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# Issues on the Optimal Financial Policy and Incentive in a Firm

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## 1 Introduction

Modigliani and Miller (1958) [6] established the financial policy of a firm that if there are no tax and transaction cost, the value of a firm is independent to the firm's liabilities. After this seminal investigation, many tried to relax its conditions. They are, for example, the debt affection to a firm's taxable capital, the bankruptcy cost, and the agency costs. In these ways, a firm's manager determines optimal liabilities taking into account its benefit and cost.

In addition to their studies, we in this paper introduce the incentive effect by firm's financial policy from both shareholders and managers. Let us imagine a venture company. For its small shareholders' equity, some additional liabilities bring it the great leverage effect. In addition, that leverage effect also brings the great incentive for its manager to make effort. On the contrary, that might be weak for a giant company because relatively, the leverage effect is weak for the large shareholders' equity. Let us also consider the case to increase their capital: to issue corporate bond or share. Generally speaking, that self-financing of a venture seems the way to provide its employees from its growth. However is the opposite true? That is; the possibility of its growth brings incentive of employees. Morellec and Smith Jr. (2004) [7] have introduced that incentive effect only from the view of shareholders. Cadenillas et al. (2004) [2] have focused on the relationship not only the shareholders but also the manager. However their work of the managers' utility does not consider the size of firm; as my previous example, it is a venture firm or a giant company. As we have described above, financial policy in ventures and giant companies might be quite different. We examine to depict it more clearly at Section 2.2 and 2.3.

We illustrate the problem as follows: a risk averse manager receives some levered shares as his compensation. This is the only source of his compensation. For a certain liabilities and compensation level, he decides a certain level of his effort and project, which is expressed by volatility, to maximize his final utility. His effort requires cost, however his choice of volatility does not. The risk neutral shareholders determine liabilities and compensation. They also aim to maximize their final utilities. In this framework, we verify the optimal liability, compensation, effort, and volatility levels required to maximize final utilities of both risk neutral shareholders and a risk averse manager. As we describe in detail later, we let the shareholders be the principal and the manager the agent.[1] These "principal" and "agent" are standard in principal-agent literatures.[2]

The rest of our paper is as follows: In section 2, we characterize the value of a firm, the effect of a manager's effort on its value and these two players: a manager and a

[1]As we describe later, we assume that all the shareholders aim to maximize their value of share. Then without loss of generality, we focus only one shareholder on her dynamics.

[2]Thus in the following contents, we often represents "she" as a shareholder and "he" as a manager according to the custom of the contract theory.

shareholder. After that, we describe how they act. In section 3, we derive the optimal choices of effort and volatility by a manager, as well as the optimal choices of compensation and liabilities by a shareholder. In addition, we find the characters of their optimal values by mainly numerical comparative statics. In section 4, we add some fixed compensation to a manager. We close this paper with some conclusions. If you have interest in the proofs and the graph of numerical comparative statics, please see Horikawa (2005) [3].

# 2 Model

First of all, we demonstrate the structure of our model. The solution is in the next section. We consider the problem of the risk neutral shareholders and a risk averse manager. Keeping our analysis simple, we ignore bankruptcy costs, credit risk, and tax of a firm. As in Morellec and Smith Jr. (2004) [7], we assume that shareholders have the right to decide the financial and compensation policy of a firm. We also assume that all the shareholders always have one policy. Hence we can consider the action of the representative shareholder only without loss of generality in the following. Then in our paper, we only pay attention to the relationship between "a principal" and "a shareholder".

Before taking up our main subject, we describe the valuation of liabilities. In this paper, we adopt the four assumptions of Merton (1974) [5]: (1) the short rate of bond yields some constant value: $r$, (2) a firm goes bankrupt when its shareholders' equity is less than its liabilities, (3) bankruptcy occurs only at the maturity of liabilities. A firm does not always go bankrupt if shareholders' equity decreases less than liabilities within the length of liabilities, and (4) clearance follows according to priority of the law.

