and $L^2\mathcal{M}$ instead of $\ell_2\Gamma$. We denote by τ the canonical tracial state on $\mathcal{L}\Gamma$ defined by

$$\tau(a) = \langle a\delta_e, \delta_e\rangle$$

Lemma 1. *Let $\varphi\colon A \to B$ be a ucp map. Then, for any $a, b \in A$, we have*

$$\|\varphi(a^*b) - \varphi(a)^*\varphi(b)\| \le \|\varphi(a^*a) - \varphi(a)^*\varphi(a)\|^{1/2}\|\varphi(b^*b) - \varphi(b)^*\varphi(b)\|^{1/2}.$$

Proof. Let $B \subset \mathbb{B}(\mathcal{H})$. By the Stinespring theorem, we have $\varphi(a) = V^*\pi(a)V$, where $\pi\colon A \to \mathbb{B}(\widehat{\mathcal{H}})$ is a $*$-homomorphism and $V\colon \mathcal{H} \to \widehat{\mathcal{H}}$ is an isometry. For any $a, b \in A$, we have that $\varphi(a^*b) - \varphi(a)^*\varphi(b) = V^*\pi(a^*)(1 - VV^*)\pi(b)V$ and that

$$\|V^*\pi(a^*)(1-VV^*)^{1/2}\| = \|V^*\pi(a^*)(1-VV^*)\pi(a)V\|^{1/2} = \|\varphi(a^*a) - \varphi(a)^*\varphi(a)\|^{1/2}.$$

The same thing for b and we are done. □

We have the following consequence.

Lemma 2 (Choi). *Let $A_0 \subset A$ and B be C^*-algebras and $\varphi\colon A \to B$ be a ucp map. Suppose that $\varphi_{|A_0}$ is a $*$-homomorphism. Then, φ is A_0-linear, i.e.,*

$$\varphi(axb) = \varphi(a)\varphi(x)\varphi(b)$$

for all $a, b \in A_0$ and $x \in A$.

We set $[a,b] = ab - ba$ and $[A,B] = \{\sum_k [a_k, b_k] : a_k \in A,\ b_k \in B\}$.

2. Main Result

Let $\bar{\Gamma}$ be a compactification of Γ, i.e., $\bar{\Gamma}$ is a compact topological space which contains Γ as a dense open subset. There is a one-to-one correspondence between $\bar{\Gamma}$ and the C*-algebra $C(\bar{\Gamma})$ such that $c_0\Gamma \subset C(\bar{\Gamma}) \subset \ell_\infty\Gamma$. We assume that the left translation action of Γ on Γ extends to a continuous action of Γ on $\bar{\Gamma}$. This is equivalent to that $C(\bar{\Gamma})$ is left translation invariant; $\alpha_s(C(\bar{\Gamma})) = C(\bar{\Gamma})$ for every $s \in \Gamma$, where $\alpha_s(f)(t) = f(s^{-1}t)$ for $s, t \in \Gamma$ and $f \in \ell_\infty\Gamma$. We say the compactification $\bar{\Gamma}$ is *amenable* if $\Gamma \ltimes_r C(\bar{\Gamma}) \cong C^*(\lambda(\Gamma) \cup C(\bar{\Gamma})) \subset \mathbb{B}(\ell_2\Gamma)$ is nuclear.

Let $\mathcal{G}$ be a non-empty family of subgroups of Γ. We denote by $c_0(\Gamma;\mathcal{G})$ the closed ideal in $\ell_\infty\Gamma$ generated by $\{\chi_{s\Lambda t} : s, t \in \Gamma,\ \Lambda \in \mathcal{G}\}$. We note that $c_0(\Gamma;\mathcal{G})$ is left and right translation invariant. For $f \in \ell_\infty\Gamma$ and $t \in \Gamma$, we define $f^t \in \ell_\infty\Gamma$ by $f^t(s) = f(st^{-1})$. That is $f^t = \rho(t)^* f \rho(t)$ in $\mathbb{B}(\ell_2\Gamma)$. Then, the C*-algebra

$$C(\Delta^{\mathcal{G}}\Gamma) := \{f \in \ell_\infty\Gamma : f^t - f \in c_0(\Gamma;\mathcal{G})\} \subset \ell_\infty\Gamma$$

is a left translation invariant and hence $\Delta^{\mathcal{G}}\Gamma$ is naturally identified with the associated compactification of Γ.

Example 3. If $\mathcal{G}$ consists of the trivial group 1, then $c_0(\Gamma;\mathcal{G}) = c_o\Gamma$.
If $\mathcal{G}$ contains Γ itself, then $c_0(\Gamma;\mathcal{G}) = \ell_\infty\Gamma$ and $\Delta^{\mathcal{G}}\Gamma = \beta\Gamma$.

Definition 4. Let $\mathcal{G}$ be a non-empty family of subgroups of Γ. We say $\mathcal{G}$ is admissible if $\Delta^{\mathcal{G}}\Gamma$ is an amenable Γ-space.

We note that Γ is exact iff the Stone-Čech compactification $\beta\Gamma$ is amenable i.e., $\mathcal{G} = \{\Gamma\}$ is admissible.

Theorem 5. *Let $\mathcal{G}$ be an admissible family of subgroups of Γ. Suppose that $\mathcal{Q} \subset \mathcal{L}\Gamma$ is an injective von Neumann subalgebra such that the relative commutant $\mathcal{Q}' \cap \mathcal{L}\Gamma$ is non-injective. Then, there exists $\Lambda \in \mathcal{G}$ such that "a piece of $\mathcal{Q}$ is conjugated into $\mathcal{L}\Lambda$".*

The meaning of a "piece" will be explained later. Under additional assumptions, we can patch the pieces and find a unitary operator $u \in \mathcal{L}\Gamma$ such that $u\mathcal{Q}u^* \subset \mathcal{L}\Lambda$.

Theorem 6. *We have the following.*

(1) *If Γ is a subgroup of a hyperbolic group, then $\mathcal{G} = \{1\}$ is admissible.*

(2) *Let* $\Gamma = \Gamma_1 \times \Gamma_2$ *and* $\mathcal{G}_i$ *be admissible for* Γ_i. *Then,* $\mathcal{G} = \{\Gamma_1\} \times \mathcal{G}_2 \bigcup \mathcal{G}_1 \times \{\Gamma_2\}$ *is admissible for* Γ.

