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A summary on the approximate approach to the exponential utility indifference valuation

Takuji Arai
Department of Economics
Keio University
2-15-45 Mita Minato-ku
Tokyo 108-8345 Japan
E-mail:arai@econ.keio.ac.jp

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Abstract

In this paper, we give a summary on the results of the approximate approach to the exponential utility indifference valuation, based on Arai (2005).

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JEL classification: G10, C60

1 Introduction

Arai (2005) have introduced an approximate approach to the exponential utility indifference valuation (EUIV, for short) by using a kind of power functions.

It is very important issue in mathematical finance that the valuation and the pricing of contingent claims in incomplete markets. Recently, there is much literature having studied the utility indifference valuation method, which is one of valuation methods for the contingent claims. We start with an incomplete market with the maturity $T > 0$, whose asset fluctuation is described by a semimartingale X. Moreover, we consider an investor having initial capital x_t at time t, and who intends to sell a contingent claim B. Let U be his/her utility function, which is an $\mathbf{R}$-valued continuous increasing concave function defined on $\mathbf{R}$. Now, we define an adapted process $C_t(B)$ by

$$\begin{aligned} &\operatorname{esssup}_{\vartheta\in\Theta} E\left[U(x_t+G_{t,T}(\vartheta))|\mathcal{F}_t\right] \\ &= \operatorname{esssup}_{\vartheta\in\Theta} E\left[U(x_t+C_t(B)+G_{t,T}(\vartheta)-B)|\mathcal{F}_t\right], \end{aligned} \tag{1}$$

where $G_{t,T}(\vartheta) := \int_t^T \vartheta_s dX_s$ and Θ a suitable set of predictable processes, represents the set of all self-financing strategies. Then, we call $C_t(B)$ the utility indifference valuation, which is one of candidates for the asking price of the contingent claim B. In addition, the valuation $C_t(B)$ strongly depends on the preference of the investor who intends to sell B. The left hand side of (1) is the expected utility maximization problem when he/she does not sell the contingent claim B. On the other hand, the right hand side is the case where he/she sells B for the price $C_t(B)$ at time t and agrees to pay B at the maturity T. In particular, there has been much literature on the exponential utility case, that is, the case where U is given by $U(x) = -e^{-\alpha x}$, for $\alpha > 0$. Remark that, for the exponential utility case, we call $C_t(B)$ the exponential utility indifference valuation(EUIV). See Becherer (2004), Frittelli (2000), Rouge and El Karoui (2000), Musiela and Zariphopoulou (2004a, 2004b), Young (2004), and so on. In particular, Musiela and Zariphopoulou (2004a) have obtained the EUIV concretely for the following model: They considered an incomplete market with two risky assets. One is driven by a geometric Brownian motion and another is nontraded, whose fluctuation is given by a diffusion process. Then, for a European type claim, they provided the dynamics of the EUIV. On the other hand, in Musiela and Zariphopoulou (2004b), they dealt with discrete time models and yielded a pricing algorithm for a multiperiod incomplete market model. In addition to this, Mania and Schweizer (2005) (MS, for short) have provided the dynamics for the case where the asset price process is given by a general continuous semimartingale.

On the other hand, when we define the EUIV, we need to assume the following strong condition with respect to the underlying contingent claim:

$$E\left[e^{\alpha B}\right] < \infty. \tag{2}$$

Indeed, MS impose the boundedness of B. For example, in the case where B is a European call option and X is given by a geometric Brownian motion, (2) does not hold, because, roughly speaking, the distribution of B is near to one of e^Y, where Y is a normal random variable. Hence, models satisfying the condition (2) do not include some typical important ones as the above example. At this, we try to reduce the condition (2) to, for a sufficient large $n \in \mathbf{N}$,

$$E[B^n] < \infty, \tag{3}$$

equivalently $E[e^{nY}] < \infty$. Now, we recall the definition of "e" as follows:

$$e^x = \lim_{n\to\infty}\left(1 + \frac{x}{n}\right)^n,$$

then, for any sufficient large n, we can say that $\left(1 + \frac{x}{n}\right)^n$ is near to e^x. If we denote, for a sufficient large number n,

$$U(x) = -\left(1 + \frac{\alpha x}{n}\right)^{-n} \quad \text{or} \quad -\left(1 - \frac{\alpha x}{n}\right)^n,$$

then we can approximate the EUIV under the condition (3). Remark that this function U is not a utility function exactly, since not concave. Although, for $x <$

n/α, U is concave, so that we can say that U is almost concave. Instead of the exponential utility, if we adopt the function U as the underlying utility function, then we may obtain an approximate approach to the EUIV. On the other hand, it is difficult for us to treat U directly. Therefore, we try to decompose the value $x_t + C_t(B) + G_{t,T}(\vartheta) - B$ into the $\mathcal{F}_t$-measurable part $x_t + C_t(B)$, the gain process part $G_{t,T}(\vartheta)$ and the contingent claim part B. Thus, instead of U, we consider, for $\alpha > 0$ and $n \in \mathbf{N}$,

$$U_{\alpha,n}(x,y,z) := -\left(1+\frac{\alpha x}{n}\right)^{-n}\left(1-\frac{\alpha y}{n}\right)^{n+1}\left(1+\frac{\alpha z}{n}\right)^{n}.$$

Note that, if n is sufficient large, then $U_{\alpha,n}(x,y,z)$ is very near to $-\left(1+\frac{\alpha}{n}(x+y-z)\right)^{-n}$ or $-\left(1-\frac{\alpha}{n}(x+y-z)\right)^{n}$. On the other hand, if we denote

$$U_{\alpha,\exp}(x,y,z) := -\exp(-\alpha(x+y-z)),$$

then the EUIV, denoted by $C_t^{\alpha,\exp}(B)$, satisfies the following:

$$\begin{aligned}&\operatorname{esssup}_{\vartheta\in\Theta} E\left[U_{\alpha,\exp}(x_t, G_{t,T}(\vartheta), 0)|\mathcal{F}_t\right]\\ &\quad = \operatorname{esssup}_{\vartheta\in\Theta} E\left[U_{\alpha,\exp}(x_t + C_t^{\alpha,\exp}(B), G_{t,T}(\vartheta), B)|\mathcal{F}_t\right].\end{aligned}$$

Remark that $C_t^{\alpha,\exp}(B)$ does not depend on the initial capital x_t. Thus, by the same way as the EUIV, we define an adapted process $C_t^{\alpha,n}(B)$ as a process satisfying

$$\begin{aligned}&\operatorname{esssup}_{\vartheta\in\Theta} E\left[U_{\alpha,n}(x_t, G_{t,T}(\vartheta), 0)|\mathcal{F}_t\right]\\ &\quad = \operatorname{esssup}_{\vartheta\in\Theta} E\left[U_{\alpha,n}(x_t + C_t^{\alpha,n}(B), G_{t,T}(\vartheta), B)|\mathcal{F}_t\right].\end{aligned}$$

This process $C_t^{\alpha,n}(B)$ may be a strong candidate of approximations to the EUIV. Remark that $C_t^{\alpha,n}(B)$ depends on x_t. Henceforth, we fix $x_t = 0$.

Remark that, in the complete market case, $C_t^{\alpha,n}(B)$ does not equal to the fair price of B. However, we can say that, if n is sufficient large, then $C_t^{\alpha,n}(B)$ is very near to the fair price.

The structure of this paper is as follows: In Section 2, we state the standing assumptions and the exact definition of our new valuation $C_t^{\alpha,n}(B)$. In particular, we need the closedness of the set of all self-financing strategies in the $\mathcal{L}^{n+1}$ sense. This closedness is close related to the $1+\frac{1}{n}$-optimal martingale measure. Thus, some standing assumptions are concerned in it. Moreover, remark that it is close related to the projection of "1" onto a suitable space of the stochastic integrations.