## 2.1 Firm's Value and Share

We assume that the value of a firm consists of two factors: shareholders' equity and liabilities. We denote $S_t$ as the shareholders' equity and $B$ as the liabilities. We also denote $V_t$ as the value of a firm, which consists of shareholders' equity and liabilities. The subscript letter $t \in [0, T]$ indicates time. $t = 0$ is the beginning of liabilities and $t = T$ is the maturity. The shareholders' equity is $S_t \equiv (V_t - Be^{rt})^+$, where $r$ is a short rate of bond in any time $t$. We also assume that the value of liabilities $B$ still yields at $t = 0$ to keep our analysis simple. Then we omit the subscript letter $t$ for $B$ in the following.

The structure of our model is as follows: we consider the relationship between one shareholder and one manager. Both would like to only maximize their expected utility of final wealth respectively. We assume that both a shareholder and a manager can observe the process in $t \in [0, T]$. At $t = 0$, the shareholder raises some capital $S_0$. For given $S_0$, she decides some liabilities $B$ and the compensation contract $p$ to the manager. No one can change both $B$ and $p$ till the maturity $T$. $B$ has a positive leverage effect to the firm's value, whereas it needs cost $e^{rt}$. $p$ $(\in [0, 1])$ indicates the ratio in a shareholders' equity: $(V_t - Be^{rt})^+$. That is to say, a shareholder grants a manager a part of her share as his compensation, then his compensation only depends on the shareholders' equity of her firm. However she has to make $p$ more than his reservation utility $R$, which is when he chooses his optimal $u = (u_t)_{t \geq 0}$ and $v = (v_t)_{t \geq 0}$. We consider the $R, u$, and $v$ in the following. At $t \in (0, T)$, the manager can change the dynamics of $V_t$ continuously by choosing his effort level $u_t$ and volatility $v_t$. His effort entails cost, however his choice of $u_t$ and $v_t$ does not. The choice depends on all information he obtained at $t$. Finally at $t = T$, a shareholder has to pay back the liabilities with interest rate $r$ and pay $p(V_T - Be^{rT})^+$ to a manager as the compensation according to the contract concluded at $t = 0$. If $V_T \leq Be^{rT}$, she and he have nothing. We study later for that condition at Remark 1: Bankruptcy condition.

Under the process of our model, we give the assumptions relating to the dynamics of the value of a firm. Let $\mu$ and $\sigma$ be some constant parameter and $(W_t)_{t\geq 0}$ be a standard Brownian motion. When a shareholder unlevers and the manager does not make effort, the dynamics $(V_t)_{t\geq 0}$ follows a geometric Brownian motion like Black and Scholes (1973) [1]:

$$dV_t = \mu V_t dt + \sigma V_t dW_t, \quad t \in [0, T] \tag{1}$$

which starts $V_0$. When both do the opposite mutually, the dynamics $(V_t)_{t\geq 0}$ follows

$$dV_t = \mu V_t dt + \delta u_t dt + \alpha v_t V_t dt + v_t V_t dW_t, \quad t \in [0, T] \tag{2}$$

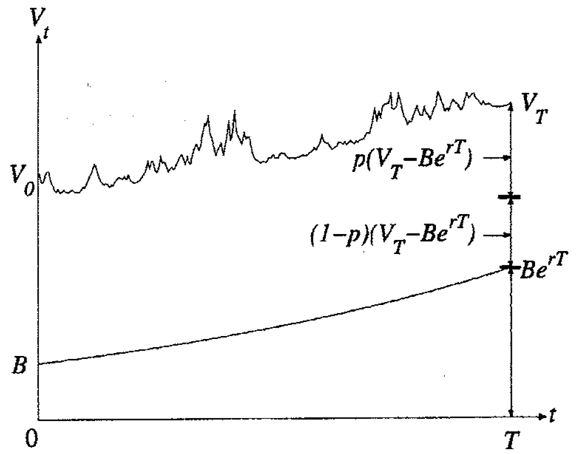
which starts $V_0$. In the following, we consider the dynamics of Equation (2). Figure 1 depicts the dynamics of Equation (2) and liabilities $B$. We assume $u$ and $v$ are adapted stochastic processes and satisfied to $E[\int_0^T |u_t|^2 dt] < \infty$ and $E[\int_0^T |v_t V_t|^2 dt] < \infty$ respectively. $u$ is the level of effort chosen by the manager. No any cost requires for the decision of $u$. Higher $u$ brings the shareholders the high expected value of a firm. $r$ and $u$ are independent because an interest rate is exogenous and his effort does not affect the determination of an interest rate $r$. $v$ is the volatility of a firm associated with the choice of the project of a firm. $\alpha$ is a measure of the benefits associated with taking more risk and satisfied to $\alpha \in (0, \infty)$. $\delta$ is a measure of the impact of the manager's effort on a value of a firm and satisfied to $\delta \in [0, \infty)$. For our argument later, we note the difference between $\alpha$ and $\delta$. Both of them indicate the ability to obtain some revenue from a firm's risk, however different the source of that ability. $\delta$ indicates the ability of a manager. On the other hand, $\alpha$ indicates the environment of a firm: scale, culture, industry segments, and so on. High growth company, industry yields high $\alpha$, while low growth does low $\alpha$. Now let us see the meaning of the right hand side of Equation (2). The first term and fourth term of the right hand side due to the assumption of geometric Brownian motion: Equation (1). The second term indicates the drift due to manager's effort. In it, $\delta$ is an ability of a manager. The third term is the drift term that dues to the environment of a firm in obtaining the revenue from risk, then it is led by the fourth term.