(3) *Let* $\Gamma = \Gamma_1 *_\Lambda \Gamma_2$ *be an amalgamated free product. If* $\mathcal{G}_i$ *is admissible for* Γ_i, *then* $\mathcal{G} = \{\Lambda\} \cup \mathcal{G}_1 \cup \mathcal{G}_2$ *is admissible for* Γ.

Corollary 7. *If* Γ *is a hyperbolic group and* $\mathcal{Q} \subset \mathcal{L}\Gamma$ *is a diffuse subalgebra, then the relative commutant* $\mathcal{Q}' \cap \mathcal{L}\Gamma$ *is injective.*

Corollary 8 (O+Popa)**.** *Let* $\Gamma_1, \ldots, \Gamma_n$ *be ICC hyperbolic groups and let* $\mathcal{N}_1, \ldots, \mathcal{N}_m$ *be type* II_1 *non-injective factors. Suppose that* $\bar{\bigotimes}\mathcal{N}_j \subset \bar{\bigotimes}\mathcal{L}\Gamma_i$. *Then, we have* $m \leq n$. *If in addition* $m = n$, *then "*$\mathcal{N}_i \subset \mathcal{L}\Gamma_i$*" modulo permutation of indices, rescaling, and unitary conjugacy.*

Corollary 9. *Let* $\Gamma = \Gamma_1 * \Gamma_2$ *be the free product of ICC exact groups* Γ_1 *and* Γ_2. *Suppose that* $\mathcal{Q} \subset \mathcal{L}\Gamma$ *is a subfactor such that the relative commutant* $\mathcal{Q}' \cap \mathcal{L}\Gamma$ *is a non-injective factor. Then, there exist* $i \in \{1,2\}$ *and a unitary operator* $u \in \mathcal{L}\Gamma$ *such that* $u\mathcal{Q}u^* \subset \mathcal{L}\Gamma_i$ *in* $\mathcal{L}\Gamma$.

Corollary 10. *Let* $\Gamma_1, \ldots, \Gamma_n$ *and* $\Lambda_1, \ldots, \Lambda_m$ *be ICC product exact non-amenable groups. Suppose that*

$$\mathcal{L}(\mathbb{F}_\infty * \Gamma_1 * \cdots * \Gamma_n) \cong \mathcal{L}(\mathbb{F}_\infty * \Lambda_1 * \cdots * \Lambda_m).$$

Then $n = m$ *and, modulo permutation of indices,* $\mathcal{L}\Gamma_i$ *and* $\mathcal{L}\Lambda_i$ *are unitarily conjugated in* $\mathcal{L}\Gamma$ *for every* $i \geq 1$.

3. Injectivity

Theorem 11 (Connes)**.** *For a von Neumann algebra* $\mathcal{M}$, *T.F.A.E.*

(1) $\mathcal{M}$ *is hyperfinite*

(2) $\mathcal{M}$ *is injective*

(3) *the* $*$*-homomorphism*

$$\mathcal{M} \otimes \mathcal{M}' \ni \sum_k a_k \otimes x_k \mapsto \sum_k a_k x_k \in \mathbb{B}(L^2\mathcal{M})$$

is continuous w.r.t. the minimal tensor norm.

In particular, $\mathcal{L}\Gamma$ *is hyperfinite iff* Γ *is amenable.*

For a von Neumann subalgebra $\mathcal{P}$ of $\mathcal{M}$, we let

$$\Phi_\mathcal{P} \colon \mathcal{M} \otimes \mathcal{M}' \ni \sum_k a_k \otimes x_k \mapsto \sum_k E_\mathcal{P}(a_k) x_k \in \mathbb{B}(L^2\mathcal{M}).$$

We note that $\Phi_\mathcal{P}$ is a ucp map on the $*$-algebra $\mathcal{M} \otimes \mathcal{M}'$ and is continuous w.r.t. the maximal tensor norm. We observe that the von Neumann subalgebra $\mathcal{P}$ is injective if the ucp map $\Phi_\mathcal{P}$ is continuous on $\mathcal{M} \otimes_{\min} \mathcal{M}'$. Indeed, by restricting $\Phi_\mathcal{P}$ to $\mathcal{P} \otimes J_\mathcal{M} \mathcal{P} J_\mathcal{M}$ and then compressing the range to $\mathbb{B}(L^2\mathcal{P})$, the ucp map $\Phi_\mathcal{P}$ gives rise to the map appearing in the above theorem. Since it is difficult to deal with general tensor products of von Neumann algebras, we reduce the problem to that of C*-algebras. We recall that a C*-algebra A is said to be *exact* if

$$(A \otimes_{\min} B)/(A \otimes_{\min} J) = A \otimes_{\min} (B/J)$$

for any C*-algebra B and its closed 2-sided ideal J. A group Γ is said to be *exact* if the reduced group C*-algebra $C^*_\lambda\Gamma$ is.

Proposition 12. *Let Γ be an exact group and $\mathcal{P} \subset \mathcal{L}\Gamma$ be a von Neumann subalgebra. Then, $\mathcal{P}$ is injective if the ucp map*

$$\Phi_{\mathcal{P}}\colon C^*_\lambda\Gamma \otimes C^*_\rho\Gamma \ni \sum_k a_k \otimes x_k \mapsto \sum_k E_{\mathcal{P}}(a_k)x_k \in \mathbb{B}(\ell_2\Gamma)$$

is continuous w.r.t. the minimal tensor norm.