In order to make sure that our new valuation is useful as an approximate approach to the EUIV, we investigate its basic properties and the asymptotic behavior as n tends to 0. In Section 3, we prove that our new valuation has same basic properties as the EUIV approximately. In particular, we show that there exists a duality relationship between a portfolio optimization problem related to our new valuation and an optimization problem among equivalent martingale measures, which is related to the $1+\frac{1}{n}$-optimal martingale measure. Furthermore, we assert in Section 4 that $C_t^{\alpha,n}(B)$ converges to the EUIV as n tends to ∞ in probability. To see this, it is worth while to notice that the p-optimal martingale measure converges to the minimal martingale measure as p tends to 1, which has been proved by Grandits and Rheinländer (2002) (GR, for short).

2 Preliminaries

In this section, we prepare mathematical preliminaries. In particular, we introduce the three standing assumptions and some notations in order to formulate the exact definition of our new valuation $C_t^{\alpha,n}(B)$, namely, give the definition of the set of all self-financing strategies.

Consider an incomplete financial market composed of one riskless asset whose price is "1" at all time, and d risky assets described by an $\mathbf{R}^d$-valued continuous semimartingale X. Suppose that the maturity is $T > 0$. Let $(\Omega, \mathcal{F}, P; \mathbf{F} = \{\mathcal{F}_t\}_{t\in[0,T]})$ be a completed filtered probability space with a right-continuous filtration $\mathbf{F}$ such that $\mathcal{F}_0$ is trivial and contains all null sets of $\mathcal{F}$, and $\mathcal{F}_T = \mathcal{F}$. Furthermore, in this paper, we treat a suitable set of $\mathbf{R}^d$-valued predictable X-integrable processes ϑ as the set of all self-financing strategies, denoted by Θ. Let B be an $\mathcal{F}_T$-measurable random variable. Throughout this paper, we regard B as a contingent claim, that is, a pay-off at the maturity T. We fix a positive real number α and a large odd number n. To simplify notations, we restrict n within odd numbers. For all unexplained notations, we refer to Dellacherie and Meyer (1982) and GR. Throughout this paper, C denotes a constant in $(0,\infty)$ which may vary from line to line.

Firstly, we give one of the standing assumptions related to the underlying contingent claim B.

Assumption 1 We assume that $B \geq 0$ and $B \in \mathcal{L}^n(P)$.

Next, we prepare some notations in order to introduce the other standing assumptions. Let P^0 be a probability measure which is equivalent to P, and $p > 1$.

Definition 2 (1) Let $S \leq T$ be a stopping time. We denote by ${}^S\mathcal{V}(P^0)$ the linear subspace of $\mathcal{L}^\infty(P^0)$ spanned by the simple stochastic integrals of the form $h^{\mathrm{tr}}(X_{T_2} - X_{T_1})$, where $S \leq T_1 \leq T_2 \leq T$ are stopping times such that the stopped process X^{T_2} is bounded, and h is a bounded $\mathbf{R}^d$-valued $\mathcal{F}_{T_1}$-measurable random variable. Set $\mathcal{V}(P^0) = {}^0\mathcal{V}(P^0)$.
(2) A signed martingale measure under P^0 is a signed measure $Q \ll P^0$ with $E_{P^0}\left[\frac{dQ}{dP^0}\right] = 1$ and $E_{P^0}\left[\frac{dQ}{dP^0}f\right] = 0$ for all $f \in \mathcal{V}(P^0)$.
(3) $\mathcal{M}^s(P^0)$ is the space of all signed martingale measures under P^0, and

