Equilibrium Refinement Problems in Cheap-talk Games

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ABSTRACT. We exemplify some problems of equilibrium refinement in cheap-talk games and show a new refinement criterion of 'suggestive domination,' which is introduced by Shirataki and Ishikawa (2005). Following this criterion, we define *suggestive dominant equilibrium* and explain how this equilibrium concept solves the problems of the refinement in cheap-talk games.

1. INTRODUCTION

Our purpose of this article is to explain the refinement problems and a new refinement concept, *the suggestive dominant equilibrium* in cheap-talk games, which is introduced by Shirataki and Ishikawa (2005).

Cheap-talk games are a kind of signaling games with costless communication. Therefore the Sender's action does not depend on the players' payoffs. As known broadly, the cheap-talk games have many equilibria and the standard refinement concepts do not work on the game. Therefore the refinement concepts for the cheap-talk games are developed. Among others, Blume and Sobel (1995) define the trumping relation in order to compare with all possible agreements between the Sender and Receiver.

While their concepts of the trumping relation can be defined for the several equilibrium concepts, the credibility of the agreements is not necessarily guaranteed. In this article, we point out this problem and define a new equilibrium, *the suggestive dominant equilibrium*, in order to overcome it.

The suggestive dominant equilibrium has the following favorable properties:(i) It can be defined more simply and can be found more easily, (ii) the existence is guaranteed, (iii) it achieves the Pareto optimal communication.

This article is organized as follows: In Section 2, we show the basic notions of cheap-talk games and exemplify the infinite many equilibria and the difficulty of the refinements in cheap-talk games. In section 3, we explain the Blume-Sobel's trumping relations and the problems of their concepts. To improve them, we introduce a new criterion, *the suggestive domination*, introduced by Shirataki and Ishikawa(2005). This domination concept guarantees the Sender's messages

and the stable communication. In section 4, we define the suggestive dominant equilibrium based on the above domination concept. We show several characters of the equilibrium.

2. Basic Characters of Cheap-talk Games

2.1. **Basic notions.** We consider a cheap-talk game with two players. $\{S, R\}$ is the set of the two players, in which S and R represent *Sender* and *Receiver*, respectively. Let T be the finite set of the Sender's types and let A be the finite set of actions available to the Receiver. In this game, the Sender can privately know his own type $t \in T$, and sends some messages to the Receiver. We denote the set of the Sender's messages as M. The cardinality of M is assumed to be sufficiently large. The Receiver decides her action after receiving the messages sent by the Sender.

In this game, their payoffs do not depend on the Sender's messages, that is, his messages are costless or *cheap talks*. Then player i's payoff is denoted as a mapping $u_i : A \times T \to \mathbb{R}$ for each $i \in \{S, R\}$. Let π be the Receiver's prior on the types: $\pi \in \Delta T$ with $\pi(t) > 0$ for any $t \in T$. Then we define a cheap-talk game $\mathcal{G}$ as

$$\mathcal{G} := \langle \{S, R\}, (T, \pi), A, M, u_S, u_R \rangle.$$

The game $\mathcal{G}$ is proceeded as follows: The Sender observes his own type $t \in T$ occurred with probability $\pi(t)$. Then he sends the message to the Receiver according to his strategy $\sigma : T \to \Delta(M)$. We denote the marginal on $m \in M$ given $t \in T$ as $\sigma(m|t)$. After the Receiver obtains his message, she decides her own action $a \in A$ according to her strategy $\alpha : M \to \Delta(A)$. Also, we denote the marginal on $a \in A$ given $m \in M$ as $\alpha(a|m)$. Finally both two players would get their payoffs depending on both the Sender's type and the Receiver's action.