Proof. Suppose that $\Phi_{\mathcal{P}}$ is continuous on $C^*_\lambda\Gamma \otimes_{\min} C^*_\rho\Gamma$. In the first step, we will show that $\Phi_{\mathcal{P}}$ is continuous on $\mathcal{L}\Gamma \otimes_{\min} C^*_\rho\Gamma$. Let I be a directed set and let

$$B = \{a \in \prod_{i\in I} C^*_\lambda\Gamma : \text{strong}^*\text{-}\lim_{i\in I} a(i) \text{ exists in } \mathcal{L}\Gamma\} \subset \prod_{i\in I} C^*_\lambda\Gamma.$$

It is not hard to see that B is a C*-subalgebra of $\prod_{i\in I} C^*_\lambda\Gamma$ and that

$$\pi\colon B \ni a \mapsto \text{strong}^*\text{-}\lim_{i\in I} a(i) \in \mathcal{L}\Gamma$$

is a $*$-homomorphism. By Kaplansky's density theorem, we may assume that the directed set I is large enough so that π is surjective. Let $J = \ker\pi$ and observe that $(B \otimes_{\min} C^*_\rho\Gamma)/(J \otimes_{\min} C^*_\rho\Gamma) = \mathcal{L}\Gamma \otimes_{\min} C^*_\rho\Gamma$ because of the exactness of $C^*_\rho\Gamma$. We consider the ucp map

$$\tilde{\Phi}_{\mathcal{P}}\colon B \otimes C^*_\rho\Gamma \ni \sum_k a_k \otimes x_k \mapsto \sum_k E_{\mathcal{P}}(\pi(a_k))x_k \in \mathbb{B}(\ell_2\Gamma)$$

The ucp map $\tilde{\Phi}_{\mathcal{P}}$ is continuous on $B \otimes_{\min} C^*_\rho\Gamma$. Indeed, we have

$$\begin{aligned}\|\tilde{\Phi}_{\mathcal{P}}(\sum_k a_k \otimes x_k)\| &= \|\text{strong}^*\text{-}\lim_{i\in I} \Phi_{\mathcal{P}}(\sum_k a_k(i) \otimes x)\| \\ &\le \sup_{i\in I} \|\sum_k a_k(i) \otimes x\|_{C^*_\lambda\Gamma\otimes_{\min}C^*_\rho\Gamma} = \|\sum_k a_k \otimes x\|_{B\otimes_{\min}C^*_\rho\Gamma}.\end{aligned}$$

Since $J \otimes C^*_\rho\Gamma \subset \ker\tilde{\Phi}_{\mathcal{P}}$, the ucp map $\tilde{\Phi}_{\mathcal{P}}$ gives rise to a continuous ucp map on $\mathcal{L}\Gamma \otimes_{\min} C^*_\rho\Gamma$, that actually coincides with $\Phi_{\mathcal{P}}$.

Now, $\Phi_{\mathcal{P}}\colon \mathcal{L}\Gamma \otimes_{\min} C^*_\rho\Gamma \to \mathbb{B}(\ell_2\Gamma)$ is a continuous ucp map. Since $\mathbb{B}(\ell_2\Gamma)$ is injective, the ucp map $\Phi_{\mathcal{P}}$ extends to a ucp map $\bar{\Phi}_{\mathcal{P}}\colon \mathbb{B}(\ell_2\Gamma) \otimes_{\min} C^*_\rho\Gamma \to \mathbb{B}(\ell_2\Gamma)$. For any $a \in \mathbb{B}(\ell_2\Gamma)$ and $x \in C^*_\rho\Gamma$, we have by Lemma 2 that

$$\bar{\Phi}_{\mathcal{P}}(a \otimes 1)x = \bar{\Phi}_{\mathcal{P}}(a \otimes x) = x\bar{\Phi}_{\mathcal{P}}(a \otimes 1),$$

i.e., $\bar{\Phi}_{\mathcal{P}}(a\otimes 1)\in C^*_\rho\Gamma'=\mathcal{L}\Gamma$. It follows that the ucp map

$$\mathbb{B}(\ell_2\Gamma)\ni a\mapsto E_{\mathcal{P}}(\bar{\Phi}_{\mathcal{P}}(a\otimes 1))\in\mathcal{P}$$

is a conditional expectation from $\mathbb{B}(\ell_2\Gamma)$ onto $\mathcal{P}$. □

It is still a difficult problem to deal with $C^*_\lambda\Gamma\otimes_{\max}C^*_\rho\Gamma$. Thus, we restrict our attention to the case where $\mathcal{P}\subset\mathcal{M}$ is a relative commutant; $\mathcal{P}=\mathcal{Q}'\cap\mathcal{M}$ for some von Neumann subalgebra. We assume that $\mathcal{Q}$ is injective. (This is a very mild assumption because of Popa's theorem.) Since $\mathcal{Q}$ is hyperfinite by Connes's theorem, we can "average" over $\mathcal{U}(\mathcal{Q})$. Hence, there exists a conditional expectation $\Psi_{\mathcal{Q}}$ from $\mathbb{B}(\mathcal{H})$ onto $\mathcal{Q}'$ such that $\Psi_{\mathcal{Q}}(x)\in\overline{\mathrm{conv}}^w\{uxu^*:u\in\mathcal{U}(\mathcal{Q})\}$ for all $x\in\mathbb{B}(\mathcal{H})$. It follows that $(\Psi_{\mathcal{Q}})_{|\mathcal{M}}$ is a trace preserving conditional expectation from $\mathcal{M}$ onto $\mathcal{Q}'\cap\mathcal{M}$. Since a trace preserving conditional expectation is unique, we have $(\Psi_{\mathcal{Q}})_{|\mathcal{M}}=E_{\mathcal{Q}'\cap\mathcal{M}}$. Moreover, since $\mathcal{M}'\subset\mathcal{Q}'$, we have

$$\Psi_{\mathcal{Q}}(\sum_k a_kx_k)=\sum_k E_{\mathcal{Q}'\cap\mathcal{M}}(a_k)x_k=\Phi_{\mathcal{Q}'\cap\mathcal{M}}(\sum_k a_k\otimes x_k).$$

for all $\sum_k a_k\otimes x_k\in\mathcal{M}\otimes\mathcal{M}'$.