$\mathcal{M}^e(P^0)$ is the subset of $\mathcal{M}^s(P^0)$ consisting of probability measures being equivalent to P^0. Moreover, we set $\mathcal{M}_p^x(P^0) := \mathcal{M}^x(P^0) \cap \mathcal{L}^p(P^0)$ for $x \in \{e, s\}$.
(4) The p-optimal martingale measure with respect to P^0 is defined as the element of $\mathcal{M}_p^s(P^0)$ which minimizes $\mathcal{L}^p(P^0)$-norm.
(5) Let Y be a uniformly integrable P^0-martingale with $Y_0 = 1$ and $Y_T > 0$. We say that Y satisfies the reverse Hölder inequality $\mathcal{R}_p(P^0)$, if there is a constant C such that for every stopping time $S \leq T$, we have

$$E_{P^0}\left[\left(\frac{Y_T}{Y_S}\right)^p \middle| \mathcal{F}_S\right] \leq C.$$

The $1+\dfrac{1}{n}$-optimal martingale measure will play a vital role, so that the following assumption is essential.

Assumption 3 We assume that the $1+\dfrac{1}{n}$-optimal martingale measure $Q^{(n)}$ exists in $\mathcal{M}^e_{1+\frac{1}{n}}(P)$, and its density process $Z^{(n)}$ satisfies the reverse Hölder inequality $\mathcal{R}_{1+\frac{1}{n}}(P)$.

Since X is a continuous semimartingale, it is special under P, and its canonical decomposition is given by $X = X_0 + M + A$ with M a local martingale, A a predictable process, and $M_0 = A_0 = 0$. Moreover, if P^0 is equivalent to P, then X is also a special semimartingale under P^0. Let us denote its canonical decomposition under P^0 as follows:

$$X = X_0 + M^0 + A^0.$$

Definition 4 (1) We denote by ${}^S\mathcal{K}_p(P^0)$ the closure in $\mathcal{L}^p(P^0)$ of ${}^S\mathcal{V}(P^0)$ for a stopping time $S \leq T$. In particular, let $\mathcal{K}_p(P^0) := {}^0\mathcal{K}_p(P^0)$.
(2) Let $L^p(M^0)$ be the space of all $\mathbf{R}^d$-valued predictable processes ϑ such that

$$||\vartheta||_{L^p(M^0)} := E_{P^0}^{1/p}\left[\left(\int \vartheta^{\mathrm{tr}} d[M^0]\vartheta\right)_T^{p/2}\right] < \infty.$$

(3) Let $L^p(A^0)$ be the space of all $\mathbf{R}^d$-valued predictable processes ϑ such that

$$||\vartheta||_{L^p(A^0)} := E_{P^0}^{1/p}\left[\left(\int |\vartheta^{\mathrm{tr}} dA^0|\right)_T^{p}\right] < \infty.$$

We define

$$\Theta^{n+1}(P^0) := L^{n+1}(A^0) \cap L^{n+1}(M^0)$$

and

$$G_{t,T}(\Theta) := \left\{\int_t^T \vartheta_s dX_s | \vartheta \in \Theta\right\},$$

for a suitable set Θ of $\mathbf{R}^d$-valued X-integrable predictable processes. In particular, we denote $G_T(\Theta) := G_{0,T}(\Theta)$. Remark that we can rearrange the definition of $\Theta^{n+1}(P^0)$ as $\Theta^{n+1}(P^0) := \{\vartheta | G(\vartheta) \in \mathcal{S}^{n+1}(P^0)\}$. By Theorem 4.1 of Grandits and Krawczyk (1998), $G_T(\Theta^{n+1}(P))$ is $\mathcal{L}^{n+1}(P)$-closed under Assumption 3. By Lemma 2.1 of GR, we have $G_{t,T}(\Theta^{n+1}(P)) = {}^t\mathcal{K}_{n+1}(P)$.