## 2.2 Manager's Problem

The manager is risk averse and requires compensation by his own efforts. We assume that the shares received from a shareholder are the only source of his compensation. He chooses his effort level $u = (u_t)_{t\geq 0}$ and the project of a firm $v = (v_t)_{t\geq 0}$ continuously to maximize his final expected utility. $v$, which is the volatility of a firm, effects to his compensation too since shares are his only compensation and its value dues to his effort $v$. Here we give three assumptions. The first is that the projects are comparable in quantity. The second is projects with higher risk bring a higher expected return. The last is his choice of risk does not influence his effort because his decision is costless. Under these assumptions, we formulate his problem as

$$\max_{u,v} E\left[\ln\{p(V_T - Be^{rT})^+\} - \frac{1}{2}\int_0^T u_t^2 dt\right]. \tag{3}$$

Figure 1: Dynamics of $V_t$ and $B$.




The first term in the expectation is the utility from his compensation as a manager. We assume that his utility function by compensation is an increasing and concave. That is, the higher the compensation is, the lower his increase of utility is. A logarithmic utility is suitable to express our assumption. The second term is the cost of his effort. $u = (u_t)_{t\geq 0}$ yields some non-negative level of his effort. We also assume that it is an increasing and convex function. That is, the higher he makes effort, the higher his dissatisfaction is. A quadratic cost function is convenient for an approximation and our calculation later.

## 2.3 Shareholder's Problem

A shareholder only pays attention to the amount of her shares. She is risk neutral and would like to maximize her shareholders' equity $S_T \equiv (V_T - Be^{rT})^+$ at the maturity of liabilities. At $t = T$, she has to pay a part of her shares to her manager as his compensation according to a contract decided at $t = 0$. That has to satisfy at least as great as his reservation utility $R$, which is the lowest utility of him to accept an offer of a firm. In our setting, $B$ has no range because we assume to ignore credit risk. When a shareholder solves this problem, she knows $u = (u_t)_{t\geq 0}$ and $v = (v_t)_{t\geq 0}$. Then her the objective function is

$$\begin{aligned}
&\max_{B,p} \quad (1-p)E\left[(V_T - Be^{rT})^+\right], \\
&\text{s.t.} \quad \max_{u,v} E\left[\ln\left\{p(V_T - Be^{rT})^+\right\} - \frac{1}{2}\int_0^T u_t^2 dt\right] \geq R, \\
&\qquad (\hat{u}, \hat{v}) \in \operatorname*{argmax}_{(u,v)} E\left[\ln\left\{p(V_T - Be^{rT})^+\right\} - \frac{1}{2}\int_0^T u_t^2 dt\right], \\
&\qquad p \in [0,1],
\end{aligned} \tag{4}$$

where $R$ represents the minimum utility of a manager to accept the offer from another firm. The first line of the condition is the individual rationality constraint (or the participation constraint). The second line is the incentive compatibility constraints. The third line is from the definition of $p$: the ratio in the total share to pay for her manager.