Given a strategy pair (σ, α), the payoffs of the type t's Sender and the Receiver are respectively given as follows:

$$U_S(\sigma, \alpha \mid t) := \sum_{m \in M} \sum_{a \in A} \sigma(m|t) \alpha(a|m) u_S(a, t) \quad \text{for each } t \in T; \tag{1}$$

$$U_R(\sigma, \alpha) := \sum_{t \in T} \pi(t) \sum_{m \in M} \sum_{a \in A} \sigma(m|t) \alpha(a|m) u_R(a, t). \tag{2}$$

Then a strategy pair (σ, α) is called a *Bayesian Nash equilibrium* of a cheap-talk game $\mathcal{G}$ if the following two conditions satisfy;

$$\text{if } \sigma(m|t) > 0, \quad \text{then} \quad m \in \arg\max_{m'} \sum_{a \in A} \alpha(a|m') u_S(a, t); \tag{3}$$

$$\text{if } \alpha(a|m) > 0, \quad \text{then} \quad a \in \arg\max_{a'} \sum_{t \in T} \beta(t|m, \pi, \sigma) u_R(a', t) \tag{4}$$

$$\text{where } \beta(t|m, \pi, \sigma) = \frac{\pi(t)\sigma(m|t)}{\sum_{s \in T} \pi(s)\sigma(m|s)}$$

	a_1	a_2	a_3	a_4	a_5
t_1	(1, 1)	(0, 0)	(-1, 3)	(3, 2)	(2, -2)
t_2	(1, 1)	(1, 5)	(-1, -7)	(2, 0)	(2, 0)
t_3	(1, 1)	(0, 0)	(-1, 3)	(2, -2)	(3, 2)
t_4	(0, 0)	(0, 0)	(2, 1)	(0, 0)	(0, 0)

TABLE 1

The condition (4) requires that the posterior β is consistent with the Bayesian rule when the Sender sends his message m following the strategy σ.

2.2. Some difficulty of the refinement. In cheap-talk games, the standard refinement concepts as the sequential equilibrium do not work well because the payoffs of both the Sender and Receiver are independent of the Sender's messages, i.e., the costless messages. As pointed out in Farrell(1993), the messages, themselves, are meaningless. If anything, the question is how the Receiver expects the Sender's type when receiving a message from the Sender. Nevertheless it is possible to construct many meaningless equilibria in cheap-talk games by considering several kinds of the Sender's strategy's mapping. To understand this, we now consider the following example:

Example 2.1. Consider $\Gamma := \langle \{S, R\}, (T, \pi), A, M, u_S, u_R \rangle$ where T consists of four types, t_1, t_2, t_3, t_4, $\pi(t) = 1/4$ for each $t \in T$, $A = \{a_1, a_2, a_3, a_4, a_5\}$, $M = \{m_1, m_2, m_3, m_4, m_5\}$, and the players' payoffs are given as Table 1. The left number in each parenthesis is the Sender's payoff and the right one is the Receiver's in the table.

Then we have three Bayesian Nash equilibria $E_0 = (\sigma_0, \alpha_0)$, $E_1 = (\sigma_1, \alpha_1)$, $E_2 = (\sigma_2, \alpha_2)$ shown as follows:The first equilibrium E_0 is a pooling equilibrium, i.e. $\sigma_0(m_2| \ t) = 1$ for all $t \in T$ and $\alpha_0(a_2| \ m_2) = 1$. The second equilibrium E_1 is given as $\sigma_1(m_2|t) = 1$ for $t = t_1, t_2, t_3$, $\sigma_1(m_3|t_4) = 1$, and $\alpha_1(a_i|m_i) = 1$ for $i = 1, 2, 3, 4, 5$. The third Bayesian Nash equilibrium E_2 is given as follows:

$$\sigma_2(m|t) = \begin{cases} \sigma_2(m_1|t) = 1 & \text{if } t = t_1, t_3 \\ \sigma_2(m_1|t) = \sigma_2(m_2|t) = \frac{1}{2} & \text{if } t = t_2 \\ \sigma_2(m_3|t) = 1 & \text{if } t = t_4 \end{cases} \quad \alpha_2(a|m) = \begin{cases} \alpha_2(a_2|m) = 1 & \text{if } m = m_2 \\ \alpha_2(a_3|m) = 1 & \text{if } m = m_3 \\ \alpha_2(a_1|m) = 1 & \text{otherwise.} \end{cases}$$