Moreover, since n is odd, Propositions 4.2 and 4.4 of GR imply, by passing to a version if necessary,

$$Z_{t,T}^{(n)} := Z_T^{(n)}/Z_t^{(n)} = C_t^{(n)} \left(1 + \frac{{}^t f_T^{(n)}}{n}\right)^n,$$

where

$$Z_t^{(n)} := E\left[\frac{dQ^{(n)}}{dP} \Big| \mathcal{F}_t\right],$$

$C_t^{(n)}$ is an $\mathcal{F}_t$-measurable positive random variable, and ${}^t f_T^{(n)} \in {}^t\mathcal{K}_{n+1}(P)$. In particular, $-{}^t f_T^{(n)}/n$ is the projection of "1" onto ${}^t\mathcal{K}_{n+1}(P)$ in $\mathcal{L}^{n+1}(P)$.

Thirdly, we define a probability measure $P^{n,B}$ as

$$\frac{dP^{n,B}}{dP} := C^{n,B}\left(1 + \frac{\alpha}{n}B\right)^n,$$

where $C^{n,B} \in \mathbf{R}_+$. Furthermore, we denote

$$Z_{t,T}^{n,B} := \frac{Z_T^{n,B}}{Z_t^{n,B}} = C_t^{n,B}\left(1 + \frac{\alpha}{n}B\right)^n \quad \text{and} \quad Z_t^{n,B} := E\left[\frac{dP^{n,B}}{dP} \Big| \mathcal{F}_t\right],$$

where $C_t^{n,B}$ is an $\mathcal{F}_t$-measurable positive random variable. Remark that X is also a semimartingale under $P^{n,B}$.

Assumption 5 We assume that the $1 + \dfrac{1}{n}$-optimal martingale measure $Q^{(n),B}$ with respect to $P^{n,B}$ exists in $\mathcal{M}^e_{1+\frac{1}{n}}(P^{n,B})$, and its density process $Z^{(n),B}$ with respect to $P^{n,B}$ satisfies $\mathcal{R}_{1+\frac{1}{n}}(P^{n,B})$, where

$$Z_t^{(n),B} := E_{P^{n,B}}\left[\frac{dQ^{(n),B}}{dP^{n,B}} \Big| \mathcal{F}_t\right].$$

We have

$$Z_{t,T}^{(n),B} := Z_T^{(n),B}/Z_t^{(n),B} = C_t^{(n),B}\left(1 + \frac{{}^t f_T^{(n),B}}{n}\right)^n,$$

where $C_t^{(n),B}$ is an $\mathcal{F}_t$-measurable positive random variable, and ${}^t f_T^{(n),B} \in {}^t\mathcal{K}_{n+1}(P^{n,B})$. In particular, $-{}^t f_T^{(n),B}/n$ is the projection of "1" onto ${}^t\mathcal{K}_{n+1}(P^{n,B})$ in $\mathcal{L}^{n+1}(P^{n,B})$.

Remark that we have $G_{t,T}(\Theta^{n+1}(P^{n,B})) = {}^t\mathcal{K}_{n+1}(P^{n,B})$. Thus, there exists a solution to the following minimization problem:

$$\max_{\vartheta \in \Theta^{n+1}(P^{n,B})} E\left[\left(1 - \frac{\alpha}{n}G_{t,T}(\vartheta)\right)^{n+1}\left(1 + \frac{\alpha}{n}B\right)^n \Big| \mathcal{F}_t\right].$$

Hence, $\Theta^{n+1}(P^{n,B})$ should be the set of all self-financing strategies.

3 Basic properties

We shall provide basic properties of $C_t^{\alpha,n}(B)$ in this section. We are interesting whether or not $C_t^{\alpha,n}(B)$ satisfies the same basic properties as the EUIV. Firstly, we prepare some notations.