Lemma 13. *Let $\mathcal{Q}\subset\mathcal{L}\Gamma$ be an injective von Neumann subalgebra. If there is a nuclear C^*-algebra A such that $C^*_\lambda\Gamma\subset A\subset\mathbb{B}(\ell_2\Gamma)$ and that $[A,C^*_\rho\Gamma]\subset\ker\Psi_{\mathcal{Q}}$, then the relative commutant $\mathcal{Q}'\cap\mathcal{L}\Gamma$ is injective.*

Proof. Since $\Psi_{\mathcal{Q}}$ is $C^*_\rho\Gamma$-linear and $[A,C^*_\rho\Gamma]\subset\ker\Psi_{\mathcal{Q}}$, we have $\Psi_{\mathcal{Q}}(A)\subset C^*_\rho\Gamma'$. Since A is nuclear, the ucp map

$$(\Psi_{\mathcal{Q}})_{|A}\times\mathrm{id}_{C^*_\rho\Gamma}\colon A\otimes C^*_\rho\Gamma\ni\sum_k a_k\otimes x_k\mapsto\sum_k\Psi_{\mathcal{Q}}(a_k)x_k\in\mathbb{B}(\ell_2\Gamma)$$

is continuous on $A\otimes_{\min}C^*_\rho\Gamma$. The restriction of $(\Psi_{\mathcal{Q}})_{|A}\times\mathrm{id}_{C^*_\rho\Gamma}$ to $C^*_\lambda\Gamma\otimes C^*_\rho\Gamma$ is nothing but $\Phi_{\mathcal{Q}'\cap\mathcal{L}\Gamma}$. Therefore, $\Phi_{\mathcal{Q}'\cap\mathcal{L}\Gamma}$ is continuous on $C^*_\lambda\Gamma\otimes_{\min}C^*_\rho\Gamma$ and $\mathcal{Q}'\cap\mathcal{L}\Gamma$ is injective by Proposition 12. (We note that Γ is exact because $C^*_\lambda\Gamma\subset A$.) □

Theorem 14. *Let $\mathcal{G}$ be an admissible family of subgroups in Γ. Let $\mathcal{Q}\subset\mathcal{L}\Gamma$ be an injective von Neumann subalgebra. If*

$$\Psi_{\mathcal{Q}}(\chi_{s\Lambda})=0$$

for every $s\in\Gamma$ and $\Lambda\in\mathcal{G}$, then the relative commutant $\mathcal{Q}'\cap\mathcal{L}\Gamma$ is injective.

Proof. Since $\Psi_{\mathcal{Q}}$ is $C^*_\rho\Gamma$-linear, the assumption implies that $c_0(\Gamma;\mathcal{G})\subset\ker\Psi_{\mathcal{Q}}$. Since $\Delta^{\mathcal{G}}\Gamma$ is amenable, $A=C^*(\lambda(\Gamma)\cup\mathbb{C}(\Delta^{\mathcal{G}}\Gamma))$ is nuclear. Now, $[A,C^*_\rho\Gamma]$ is spanned by

$$[\lambda(s)f,\rho(t)]=\lambda(s)\rho(t)(f^t-f),$$

where $s,t\in\Gamma$ and $f\in C(\Delta^{\mathcal{G}}\Gamma)$. Since $f^t-f\in c_0(\Gamma;\mathcal{G})$, the conclusion follows from the previous lemma. We note that $\|\psi(a^*b)\|\leq\|\psi(a^*a)\|^{1/2}\|\psi(b^*b)\|^{1/2}$ for any ucp map ψ and a,b. □

Let $\mathbb{F}_r = \langle g_1, \ldots, g_r \rangle$ be a free group on r generators. The ideal boundary

$$\partial\mathbb{F}_r \subset \{g_1, g_1^{-1} \ldots, g_r, g_r^{-1}\}^{\mathbb{N}}$$

of $\mathbb{F}_r$ is the set of all infinite reduced words. The space $\bar{\mathbb{F}}_r = \mathbb{F}_r \cup \partial\mathbb{F}_r$ is compact in the relative product topology; an open neighborhood basis of $x = s_1 s_2 \cdots \in \partial\mathbb{F}_r$ is given by

$$U(x, n) = \{y = t_1 t_2 \cdots \in \bar{\mathbb{F}}_r : s_k = t_k \text{ for } k \leq n\}.$$

The free group $\mathbb{F}_r$ acts on $\bar{\mathbb{F}}_r$ by left multiplication (and rectifying possible redundancy). It was shown by Spielberg that $\Gamma \ltimes_r C(\bar{\mathbb{F}}_r)$ is $*$-isomorphic to an extended Cuntz-Krieger algebra and hence is nuclear. It is not hard to see that if $\{x_n\}$ is a sequence in $\mathbb{F}_r$ which converges to $x \in \partial\mathbb{F}_r$, then $x_n t \to x$ for every $t \in \mathbb{F}_r$. This means that $f^t - f \in c_o\Gamma$ for every $f \in C(\bar{\mathbb{F}}_r)$ and $t \in \mathbb{F}_r$. Since the conditional expectation $\Psi_{\mathcal{Q}}$ is singular (i.e., $\mathbb{K}(\ell_2\Gamma) \subset \ker \Psi_{\mathcal{Q}}$) whenever $\mathcal{Q} \subset \mathcal{M}$ is diffuse (i.e., contains no minimal projection), we obtain the following corollary.

Corollary 15. *If $\mathcal{Q} \subset \mathcal{L}\mathbb{F}_r$ is a diffuse subalgebra, then $\mathcal{Q}' \cap \mathcal{L}\mathbb{F}_r$ is injective.*

4. Popa's Machinery

Let $(\mathcal{M}, \tau)$ be a finite von Neumann algebra, e.g. $\mathcal{M} = \mathcal{L}\Gamma$. We denote by $L^2\mathcal{M}$ the GNS Hilbert space and by $\widehat{x}$ the vector in $L^2\mathcal{M}$ corresponding to $x \in \mathcal{M}$. The conjugation $J = J_{\mathcal{M}}$ on $L^2\mathcal{M}$ is given by

$$J\widehat{x} = \widehat{x^*}$$

for $x \in \mathcal{M}$. The von Neumann algebra $\mathcal{M}$ acts on $L^2\mathcal{M}$ from the left and from the right;

$$a\widehat{x} = \widehat{ax} \quad \text{and} \quad \widehat{x}a = Ja^*J\widehat{x} = \widehat{xa}$$