Then, for instance, we construct other Bayesian Nash equilibrium $E_{21} = (\sigma_{21}, \alpha_2)$ by replacing σ_2 of E_2 with the following σ_{21}: For any $x \in (0, 1]$,

$$\sigma_{21}(m|t_1) = \sigma_{21}(m|t_3) = \begin{cases} \sigma(m_1|t_1) = 1 - x \\ \sigma(m_4|t_1) = x, \end{cases} \quad \sigma_{21}(m|t_2) = \begin{cases} \sigma(m_1|t_2) = \frac{1}{2}(1 - x) \\ \sigma(m_2|t_2) = \frac{1}{2} \\ \sigma(m_4|t_2) = \frac{1}{2}x, \end{cases}$$

and $\sigma_{21}(m_3|t_4) = 1$.

However, this equilibrium E_{21} is essentially the same one as E_2 because E_{21} is the equilibrium obtained just by replacing the probability on m_1 with the probability $(1 - x)$. Indeed σ_{21} lead the same expectations as σ_2 to the Receiver and then it is the best response for her to play the same strategy α_2.

Therefore we need to consider such different strategies as the same one when they are essentially same. Now we focus on the *outcome* following Park(1997). Given a pair (σ, α), the outcome $o_{(\sigma,\alpha)} : T \to \Delta A$ is defined as follows: For each $t \in T$,

$$o_{(\sigma,\alpha)}(a|t) \equiv o(a|t, \sigma, \alpha) = \sum_{m \in M} \sigma(m|t)\alpha(a|m) \text{ for each } a \in A.$$

This distribution represents the realized probability of each action a no matter which messages are sent. Look at the previous example again, and we observe that both E_2 and E_{21} lead the same outcome as follow:

$$o(a|t, \sigma_2, \alpha_2) = \begin{cases} o(a_1|t_1, \sigma_2, \alpha_2) = o(a_1|t_3, \sigma_2, \alpha_2) = 1 \\ o(a_1|t_2, \sigma_2, \alpha_2) = o(a_2|t_2, \sigma_2, \alpha_2) = \frac{1}{2} \\ o(a_3|t_4, \sigma_2, \alpha_2) = 1 \end{cases}$$
$$= o(a|t, \sigma_{21}, \alpha_2)$$

Park(1997) defined the reduce-form equilibrium and refined the equilibrium led to the same outcomes. However, even when we focus on the difference of the outcomes, we need refine the outcomes in cheap-talk games. In the following, we regard the equilibria leading the same outcome as the same equilibrium and discuss the refinement among the different outcomes.

3. NEW CRITERIA OF EQUILIBRIUM REFINEMENT

There are many refinement concepts in cheap-talk games (E.g. see Farrell(1993)). Blume and Sobel (1995) especially give the comprehensive concept to compare with two different equilibria, called *the trumping relations*. Nevertheless their equilibrium concept based on the relations seems to have some problems. Therefore we define another refinement concept, *the suggestive domination*, and reconsider the refinement in cheap-talk games.

3.1. Blume-Sobel's trumping relations. Blume and Sobel (1995) consider a situation that the Receiver revises her prior π on types after receiving a message from the Sender. The *game given p* is the game where the Receiver places probability $p(t)$ on each type $t \in T$. Then (σ, α) is called an *equilibrium given p* if (3) and (4) hold for $\pi(\cdot) \equiv p(\cdot)$. Then, the *agreement* is defined as a triple of the form $\mathcal{A} = (\sigma, \alpha, p)$, where p is a probability distribution over types and (σ, α) is a perfect Bayesian equilibrium for the game given p.