For an $\mathcal{F}_t$-measurable random variable x_t, we define

$$V_t^{\alpha,n,B}(x_t) := \operatorname{esssup}_{\vartheta\in\Theta_B^{(n)}} E[U_{\alpha,n}(x_t, G_{t,T}(\vartheta), B)|\mathcal{F}_t].$$

Then, we can rewrite the definition of $C_t^{\alpha,n}(B)$ as

$$V_t^{\alpha,n,0}(0) = V_t^{\alpha,n,B}(C_t^{\alpha,n}(B)).$$

We have

$$\frac{V_t^{\alpha,n,B}(0)}{V_t^{\alpha,n,0}(0)} = \frac{V_t^{\alpha,n,B}(0)}{V_t^{\alpha,n,B}(C_t^{\alpha,n}(B))} = \left(1+\frac{\alpha}{n}C_t^{\alpha,n}(B)\right)^n,$$

namely,

$$C_t^{\alpha,n}(B) = \frac{n}{\alpha}\left\{\left(\frac{V_t^{\alpha,n,B}(0)}{V_t^{\alpha,n,0}(0)}\right)^{\frac{1}{n}} - 1\right\}.$$

Remark that, by Proposition 4.4 of GR and Assumption 3, we have

$$\begin{aligned} V_t^{\alpha,n,0}(0) &= \operatorname{esssup}_{\vartheta\in\Theta_0^{(n)}} E\left[U_{\alpha,n}(0, G_{t,T}(\vartheta), 0)|\mathcal{F}_t\right] \\ &= \operatorname{esssup}_{\vartheta\in\Theta^{n+1}(P)} E\left[-\left(1-\frac{\alpha}{n}G_{t,T}(\vartheta)\right)^{n+1}\middle|\mathcal{F}_t\right] \\ &= -E\left[\left(1+\frac{{}^t f_T^{(n)}}{n}\right)^{n+1}\middle|\mathcal{F}_t\right]. \end{aligned}$$

Remark $V_t^{\alpha,n,0}(0) < 0$. For any $Q \in \mathcal{M}^e_{1+\frac{1}{n}}(P^{n,B})$, we denote

$$Z_{t,T}^Q := \frac{Z_T^Q}{Z_t^Q}, \quad \text{and} \quad Z_t^Q := E\left[\frac{dQ}{dP}\middle|\mathcal{F}_t\right].$$

Moreover, we define

$$\widetilde{V}_t^{\alpha,n,B} := \operatorname{essinf}_{Q\in\mathcal{M}^e_{1+\frac{1}{n}}(P^{n,B})} E_Q\left[\left(Z_{t,T}^Q\right)^{\frac{1}{n}}\left(1+\frac{\alpha}{n}B\right)^{-1}\middle|\mathcal{F}_t\right].$$

Remark that we have

$$\widetilde{V}_t^{\alpha,n,0} = \operatorname{essinf}_{Q\in\mathcal{M}^e_{1+\frac{1}{n}}(P)} E_Q\left[\left(Z_{t,T}^Q\right)^{\frac{1}{n}}\middle|\mathcal{F}_t\right] = \left(C_t^{(n)}\right)^{\frac{1}{n}}.$$

In order to investigate basic properties, we need to show a duality relationship between a portfolio optimization problem and an optimization problem with respect to equivalent martingale measures.

Theorem 6 *We have the following duality relationship:*

$$\begin{aligned}&\mathrm{esssup}_{\vartheta\in\Theta_B^{(n)}} E\left[-\left(1-\frac{\alpha}{n}G_{t,T}(\vartheta)\right)^{n+1}\left(1+\frac{\alpha}{n}B\right)^n\Big|\mathcal{F}_t\right]\\ &= -\left\{\mathrm{essinf}_{Q\in\mathcal{M}^e_{1+\frac{1}{n}}(P^{n,B})} E_Q\left[\left(Z^Q_{t,T}\right)^{\frac{1}{n}}\left(1+\frac{\alpha}{n}B\right)^{-1}\Big|\mathcal{F}_t\right]\right\}^{-n} \quad (4)\end{aligned}$$

Theorem 6 provides the following representation of $C_t^{\alpha,n}(B)$:

Corollary 7 *By the result of Theorem 6, we obtain*

$$V_t^{\alpha,n,B}(0) = -(\widetilde{V}_t^{\alpha,n,B})^{-n}$$

and

$$C_t^{\alpha,n}(B) = \frac{n}{\alpha}\left\{\frac{\widetilde{V}_t^{\alpha,n,0}}{\widetilde{V}_t^{\alpha,n,B}} - 1\right\}.$$

Next, we study basic properties of $C_t^{\alpha,n}(B)$ by using the above duality relation. First of all, we introduce the basic properties of the EUIV, which have been introduced in MS.