Let $\mathcal{N} \subset \mathcal{M}$ be a von Neumann subalgebra, e.g. $\mathcal{N} = \mathcal{L}\Lambda \subset \mathcal{L}\Gamma = \mathcal{M}$ for some subgroup $\Lambda \leq \Gamma$. (The tracial state on $\mathcal{N}$ is given by the restriction of τ.) Then, (assuming the index is infinite) there exists a Hilbert space $\mathcal{K}$ such that

$$L^2\mathcal{M} = \mathcal{K} \otimes L^2\mathcal{N} \qquad \text{e.g.,} \quad \ell_2\Gamma = \ell_2(\Gamma/\Lambda) \otimes \ell_2\Lambda$$

as a right $\mathcal{N}$-module. Suppose now that $\mathcal{Q} \subset \mathcal{M}$ is an injective von Neumann subalgebra such that

$$\mathbb{K}(\mathcal{K}) \otimes \mathbb{B}(L^2\mathcal{N}) \not\subset \ker \Psi_{\mathcal{Q}}.$$

Then, there is a rank one projection x on $\mathcal{K}$ such that $b = \Psi_{\mathcal{Q}}(x \otimes 1_{\mathcal{N}}) \neq 0$. Since $b \in \overline{\mathrm{conv}}^w\{u(x \otimes 1)u^* : u \in \mathcal{Q}\}$, the element b commutes with the right $\mathcal{N}$ action, or equivalently $b \in (\mathbb{B}(\mathcal{K})\bar{\otimes}\mathcal{N}) \cap \mathcal{Q}'$. Moreover, we have $(\mathrm{Tr} \otimes \tau_{\mathcal{N}})(b) \leq \mathrm{Tr}(x) < \infty$. Thus, there is a non-zero spectral projection p of b with $(\mathrm{Tr} \otimes \tau_{\mathcal{N}})(p) < \infty$. It follows that $\mathcal{H} = pL^2\mathcal{M}$ is a $\mathcal{Q}$-$\mathcal{N}$ sub-bimodule of $L^2\mathcal{M}$ with $\dim_{\mathcal{N}} \mathcal{H}_{\mathcal{N}} < \infty$. (Strictly speaking, $\mathcal{H}$ is a $\mathcal{Q}p$-$\mathcal{N}p'$ bimodule where $p' = J_{\mathcal{M}} p J_{\mathcal{M}}$.)

Theorem 16 (Popa). *Let $(\mathcal{M}, \tau)$ be a finite von Neumann algebra, $\mathcal{N}$ and $\mathcal{Q}$ be von Neumann subalgebras. Suppose that there exists a non-zero $\mathcal{Q}$-$\mathcal{N}$ sub-bimodule $\mathcal{H}$ of $L^2\mathcal{M}$ such that* $\dim_{\mathcal{N}} \mathcal{H}_{\mathcal{N}} < \infty$. *Then, there exist projections $e \in \mathcal{N}$ and $q \in \mathcal{Q}$, a non-zero partial isometry $v \in \mathcal{M}$ and a homomorphism $\theta \colon q\mathcal{Q}q \to e\mathcal{N}e$ such that*

$$vv^* \in (q\mathcal{Q}q)' \cap q\mathcal{M}q, \ v^*v \in \theta(q\mathcal{Q}q)' \cap e\mathcal{M}e \text{ and } xv = v\theta(x) \text{ for } x \in q\mathcal{Q}q.$$

Proof. For simplicity, we assume that $\mathcal{Q}$ is diffuse and $\mathcal{N}$ is a factor. Let $\mathcal{H}$ be a non-zero $\mathcal{Q}$-$\mathcal{N}$ sub-bimodule of $L^2\mathcal{M}$ such that $d = \dim_{\mathcal{N}} \mathcal{H}_{\mathcal{N}} < \infty$. Let $q \in \mathcal{Q}$ be a non-zero projection such that $\tau(q)d \leq 1$ and take a projection $e \in \mathcal{N}$ so that $\tau(e) = \tau(q)d$. Then, the $q\mathcal{Q}q$-$\mathcal{N}$ module $q\mathcal{H}$ is isomorphic to $eL^2\mathcal{N}$ as a right $\mathcal{N}$-module. Let $U \colon q\mathcal{H} \to eL^2\mathcal{N}$ be a unitary operator which intertwines the right $\mathcal{N}$ actions. Then, $\theta = \operatorname{Ad} U$ is a unital $*$-homomorphism from $q\mathcal{Q}q$ into $e\mathcal{N}e$. Moreover, if we denote $\xi = U^* e\hat{1}_{\mathcal{N}} \in q\mathcal{H}$, then $x\xi = \xi\theta(x)$ for every $x \in q\mathcal{Q}q$. Since $\xi \in q\mathcal{H} \subset L^2\mathcal{M}$, we may regard ξ as a square-summable operator affiliated with $\mathcal{M}$ and do the polar decomposition $\xi = v|\xi|$. Then, $|\xi|$ commutes with $\theta(q\mathcal{Q}q)$ and $xv = v\theta(x)$ for every $x \in q\mathcal{Q}q$. □

Proof of Theorem 5. Let $\mathcal{G}$ be an admissible family of subgroups in Γ. If $\mathcal{Q} \subset \mathcal{L}\Gamma$ is a von Neumann subalgebra such that $\mathcal{Q}' \cap \mathcal{L}\Gamma$ is non-injective, then Theorem 14 implies that $\Psi_{\mathcal{Q}}(s\chi_\Lambda) \neq 0$ for some $\Lambda \in \mathcal{G}$ and $s \in \Gamma$. Since $s\chi_\Lambda \in \mathbb{K}(\ell_2(\Gamma/\Lambda)) \otimes \mathbb{B}(\ell_2\Lambda)$, Theorem 16 is applicable for $\mathcal{N} = \mathcal{L}\Lambda$. □

5. Amenable Action

For a group Γ, we write $\operatorname{Prob}(\Gamma)$ for the space of all probability measures on Γ;

$$\operatorname{Prob}(\Gamma) = \{\mu \in \ell^1(\Gamma) : \mu \geq 0 \text{ and } \sum_{t\in\Gamma} \mu(t) = 1\}.$$

The group Γ acts on $\operatorname{Prob}(\Gamma)$ by the left translation; $(s\mu)(t) = \mu(s^{-1}t)$ for $s, t \in \Gamma$ and $\mu \in \operatorname{Prob}(\Gamma)$. We equip $\operatorname{Prob}(\Gamma)$ with the pointwise convergence topology. It is not hard to see that the pointwise convergence topology coincides with the norm topology on $\operatorname{Prob}(\Gamma)$.