Their idea of the refinements means to consider the deviation from the original agreement to another agreement. This idea is based on the stable set in cooperative games. Now we show Blume-Sobel's *communication proof trumping relation* (CP-trumping relation).

Definition 3.1. (σ, α, p) is CP-trumped by (σ', α', p') at m^* if and only if there exists a message m^* such that

(i) there exists t' such that $p(t')\sigma(m^*|t') > 0$, and for all $t \in T$, $p'(t) = \beta(t|m^*, p, \sigma)$

(ii) for all $t \in T$ with $\sigma(m^*|t) > 0$, $U_S(\sigma, \alpha|\ t) < U_S(\sigma', \alpha'|\ t)$.

Condition (i) requires that the new prior p' is consistent with the revision of p based on the Bayesian rule given $\sigma(m^*)$. Condition (ii) guarantees that the new agreement leads the higher expected payoff than the original agreement to the Sender when the message m^* is sent with positive probabilities. When m^* is sent under σ', it implies that the Sender has the incentive to deviate from the original agreement by changing the Receiver's strategies from α to α'. Note that all the agreements are Bayesian Nash equilibrium and then α' is the best response for the Receiver when the Sender plays on σ'. In the previous example, the equilibrium E_2 is CP-trumped at m_1 by following agreement $A" = (\sigma", \alpha", p")$:

$$\sigma"(m|t) = \begin{cases} \sigma"(m_4|t_1) = 1 \\ \sigma"(m_4|t_2) = \sigma"(m_5|t_2) = \frac{1}{2} \\ \sigma"(m_5|t_3) = 1, \end{cases} \quad \alpha"(a|m) = \begin{cases} \alpha"(a_5|m) = 1 & \text{if } m = m_5 \\ \alpha"(a_4|m) = 1 & \text{otherwise,} \end{cases}$$

$$p"(t) = (p"(t_1), p"(t_2), p"(t_3), p"(t_4)) = (\frac{2}{5}, \frac{1}{5}, \frac{2}{5}, 0).$$

3.2. Problems of CP-trumping relation and suggestive domination. While the CP-trumping relation enables to refine the equilibria in cheap-talk games, we have a doubt that the Receiver believes the message credible. Indeed, Condition (ii) doesn't restrict on $U_S(|\ t)$ if type t doesn't send the message m^* i.e., $\sigma(m^*|\ t) = 0$. Consider, for example, that the Receiver conceives the belief $p" = \beta(m_1, \pi, \sigma_2)$ when she merely get the message m_1. In addition, when she knows the Sender agrees to the new agreement $\mathcal{A}"$, she can reconsider the belief: "Is it impossible that the Sender is type t_4?" This fact has been pointed out by a referee of Blume-Sobel's paper. The referee has suggested appending the following condition to the definition of the CP-trumping relation:

(iii) For all $t \in T$ with $\sigma(m^*|t) = 0$,

$$U_S(\sigma, \alpha|\ t) \geq \max_{\hat{m} \in M} \sum_{a \in A} \alpha'(a|\hat{m}) u_S(a, t) \qquad (5)$$

Blume and Sobel call the trumping relation satisfying (i), (ii), and (iii) the *R-trumping relation*.

In the contrary to the condition (ii), (iii) requires that the type which doesn't send the message m^* does not deviate from $\mathcal{A}$. In the previous example of the game Γ, if the Sender of type t_4 preferred the action a_4 or a_5 to a_3, the Receiver possibly thinks that the Sender at t_4 sent the message m_1 in order to let the Receiver take the action a_4 or a_5. In that case, the Receiver might revise her belief from $p"$ and we can no longer describe the situation by game with $p"$

As stated before, Condition (ii) requires that all the types who deviate from the trumped agreement can get higher or equal payoff in the new trumping agreement. Furthermore, Condition (iii) guarantees that the types who don't deviate can not get higher payoff. These conditions

can be taken for "credibility" in the sense that they don't have any incentive to tell a lie on his types. This concept often has been referred as *incentive compatibility* when discussing the communication among players. Therefore the R-trumping relation is the criterion that guarantees the credibility about p' for the CP-trumping relation. However we still think that the condition (iii) does not sufficiently guarantee the credibility.