# 3 Optimal Strategies and Their Properties

In the previous section, we set the framework. In this section, we derive an optimal activities of a manager: effort $u_t$ and volatility $v_t$, and a decision of a shareholder: liability $B$ and ratio of share $p$ to give as compensation. In addition, we study their properties.

## 3.1 Optimal Strategies

At first, we derive a manager's optimal effort $\hat{u}$ and volatility $\hat{v}$. Let an exponential martingale by $Z_t := \exp(-(\alpha^2 t)/2 - \alpha W_t)$, where $\alpha$ is the parameter as we described in Equation (2), a measure of the benefits associated with taking some additional risk. Let $\check{z}_t$ as the positive solution of

$$\delta^2 e^{-2\mu T+\mu t}\left(e^{\alpha^2 T} - e^{\alpha^2 t}\right) Z_t^2 z^2 + \alpha^2\left(V_t - Be^{(r-\mu)T+\mu t}\right) Z_t z - \alpha^2 e^{\mu t} = 0 \qquad (5)$$

in $z$ for each $t \in (0, T)$. Using these notations, we can write the optimal effort and volatility of a manager, and the bankruptcy condition:

**Theorem 1** (Optimal effort and volatility).
*Consider the manager's problem Equation* (3). *Define $Z_t$ and $\check{z}_t$ as above.*

(*I*) *When $\delta > 0$, his optimal effort $\hat{u}$ is $\hat{u}_t = \delta \check{z}_t e^{-\mu t} Z_t$, and volatility $\hat{v}$ is*

$$\hat{v}_t V_t = \frac{\alpha e^{\mu t}}{\check{z}_t Z_t} - \frac{\check{z}_t \delta Z_t}{\alpha} e^{-2\mu T+\mu t}\left(e^{\alpha^2 T} - e^{\alpha^2 t}\right).$$

*Given $\hat{u}$ and $\hat{v}$, the value of a firm yields*

$$V_t = \frac{e^{\mu t}}{\check{z}_t Z_t} + Be^{(r-\mu)T+\mu t} - \frac{\check{z}_t \delta^2 Z_t}{\alpha^2} e^{-2\mu T+\mu t}\left(e^{\alpha^2 T} - e^{\alpha^2 t}\right). \qquad (6)$$

(*II*) *When $\delta = 0$, if $V_t > Be^{(r-\mu)T}$, the results are the same except the value of $\check{z}_t$. If $V_t \leq Be^{(r-\mu)T}$, $\hat{u}_t$ and $\hat{v}_t$ do not exist.*

*Proof.* See Horikawa (2005) [3], Appendix A, Proof of Theorem 1. □

**Remark 1** (Bankruptcy condition).
*We find that if $\delta = 0$ and $V_t - Be^{(r-\mu)T} > 0$, then $V_T - Be^{rT} > 0$ at proof of Theorem 1. That is, when the manager's effort is no influence on the value of a firm, bankruptcy never occurs so long as a shareholder decides the liabilities $B$ is $B < e^{-(r-\mu)T} V_0$ at $t = 0$.*

*How about the case of $\delta > 0$? We can obtain by Equation* (6) *that firm's value is*

$$V_t - Be^{rt} = \frac{e^{\mu t}}{\check{z}_t Z_t} + Be^{(r-\mu)T+\mu t} - Be^{rT} - \frac{\check{z}_t \delta^2 Z_t}{\alpha^2} e^{-2\mu T+\mu t}\left(e^{\alpha^2 T} - e^{\alpha^2 t}\right).$$

*It might yields $-\infty$. However at $t = T$, $V_T - Be^{rT} = e^{\mu T}/(\check{z}_T Z_T) > 0$. Note that $\check{z}_T$ is the positive solution of the quadratic equation* (5). *Therefore we make out that as long as manager's effort is valid to the firm's value, bankruptcy never occurs at $t = T$ if a firm is under bankruptcy at $t \in [0, T)$.*

Given the optimal effort $\hat{u}$ and volatility $\hat{v}$ as Theorem 1, we can verify the shareholder's optimal liabilities $\hat{B}$ and compensation contract $\hat{p}$.