Definition 17. A (topological) Γ-space is a topological space X together with a continuous Γ-action on X. A compact Γ-space X is said to be *amenable* if for any finite subset $E \subset \Gamma$ and $\varepsilon > 0$, there exists a continuous map

$$\mu \colon X \to \operatorname{Prob}(\Gamma)$$

such that

$$\max_{s\in E} \sup_{x\in X} \|s\mu_x - \mu_{sx}\| < \varepsilon.$$

We say a group Γ is exact if there exists an amenable compact Γ-space.

Suppose that X and Y are compact Γ-spaces and that there exists a Γ-equivariant continuous map from Y into X. Then, Y is amenable if X is amenable. Let X be a compact Γ-space and fix a point $o \in X$. For $f \in C(X)$, we define $f_o \in \ell^\infty(\Gamma)$ by $f_o(s) = f(so)$. Let

$$C(\bar{\Gamma}) = c_0\Gamma + \{f_o : f \in C(\bar{\Gamma})\} \subset \ell_\infty\Gamma.$$

Since the map $\Gamma \ni s \mapsto so \in X$ extends to a Γ-equivariant continuous map $\bar{\Gamma} \to X$, the amenability of X implies that of $\bar{\Gamma}$. We note that Γ is exact iff the Stone-Čech compactification $\beta\Gamma$ is amenable.

Theorem 18 (Anantharaman-Delaroche). *Let X be a compact Γ-space. Then, the reduced crossed product $\Gamma \ltimes_r C(X)$ is nuclear iff the Γ-space X is amenable.*

Example 19 (Connes). Let Γ be a discrete subgroup in a locally compact group G and $H \leq G$ be a closed amenable subgroup such that $X = G/H$ is compact. Then, Γ acts amenably on the compact homogeneous space X from the left.

This applies to, for instance, $G = \mathrm{SL}(n, \mathbb{R})$ (or any group which admits an Iwasawa decomposition). The subgroup $P \leq G$ of upper triangular matrices is solvable and hence is amenable. Thus, a discrete subgroup $\Gamma \leq G$ (e.g., $\Gamma = \mathrm{SL}(n, \mathbb{Z})$) acts amenably on $X = G/P = \mathrm{SO}(n, \mathbb{R})/\{\pm 1\}$.

We denote by $\mathrm{Prob}(X)$ the set of all regular Borel probability measures on X. For any Γ-space X, we denote the stabilizer subgroup of $x \in X$ by $\Gamma^x = \{s \in \Gamma : sx = x\}$. The following technical result is useful.

Proposition 20. *Let Γ a countable group, X be a compact Γ-space, K be a countable Γ-set. Assume that for any finite subset $E \subset \Gamma$ and $\varepsilon > 0$ there exists a Borel map $\zeta\colon X \to \mathrm{Prob}(K)$ (i.e., the function $X \ni x \mapsto \zeta_x(a) \in \mathbb{R}$ is Borel for every $a \in K$) such that*

$$\max_{s\in E} \sup_{x\in X} \|s\zeta_x - \zeta_{sx}\| < \varepsilon.$$

Let Y be a compact Γ-space which is amenable as a Γ^a-space for every $a \in K$. Then, $X \times Y$ (with the diagonal Γ-action) is an amenable Γ-space.

Proof. We first claim that we may take ζ in the statement to be continuous rather than Borel. Fix a finite symmetric subset $E \subset \Gamma$. For every continuous map $\zeta\colon X \to \mathrm{Prob}(K)$, we define $f_\zeta \in C(X)$ by

$$f_\zeta(x) = \sum_{s\in E} \|s\zeta_x - \zeta_{sx}\| = \sum_{s\in E} \sum_{a\in K} |\zeta_x(s^{-1}a) - \zeta_{sx}(a)|.$$

Since $f_{\sum_k \alpha_k\zeta_k} \leq \sum_k \alpha_k f_{\zeta_k}$ for every $\alpha_k \geq 0$ with $\sum_k \alpha_k = 1$, if 0 is in the weak closure of $\{f_\zeta : \zeta\}$ in $C(X)$, then 0 is in the norm closure of $\{f_\zeta : \zeta\}$ by the Hahn-Banach separation theorem. We note that the dual of $C(X)$ is the space of finite regular Borel measures on X by the Riesz representation theorem. Let $\varepsilon > 0$ and $m \in \mathrm{Prob}(X)$ be given. By assumption, there exists a Borel map $\eta\colon X \to \mathrm{Prob}(K)$

such that $\sup_{x\in X}\|s\eta_x - \eta_{sx}\| < \varepsilon/|E|$. By the countable additivity of the measure, there exists a finite subset $F \subset K$ such that $\int_X \sum_{a\in F}\eta_x(a)\,dm(x) > 1-\varepsilon/|E|$. We approximate, for each $a \in F$, the Borel function $x \mapsto \eta_x(a)$ by a continuous function and obtain a continuous map $\zeta\colon X \to \mathrm{Prob}(K)$ such that $\operatorname{supp}\zeta_x \subset F$ for all $x \in X$ and

$$\int_X \|\zeta_x - \eta_x\|\,dm(x) = \int_X \sum_{a\in K}|\zeta_x(a) - \eta_x(a)|\,dm(x) < 2\varepsilon/|E|.$$

It follows that

$$\int_X f_\zeta(x)\,dm(x) < \int_X \sum_{s\in E}\|s\eta_x - \eta_{sx}\|\,dm(x) + 4\varepsilon < 5\varepsilon.$$

Thus, we proved our claim.