Proposition 8 (Proposition 4 of MS) *We assume that B and B' are bounded (not necessarily positive). For fixed $t\in[0,T]$ and $\alpha>0$, $C_t^{\alpha,\exp}(B)$ has the following properties:*
(1) $-\|B\|_\infty \le C_t^{\alpha,\exp}(B) \le \|B\|_\infty$,
(2) if $B\le B'$, then $C_t^{\alpha,\exp}(B) \le C_t^{\alpha,\exp}(B')$,
(3) $C_t^{\alpha,n}(\lambda B+(1-\lambda)B') \le \lambda C_t^{\alpha,n}(B)+(1-\lambda)C_t^{\alpha,n}(B')$, for any $\lambda\in[0,1]$,
(4) $C_t^{\alpha,n}(B+x_t) = C_t^{\alpha,n}(B)+x_t$, for any $x_t\in\mathcal{L}^\infty(\mathcal{F}_t)$.

MS called $C_t^{\alpha,\exp}(B)$ a convex monetary utility functional. Furthermore, they remarked that $C_t^{\alpha,\exp}(-B)$ is close related to a convex monetary risk measure (see Cheridito, Delbaen and Kupper (2004)).

In order to see that our new valuation $C_t^{\alpha,n}(B)$ is available as one of approximate approaches to the EUIV, we wish to prove that $C_t^{\alpha,n}(B)$ satisfies Proposition 8. Henceforth, we shall illustrate that this fact holds approximately.

Proposition 9 *For any $t\in[0,T]$, we have the following:*
(1) For $B\in\mathcal{L}_+^\infty(P)$, $0\le C_t^{\alpha,n}(B)\le\|B\|_\infty$.
(2) Under Assumptions 1 and 5 for B', $B\le B' \Longrightarrow C_t^{\alpha,n}(B)\le C_t^{\alpha,n}(B')$.
(3) Suppose that $\lambda\in[0,1]$ and B, $B'\in\mathcal{L}_+^\infty(P)$. For sufficient large n, there exists a constant $C>0$ depending on $\|B\|_\infty$ and $\|B'\|_\infty$ such that

$$C_t^{\alpha,n}(\lambda B+(1-\lambda)B') \le \lambda C_t^{\alpha,n}(B)+(1-\lambda)C_t^{\alpha,n}(B')+C\frac{\alpha}{n}.$$

(4) Let x_t be a bounded $\mathcal{F}_t$-measurable random variable. For any sufficient large n, there exists a constant $C>0$ depending on $\|x_t\|_\infty$ and $\|B\|_\infty$ such that

$$|C_t^{\alpha,n}(B+x_t) - C_t^{\alpha,n}(B) - x_t| \le C\frac{\alpha}{n}. \quad (5)$$

4 Asymptotic behavior

In this section, we treat asymptotic behavior of $C_t^{\alpha,n}(B)$ as n tends to ∞. Our aim of studying such asymptotic behavior is to make sure that $C_t^{\alpha;n}(B)$ is justified as an approximate approach to the EUIV. We prove that $C_t^{\alpha,n}(B)$ converges to the EUIV in probability. Remark that GR have proved that the p-optimal martingale measure converges to the minimal entropy martingale measure as p tends to 1. In the proof of the following theorem, this asymptotic behavior will be essential.

Theorem 10 *Suppose that $B \in \mathcal{L}_+^\infty(P)$. For fixed $t \in [0,T]$, $C_t^{\alpha,n}(B)$ converges to $C_t^{\alpha,\exp}(B)$ in probability as $n \to \infty$.*

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