**Theorem 2** (Optimal liabilities and compensation).
*Consider the shareholder's problem Equation* (4).

(*I*) *When* $\delta > 0$, (*a*) *if* $R$*: the reservation utility of the manager is*

$$R \leq \ln(1-\check{z}_0) + (\alpha^2+\mu)T - \frac{\delta^2 \check{z}_0^2}{2\alpha^2} e^{-2\mu T}\left(e^{\alpha^2 T} - 1\right), \tag{7}$$

$\hat{B}$*: the optimal liabilities a shareholder decides is*

$$\hat{B} = e^{-(r-\mu)T}\left\{V_0 - \frac{1}{\check{z}_0} + \frac{\delta^2 \check{z}_0^2}{\alpha^2} e^{-2\mu T}\left(e^{\alpha^2 T} - 1\right)\right\},$$

*and* $\hat{p}$*: the optimal compensation contract is*

$$\hat{p} = \check{z}_0 + \exp\left\{R - (\alpha^2+\mu)T + \frac{\delta^2 \check{z}_0^2}{2\alpha^2} e^{-2\mu T}\left(e^{\alpha^2 T} - 1\right)\right\}.$$

(*b*) *if* $R$ *is elsewhere of Equation* (7), *both the optimal* $\hat{B}$ *and* $\hat{p}$ *do not exist either.*

(*II*) *When* $\delta = 0$, *if both* $V_t > Be^{(r-\mu)T}$ *and Equation* (7) *are satisfied, the results are the same to* (*I*) *except* $\check{z}_0$. *If not, both the optimal* $\hat{B}$ *and* $\hat{p}$ *do not exist either.*

*Proof.* See Horikawa (2005) [3], Appendix A, Proof of Theorem 2. □

## 3.2 Numerical Comparative Statics

In this section, we study the properties of $\hat{u}$, $\hat{v}$, $\hat{B}$, and $\hat{p}$ whom we obtained in the previous section using comparative statics mainly numerically. In addition, we verify whether the results are adjusted to the rational action of a risk neutral shareholder and a risk averse manager or not. To keep our analysis simple, we give two assumptions in this section. One is $r = \mu$, and another is $\delta > 0$. Then we can express the values of parameters as

$$\hat{u}_t = \frac{e^{2\mu(T-t)} \cdot \alpha Y_t}{2\delta\left(e^T - e^t\right) \cdot e^{\alpha^2}} = \frac{\alpha \cdot Y_t e^{2r(T-t)}}{2\delta\left(e^{\alpha^2 T} - e^{\alpha^2 t}\right)},$$

$$\hat{v}_t = \frac{1}{V_t}\left[\frac{\alpha\delta}{\hat{u}_t} + \frac{\hat{u}_t}{\alpha} e^{-2r(T-t)}\left(e^{\alpha^2 T} - e^{\alpha^2 t}\right)\right],$$

$$\hat{B} = V_0 - \frac{1}{\check{z}_0} + \frac{\delta^2 \check{z}_0^2}{\alpha^2} e^{-2rT}\left(e^{\alpha^2 T} - 1\right),$$

$$\hat{p} = \check{z}_0 + \exp\left\{R - (\alpha^2 + r)T + \frac{\delta^2 \check{z}_0^2}{2\alpha^2} e^{-2rT}\left(e^{\alpha^2 T} - 1\right)\right\},$$

where

$$Y_t = \alpha\left(Be^{rt} - V_t\right) + \sqrt{\alpha^2\left(Be^{rt} - V_t\right)^2 + 4\delta^2 e^{-2r(T-t)}\left(e^{\alpha^2 T} - e^{\alpha^2 t}\right)},$$

$$\check{z}_0 = \frac{\alpha Y_0}{2\delta^2 e^{-2rT}\left(e^{\alpha^2 T} - 1\right)}.$$

Most of properties are too complex to find them. Then we examine numerical comparative statics to find them. We compute $\hat{u}_t, \hat{v}_t, \hat{B}$, and $\hat{p}$ shifting $\alpha, r, \delta, B$, and $t$ for fixed $V_0, T$, and $R$. Graphs and properties in detail are in Horikawa (2005) [3]. Numerical results are as Table 1 and 2. The remarkable constructions are in conclusion.