Now, let a finite subset $E \subset \Gamma$ and $\varepsilon > 0$ be given. By the previous result, there exists a continuous ζ such that $\sup_{x\in X}\|s\zeta_x - \zeta_{sx}\| < \varepsilon$ for every $s \in E$. We may assume that there exists a finite subset $F \subset \Gamma$ such that $\operatorname{supp}\zeta_x \subset F$ for all $x \in X$. We fix a Γ-fundamental domain $V \subset K$ with a projection $v\colon K \to V$ and a cross section $\sigma\colon K \to \Gamma$, i.e., K decomposes into the disjoint union $\bigsqcup_{v\in V}\Gamma v$ and $a = \sigma(a)v(a)$ for every $a \in K$. We note that $\sigma(sa)^{-1}s\sigma(a) \in \Gamma^{v(a)}$ for every $s \in \Gamma$ and $a \in K$. For each $v \in V$, we set

$$E^v = \{\sigma(sa)^{-1}s\sigma(a) : a \in F\cap\Gamma v \text{ and } s \in E\} \subset \Gamma^v.$$

Since Y is Γ^v-amenable and E^v is finite, there exists a continuous map $\nu^v\colon Y \to \mathrm{Prob}(\Gamma)$ such that

$$\max_{s\in E^v}\sup_{y\in Y}\|s\nu^v_y - \nu^v_{sy}\| < \varepsilon.$$

Now, we define $\mu\colon X \times Y \to \mathrm{Prob}(\Gamma)$ by

$$\mu_{x,y} = \sum_{a\in K}\zeta_x(a)\,\sigma(a)\nu^{v(a)}_{\sigma(a)^{-1}y}.$$

The map μ is clearly continuous. Moreover, we have

$$\begin{aligned}
s\mu_{x,y} &= \sum_{a\in K}\zeta_x(a)\,s\sigma(a)\nu^{v(a)}_{\sigma(a)^{-1}y}\\
&= \sum_{a\in K}\zeta_x(a)\,\sigma(sa)\big(\sigma(sa)^{-1}s\sigma(a)\nu^{v(a)}_{\sigma(a)^{-1}y}\big)\\
&\approx_\varepsilon \sum_{a\in K}\zeta_x(a)\,\sigma(sa)\nu^{v(sa)}_{\sigma(sa)^{-1}sy}\\
&\approx_\varepsilon \sum_{a\in K}\zeta_{sx}(sa)\,\sigma(sa)\nu^{v(sa)}_{\sigma(sa)^{-1}sy} = \mu_{sx,sy}
\end{aligned}$$

for every $s \in E$ and $(x,y) \in X \times Y$. □

We consider a group action on a countable tree T. We will identify T with its vertex set. For example, the Cayley graph of a finitely generated free group $\mathbb{F}_r$ w.r.t. the standard generating set is a homogeneous tree of degree $2r-1$. Another example is the Bass-Serre tree T for an amalgamated free product $\Gamma = \Gamma_1 *_\Lambda \Gamma_2$. This is defined as $T = \Gamma/\Gamma_1 \sqcup \Gamma/\Gamma_2$, where $s\Gamma_1$ and $t\Gamma_2$ are adjacent iff $s\Gamma_1 \cap t\Gamma_2 \neq \emptyset$. The group Γ acts on T by the left multiplication. The vertex stabilizer $\Gamma^x = \{s \in \Gamma : sx = x\}$ for each vertex $x \in T$ is conjugate to either Γ_1 or Γ_2. The edge stabilizers are conjugate to Λ. We note that T is not locally finite unless both Γ_1 and Γ_2 are finite.

A path is a finite or infinite sequence $x_0x_1\cdots$ in T such that x_k is adjacent to x_{k+1} for each k. It is a geodesic path if there is no backtracking, i.e., all x_k's are distinct. We say a path $x_0\cdots x_n$ connects x_0 to x_n. Two infinite geodesic paths x and y are said to be *equivalent* if they eventually flow together, i.e., if there exist $m \in \mathbb{Z}$ and $N \geq |m|$ such that $x_n = y_{m+n}$ for every $n \geq N$. We define the ideal boundary ∂T of the tree T as the set of all equivalence classes of infinite geodesic paths. If x is a boundary point which is represented by $x_0x_1\cdots$, then we say the infinite geodesic path $x_0x_1\cdots$ connects x_0 to x. Likewise, we say a biinfinite geodesic path $\cdots x_{-1}x_0x_1\cdots$ connects the boundary point $x_0x_{-1}\cdots$ to the boundary point $x_0x_1\cdots$. Let $\bar{T} = T \cup \partial T$. It is not too hard to show that for any two points $x, y \in \bar{T}$, there exists a unique geodesic path $[x,y]$ which connects x to y. The topology of $\bar{T}$ is defined by declaring that

$$U(x,F) = \{y \in \bar{T} : [x,y] \cap F \subset \{x\}\}$$

are open for every $x \in \bar{T}$ and every finite subset $F \subset T$. It is not hard to see that the family $\{U(x,F)\}_F$ is an open neighborhood of $x \in \bar{T}$. We omit the proof of the following theorem.

Theorem 21. *The topological space $\bar{T}$ is compact and contains T as a dense subset. Every graph automorphism on T extends uniquely to a homeomorphism on $\bar{T}$.*

We are including the Hausdorff property in the definition of compactness. The space $\bar{T}$ is metrizable when T is countable.

Theorem 22. *Let Γ be a group and T be a countable tree on which Γ acts. Let Y be a compact Γ-space such that Y is amenable as a Γ^x-space for every vertex stabilizer Γ^x. Then, $\bar{T} \times Y$ is an amenable Γ-space.*

We note that if $\Lambda \leq \Gamma$ is an exact subgroup, then the Stone-Čech compactification $\beta\Gamma$ is an amenable Λ-space. Therefore, above theorem implies the result of Dykema and Tu that a group Γ acting on a (countable) tree is exact provided that all the vertex stabilizers are exact. By Proposition 20, Theorem follows from the following lemma. (Fix an origin $o \in T$ and set $\zeta\colon \bar{T} \to \mathrm{Prob}(T)$ by $\zeta_x = \xi(o,x)$.)