Consider the agreement $\mathcal{A}''in$Example refEX. The Receiver believes that the Sender's type cannot be t_4 according the condition (iii). As a result, the Receiver just obtain the information that the Sender's type is t_1, t_2, or t_3 and then there can be innumerable distributions of the belief. To avoid this ambiguity, we present a new criterion, *suggestive domination*, in order to require the credibility of the distribution of p'.

As the first step, we revise (ii) and (iii) as follows:

(ii)' For all $t \in T$ with $\sigma(m^*|t) \geq 0$, $U_S(\sigma, \alpha| t) < U_S(\sigma', \alpha'| t)$, and the inequality is strictly held for at least one type;

(iii)' For all $t \in T$ with $\sigma(m^*|t) < 1$, $U_S(\sigma, \alpha| t) \geq \max_{\hat{m} \in M} \sum_{a \in A} \alpha'(a|\hat{m}) u_S(a, t)$.

We require the strict inequality not for the all types but for at least one type in (ii)'. On the other hand, we impose the same condition as (5) on all the mixed strategies as well as the zero-probability strategy in the messages m^*. That is, the types who deviate with some positive probabilities of mixed actions get the equal payoff between two agreements with satisfying both (ii)' and (iii)'.

In Γ of Example 2.1, the Receiver's belief p'' in $\mathcal{A}''$ is based on the Sender's action that the Sender at t_2 would deviate with the probability $1/2$. But the Sender at t_2 gets his expected payoff 1 by either m_1 or m_2 in E_2, and can get 2 in $\mathcal{A}''$. Then he could always get more by sending m_1 and deviating from E_2 to $\mathcal{A}''$.

However this deviation seems to be insufficient. Consider the equilibrium E_{21} in the game Γ again. As is discussed before, E_{21} is essentially equivalent to E_2. Nevertheless, Conditions (ii)' and (iii)' are independently required for the same outcomes E_2 and E_{21}. To avoid this problem, we redefine the suggestive domination for the set of messages used with positive probabilities.

Now we denote the set of messages used with a positive probability on a pair (σ, p) as $M(\sigma, p) := \{m \in M \mid \sum_{t \in T} p(t)\sigma(m|t) > 0\}$. Then we formally define the following concept of dominated agreements:

Definition 3.2. An agreement $\mathcal{A} = (\sigma, \alpha, p)$ is *suggestively dominated* (*S-dominated*) by another agreement $\mathcal{A}' = (\sigma', \alpha', p')$ in M^* if and only if

(I) there is a nonempty set M^* as follows:

$$M^* := \left\{ m^* \in M(\sigma, p) \mid p' = \beta(m^*, p, \sigma) \text{ for all } t \in T \right\};$$

(II) for all $t \in T$ with $\sum_{m^* \in M^*} \sigma(m^*|t) > 0$,

$$U_S(\sigma, \alpha| t) \leq U_S(\sigma', \alpha'| t),$$

and the inequality is strictly held for at least one type;

(III) for all $t \in T$ with $\sum_{m^* \in M^*} \sigma(m^*|t) < 1$,

$$U_S(\sigma, \alpha|\ t) \geq \max_{\hat{m} \in M} \sum_{a \in A} \alpha'(a|\hat{m}) u_S(a, t). \tag{6}$$

We require (II) or (III) for the messages that the Sender sends with positive probabilities p' except $p' = 1$. In fact, it makes no difference if we revise the definition of CP-trumping or R-trumping in the same way as the second step.

4. SUGGESTIVE DOMINANT EQUILIBRIUM

First of all we define the equilibria based on the trumping relations by Blume and Sobel. Their concept makes use of the idea of stable sets in cooperative games. The problem of their equilibrium is due to the partial use of the stable sets. We make it clear in this section and define our new equilibrium concept, *the suggestive dominant equilibrium*.