Table 1: Liability and compensation

| Out \ In | $R$ | $\delta$ | $\alpha$ | $V_0$ | $T$ | $r$ |
|---|---|---|---|---|---|---|
| $\hat{B}$ | × | − | − | + | − | + |
| $\hat{p}$ | ↗ | + | − | − | − | − |

Table 2: Effort and volatility

| Out \ In | $\alpha$ | $B$ | $r$ | $V_t$ | $\delta$ | $T-t$ |
|---|---|---|---|---|---|---|
| $\hat{u}_t$ | ↘ | ↗ | ↗ | ↘ | + | − |
| $\hat{v}_t$ | + | − | − | ? | + | + |

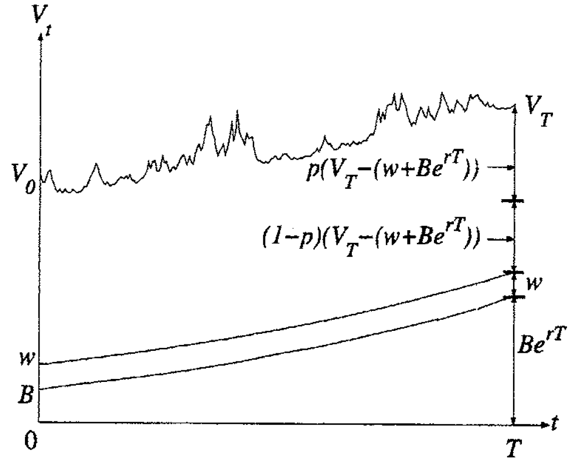
- Arrows represent the analytical results. ? means not to yield some tendency. Others express the numerical results.

## 4 Cash and Share Compensation

In this section, we study more realistic compensation case: it consists of fixed and variable factors. The case only variable term is we have studied in the previous sections.

We let the priority for a shareholder at the maturity of liabilities as follows. A shareholder first pay back her liabilities with interest rate $r$, that is, $Be^{rT}$. Next she pays her manager some fixed compensation $w$ from her shareholders' equity. Here let $p \in [0, 1]$ the ratio for her residual share. After she pays $w$ to him, she pays $100p\,\%$ of share as his "incentive bonus." Therefore we cannot just compare "$p$" of this section to the one of the previous sections. These are adjusted to the assumption of Merton (1974) [5] at Section 2. We keep all the other setting and notations of our model as we have used. At last, we assume $\delta > 0$ because of keeping our analysis simple.

Figure 2: Dynamics of $V_t$, $B$, and $w$.



Then let us consider the problems of a manager and a shareholder respectively like the section 3. The manager's problem is

$$\max_{u,v} E\left[\ln\left\{w+p\left(V_T-\left(w+Be^{rT}\right)\right)^+\right\}-\frac{1}{2}\int_0^T u_t^2 dt\right]. \tag{8}$$

The shareholder's problem is

$$\begin{aligned}
\max_{B,p,w}\quad & (1-p)E\left[\left(V_T-\left(w+Be^{rT}\right)\right)^+\right],\\
\text{s.t.}\quad & \max_{u,v} E\left[\ln\left\{w+p\left(V_T-\left(w+Be^{rT}\right)\right)^+\right\}-\frac{1}{2}\int_0^T u_t^2 dt\right]\geq R,\\
& (\hat{u},\hat{v})\in\operatorname*{argmax}_{(u,v)} E\left[\ln\left\{w+p\left(V_T-\left(w+Be^{rT}\right)\right)^+\right\}-\frac{1}{2}\int_0^T u_t^2 dt\right],\\
& p\in[0,1].
\end{aligned} \tag{9}$$

Calculated Equation (8) and (9), we obtain the optimal solution as Theorem 3.