Lemma 23. *Let T be a countable tree. Then, there exists a sequence of Borel maps $\xi_n\colon T \times \bar{T} \to \mathrm{Prob}(T)$ such that*

$$\xi_n(sx, sz) = s \cdot \zeta_n(x,z)$$

for all graph automorphism s on T and

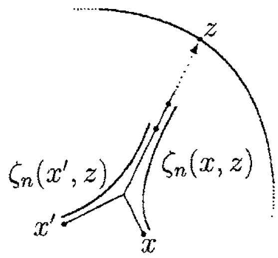
$$\lim_{n\to\infty} \sup_{z\in\bar{T}} \|\xi_n(x,z) - \xi_n(x',z)\| = 0$$

for every $x, x' \in T$.

Proof. Let $n \in \mathbb{N}$ be given. Let $x \in T$ and $z \in \bar{T}$. Then, there exists a unique geodesic path $a_0a_1a_2\cdots$ connecting x to z. We define $\zeta_n(x,z) \in \mathrm{Prob}(T)$ as

$$\zeta_n(x,z) = \frac{1}{n}\sum_{k=0}^{n-1} \delta_{a_0\cdots a_k},$$

the normalized characteristic function of the first n segment of the geodesic path $[x,z]$. (When $k > d = d(x,z)$, we use $a_0\cdots a_d$ instead of $a_0\cdots a_k$.)



It is not hard to see that ζ_n is Borel and

$$\sup_{z\in\bar{T}} \|\xi_n(x,z) - \xi_n(x',z)\| \le \frac{d(x,x')}{n}$$

for every $x, x' \in T$. See the figure. □

Lemma 24. *Let Γ be a group and T be a countable tree on which Γ acts. We denote by $\mathcal{G}$ the family of all edge stabilizers $\Gamma^{(x,y)}$ of Γ. Let $o \in T$ be fixed. Then, we have*

$$\{f_o^t - f_o : f \in C(\bar{T}),\ t \in \Gamma\} \subset c_0(\Gamma;\mathcal{G}).$$

Proof. We recall that $f_o \in \ell_\infty\Gamma$ is defined by $f_o(s) = f(so)$. Suppose by contradiction that there exist $f \in C(\bar{T})$, $t \in \Gamma$ and $\varepsilon > 0$ such that $S = \{s \in \Gamma : |f_o^t(s) - f_o(s)| > \varepsilon\}$ is not contained in a finite union of $s\Lambda t$'s. Then, there exists a sequence $\{s_n\}$ in S such that every $s\Lambda t$ contains only finitely many s_n's. Since $\bar{T}$ is compact, we may assume that $s_n o \to x \in \bar{T}$ and $s_n t^{-1} o \to y \in \bar{T}$. Since $f(x) \ne f(y)$, we have $x \ne y$. Since $d(s_n o, s_n t^{-1} o) = d(o, t^{-1}o)$ for all n, if x is a boundary point, then so is y and $x = y$ in contradiction. Therefore, we must have $x, y \in T$. Since $s_n o \to x$, we have $[x, s_n o] \cap [x, y] = \{x\}$ eventually. Likewise, $[y, s_n t^{-1} o] \cap [y, x] = \{y\}$ eventually. These imply that $s_n[o, t^{-1}o] = [s_n o, s_n t^{-1} o]$ intersects with $[x, y]$ for every sufficiently large n, in contradiction to that $s_n \to \infty$ w.r.t. $s\Lambda t$'s. □

Corollary 25. *Let $\Gamma = \Gamma_1 *_\Lambda \Gamma_2$ be an amalgamated free product. For each i, let $\mathcal{G}_i$ be an admissible family of subgroups of Γ_i. Then, $\mathcal{G} = \{\Lambda\} \cup \mathcal{G}_1 \cup \mathcal{G}_2$ is admissible for Γ.*

Proof. Let $\Gamma = \Gamma_1\mathfrak{X}_1$ be a coset decomposition. We have the corresponding Γ_1-equivariant map from Γ (resp. $\beta\Gamma$) onto Γ_1 (resp. $\beta\Gamma_1$). Equivalently, we have the Γ_1-equivariant embedding

$$\pi_1\colon \ell_\infty\Gamma_1 \ni f \mapsto \pi_1(f) \in \ell_\infty\Gamma, \quad \pi_1(f)(sx) = f(s) \text{ for } sx \in \Gamma_1\mathfrak{X}_1.$$

We note that if $f \in \pi_1(C(\Delta^{\mathcal{G}_1}\Gamma_1))$, then we have $\mathrm{supp}(f^t - f) \subset c_0(\Gamma_1;\mathcal{G}_1) \subset \ell_\infty\Gamma$ for $t \in \Gamma_1$, and $f^t - f = 0$ for $t \in \Gamma_2 \setminus \Lambda$. Thus, we have

$$f^{t_1\cdots t_n} - f = (f^{t_1} - f)^{t_2\cdots t_n} + (f^{t_2} - f)^{t_3\cdots t_n} + \cdots + (f^{t_n} - f) \in c_0(\Gamma;\mathcal{G})$$

for all $f \in \pi_1(C(\Delta^{\mathcal{G}_1}\Gamma_1))$ and $t_1\cdots t_n \in \Gamma$. It follows that $\pi_1(C(\Delta^{\mathcal{G}_1}\Gamma_1)) \subset C(\Delta^{\mathcal{G}}\Gamma)$. This means that there exists a Γ_1-equivariant map from $\Delta^{\mathcal{G}}\Gamma$ into $\Delta^{\mathcal{G}_1}\Gamma_1$ and hence $\Delta^{\mathcal{G}}\Gamma$ is an amenable Γ_1-space. The same thing for Γ_2. Hence, if we denote by T the Bass-Serre tree associated with the amalgamated free product, then $\bar{T} \times \Delta^{\mathcal{G}}\Gamma$ is an amenable Γ-space by Theorem 22. But by Lemma 24, there exist Γ-equivariant maps from $\Delta^{\mathcal{G}}\Gamma$ into $\bar{T}$ and hence from $\Delta^{\mathcal{G}}\Gamma$ into $\bar{T} \times \Delta^{\mathcal{G}}\Gamma$. It follows that $\Delta^{\mathcal{G}}\Gamma$ is an amenable Γ-space. □

For fine hyperbolic graphs, see [4].

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