4.1. Equilibrium concepts on the trumping relations. In order to define the equilibrium based on the trumping relations, Blume and Sobel introduce *the consistent partition* on the set of all agreements.

Definition 4.1. $\{G, B\}$ is a *consistent partition* of the set of agreements relative to a trumping relation if and only if

- every agreement in G is trumped only by agreements in B;
- every agreement in B is trumped by some agreement in G.

For a consistent partition $\{G, B\}$, we call the elements in G *good agreements* and those in B *bad agreements* relative to the trumping relation, respectively. Consistent partition means the following situation. First a good agreement cannot be trumped by any other good agreement, i.e. there is no trumping relation among good agreements. Secondly, a bad agreement must be trumped by some good agreement, i.e. even if a good agreement is trumped by a bad agreement, there exists another good agreement which trumps the bad agreement.

For the CP-trumping relation, a unique consistent partition is guaranteed (Blume and Sobel (1995, Proposition 1, p. 366)). Then they define the following equilibrium with the consistent partition.

Definition 4.2. An equilibrium (σ, α) is *communication-proof* if and only if $(\sigma, \alpha; \pi)$ is a good agreement relative to the CP-trumping relation.

The existence of communication-proof equilibria is also guaranteed by Proposition 2 in Blume and Sobel [1, p. 368]. We also define the R-proof equilibrium by substituting the R-trumping relation for the CP-trumping relation. These definitions may induce unnatural results because of the partial use of stable sets. Indeed, according to the definition of the consistent partition, the

whole set of good agreements can be stable against bad agreements. However, the agreements of the equilibria are a subset of the good agreements. Then, when a communication-proof or R-proof equilibrium is trumped by a bad agreement, there are no good agreements which trump the bad agreement in the set of equilibria. While the whole use of good agreements keeps the stability in the stable set, the partial use does not keep the stability any longer. Therefore, in the next section, we define the " the suggestive dominant equilibrium" without the consistent partition.

Before proceeding to the next section, we exemplify the instability due to the partial use of stable sets in Example 2.1. We has shown three Bayesian Nash equilibria E_0, E_1, E_2, and another agreement $\mathcal{A}''$. In this game we can have one more agreement $\mathcal{A}' = (\sigma', \alpha', p')$ as follows:

$$\sigma'(m|\ t) = \begin{cases} 1 & \text{for } m_1 \in M \text{ at } t_1, t_3 \in T \\ 1/2 & \text{for } m_1, m_2 \in M \text{ at } t_2 \in T \\ 0 & \text{otherwise;} \end{cases} \qquad \alpha'(a|\ m) = \begin{cases} 1 & \text{for } a_1 \in A \text{ at } m_1 \in M \\ 1 & \text{for } a_2 \in A \text{ at } m \neq m_1 \\ 0 & \text{otherwise;} \end{cases}$$

$$p'(t) = (p'(t_1), p'(t_2), p'(t_3), p'(t_4)) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0).$$

Among these agreements, $\mathcal{A}''$ achieves the highest expected payoff and then it is a good agreement. Since both $\mathcal{A}'$ and E_2 are CP-trumped and R-trumped by $\mathcal{A}''$ at message m_1, these two are bad agreements. Moreover, since E_0 is trumped by E_1 and E_1 is not trumped by any other agreements than $\mathcal{A}'$, E_2 is a good agreement. Then E_1 is both a communication-proof and R-proof equilibrium. It is, however, doubtful that this equilibrium is unstable because $\mathcal{A}''$ is also a good agreement and E_1 is trumped by $\mathcal{A}'$. Furthermore, both the Sender at every type and the Receiver could get higher expected payoffs in E_2 than those in E_1. Nevertheless, Why is E_1 preferred to E_2? We think it might be due to the partial use of the stable sets. In the following we define our equilibrium concept in order to improve this problem.