**Theorem 3** (Optimal values when compensation includes fixed term).
*Consider the manager's and shareholder's problem: Equation* (8) *and* (9). *We assume* $\delta > 0$. *The optimal effort, volatility, the value of a firm, and liabilities are*

$$\begin{aligned}
\hat{u}_t &= \delta\check{z}_t e^{-\mu t}Z_t,\\
\hat{v}_t V_t &= \alpha\tilde{H}_t\frac{\partial}{\partial y}g\left(t,\tilde{H}_t\right)+\frac{\check{z}_t\delta^2 Z_t}{\alpha}\left[\exp(s-t)-1\right],\\
V_t &= g\left(t,\tilde{H}_t\right)-\frac{\check{z}_t\delta^2 Z_t}{\alpha^2}\left[\exp\left\{\alpha^2(s-t)\right\}-1\right],\\
\hat{B} &= e^{-rT}\left[\frac{w}{p}+\frac{1}{N\left(d_2\left(t,\tilde{H}_t\right)\right)}\right.\\
&\qquad\left.\left[V_t-\tilde{H}_tN\left(d_1\left(t,\tilde{H}_t\right)\right)-\frac{\check{z}_t\delta^2 Z_t}{\alpha^2}\left[\exp\left\{\alpha^2(s-t)\right\}-1\right]\right]\right].
\end{aligned}$$

*where*

$$\begin{aligned}
g(t,y) &:= \left(Be^{rT}-\frac{w}{p}\right)N\left(d_2(t,y)\right)+yN\left(d_1(t,y)\right),\\
d_1(t,y) &:= \frac{\ln(py/cw)+\alpha^2(T-t)/2}{\alpha\sqrt{T-t}},\\
d_2(t,y) &:= \frac{\ln(py/cw)-\alpha^2(T-t)/2}{\alpha\sqrt{T-t}},\\
\tilde{H}_t &:= \frac{1}{\check{z}_t Z_t},\quad N(x):=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)dz,
\end{aligned}$$

**Remark 2.**
*We cannot compute the optimal compensation* $\hat{p}$, *fixed compensation* $\hat{w}$, *and the range of reservation utility* $R$ *in Theorem* 3.

*Proof.* See Horikawa (2005) [3], Appendix A, Proof of Theorem 3. □

# 5 Conclusion

In this paper, we study the optimal liabilities of a firm, taking into account the incentive of a manager. The risk neutral shareholders aim to maximize the value of a firm by determining the level of liabilities and compensation to a manager. For these two factors, a risk averse manager can improve the shareholders' equity through his choice of effort and volatility. Effort entails cost whereas volatility does not. We derive the optimal effort, volatility, liabilities, and compensation by use of a dynamic principal agent model.

We mainly find the following three facts. Firstly, a smart manager decreases liabilities because he makes a large effort. Secondly, an efficient firm also decreases liabilities, however it encourages a manager to make less effort. Finally, liabilities has an incentive if effort of a manager is valid, however it decreases as a firm grows.

These are some directions that we could extend in this paper. The first direction is the analysis when compensation includes some fixed term $w$. Our difficulty dues to Remark 2. It would bring more fruitful result that whether to try within the limitation of Remark 2 or to try more relax condition. Other idea to be more realistic form is a non-linear contract form. We assume that the compensation in our model is a linear contract. The more manager achieves, the more he obtain shares. That contracts yields a "call-option form contract" in addition to a linear one. Now how do we solve it? We leave these problems for our further study.

# References

[1] F. Black and M. Scholes. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3):637–654, 1973.

[2] A. Cadenillas, J. Cvitanić, and F. Zapatero. Leverage decision and manager compensation with choice of effort and volatility. *Journal of Financial Economics*, 73:71–92, 2004.

[3] K. Horikawa. Optimal financial policy and incentive in a firm. *Working Paper, Osaka University*, 2005.

[4] I. Karatzas and S. Shreve. *Brownian Motion and Stochastic Calculus.* Springer-Verlag, NY, second edition, 1991.

[5] R. C. Merton. On the pricing of corporate debt: The risk structure of interest rates. *The Journal of Finance*, 29(2):449–470, May 1974.

[6] F. Modigliani and M. H. Miller. The cost of capital, corporation finance and the theory of investment. *The American Economic Review*, 48(3):261–297, June 1958.

[7] E. Morellec and C. W. Smith Jr. Investment policy, financial policies, and the control of agency conflicts. *Working Paper, University of Rochester*, April 2004.

[8] B. Salanié. *The Economics of Contracts: A Primer.* MIT Press, MA, second edition, 2005.