4.2. Suggestive dominant equilibrium. We now define the equilibrium based on the S-domination as follows.

Definition 4.3. A Bayesian Nash equilibrium (σ, α) is a *suggestive dominant* equilibrium if there is no message such that (σ, α, π) is S-dominated by any agreement.

In contrast to both the communication-proof and R-proof equilibria, the suggestive dominant equilibrium considers the maximal elements of all the agreements. To consider whether an equilibrium (σ, α) is suggestive dominant, we don't need consider all agreements, but only need to examine agreements (σ', α', p') such that $p' = \beta(m, \pi, \sigma)$ for $m \in M(\sigma, \pi)$. Moreover, our main theorem guarantees the existence of an S-dominant equilibrium as follows:

Theorem 4.4 (Shirataki and Ishikawa(2005)). *There exists a suggestive dominant equilibrium in any chap-talk game.*

We here give a sketch of the proof for the theorem. Now suppose that a Bayesian Nash equilibrium $\mathcal{A} := (\sigma, \alpha, \pi)$ is S-dominated by $\mathcal{A}' = (\sigma', \alpha', p')$ at M^*. In this proof, we suppose $M(\sigma, \pi) \cap M(\sigma', p') = \emptyset$. Note that this assumption does not lose the generality because many messages achieve the same equilibrium outcome in a cheap-talk game. Then we construct another agreement $\mathcal{A}^*$ by using both $\mathcal{A}$ and $\mathcal{A}'$. Then we consider the other pair of probability distributions $\mathcal{A}^* = (\sigma^*, \alpha^*, \pi)$ constructed as follows: For each $t \in T$,

$$\sigma^*(m\mid t) = \begin{cases} \sigma'(m\mid t) \displaystyle\sum_{m^* \in M^*} \sigma(m^*\mid t) & \text{if } m \in M(\sigma', p') \\ 0 & \text{if } m \in M^* \\ \sigma(m\mid t) & \text{otherwise;} \end{cases}$$

$$\alpha^*(m) = \begin{cases} \alpha'(m) & \text{if } m \in M(\sigma', p') \\ \alpha(m) & \text{otherwise.} \end{cases}$$

First we prove that $\mathcal{A}^*$ is a Bayesian Nash equilibrium and second that $\mathcal{A}^*$ is not S-dominated by $\mathcal{A}'$. If $\mathcal{A}^*$ is S-dominated by another agreement, we repeat the construction of an agreement. Finally we show that the repetitive construction is completed in the finite steps.

4.3. Efficiency of S-dominant equilibrium messages. The suggestive dominant equilibrium is a maximal element which is not S-dominated by any agreements at any message. Therefore the Sender follows the equilibrium agreement even when he has an additional communication. We also show below that the suggestive domination achieves a Pareto improvement. Therefore the agreement of the equilibrium is suggestive for the Receiver.

Let us consider again two agreements $\mathcal{A}, \mathcal{A}'$ such that $\mathcal{A} = (\sigma, \alpha, \pi)$ is S-dominated by $\mathcal{A}' = (\sigma', \alpha', p')$ at M^*. We also consider the other agreement $\mathcal{A}^*$ as constructed above. We can find that $\mathcal{A}^*$ is constructed from $\mathcal{A}'$ for the type such that $U_S(\sigma, \alpha\mid t) \leq U_S(\sigma', \alpha'\mid t)$ and from $\mathcal{A}$ for the type such that $U_S(\sigma, \alpha\mid t) \leq U_S(\sigma', \alpha'\mid t)$ Then the following proposition guarantees the higher or equal expected payoff for the Receiver as follows:

Proposition 4.5. *The constructed agreement $\mathcal{A}^* = (\sigma^*, \alpha^*, \pi)$ guarantees that the expected payoffs of both the players are higher or equal in comparison with $\mathcal{A} = (\sigma, \alpha, \pi)$.*

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