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The emergence of economic fluctuations and the possibility of economic slumps: Two sticky-price models

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Abstract The present paper summarizes the results obtained in Yoshida (2004, 2005). These papers develop simple monetary optimizing models with sticky prices. We show the possibility of economic slumps with liquidity trap and the emergence of persistent economic fluctuations. These results imply that the rate of money growth has much influence on real economic activity in a sticky-price environment.

1 Introduction

In this paper we consider an important topic in monetary economics: we shall examine the question whether the nominal money supply has a real effect or not. If real economic activity is independent of the rate of money growth, money is said to be "superneutral." The seminal work by Sidrauski (1967) shows that the superneutrality of money is valid in the long-run steady state using the money-in-the-utility-function (MIUF) approach. After his contribution, the validity of the Sidrauski result has been reexamined from many different viewpoints.

The present paper summarizes the results obtained in Yoshida (2004, 2005). These papers departs from the above literature by considering a sticky-price environment. As pointed out by Ball and Mankiw (1994, p. 127), prominent twentieth-century economists such as John Maynard Keynes, Milton Friedman, Franco Modigliani, and James Tobin believe that price stickiness plays an important role in explaining economic fluctuations.

The organization of the present paper is as follows. Section 2 explains the new Keynesian Phillips curve. The analytical framework is presented in Section 3. Section 4 considers a monetary optimizing model with the backward-looking Phillips curve. Section 5 investigates a model with the core-inflation augmented Phillips curve. Section 5 concludes the major findings of our analysis.

2 New Keynesian Phillips curve

We introduce price stickiness by assuming the new Keynesian Phillips curve, which possesses essentially the same properties as the expectations-augmented Phillips curve of Friedman and Phelps. Several authors have derived the new Keynesian Phillips curve on the basis of the micro-foundations. For example, Rotemberg (1982) has formulated the quadratic price adjustment cost approach and Taylor (1980) and Calvo (1983) have developed the staggered contracts approach. The two approaches share two common features. The first feature is that a high level of aggregate demand is associated with a high rate of inflation. The second is that the expected rate of inflation $E_t(\pi_{t+1})$ affects the current rate of inflation π_t. The following new Keynesian Phillips curve is derived:

$$\pi_t = E_t(\pi_{t+1}) + \beta(c_t - y^e),\ \beta > 0 \tag{1}$$

where c_t represents consumption in period t, and y^e is the efficient level of output, which is obtained when the existing capital stock is fully utilized.

Three types of specification are considered. The first specification is employed by Calvo (1983) and Buiter and Panigirtzoglou (2003):

$$\pi_t = \pi_{t+1} + \beta(c_t - y^e), \tag{2}$$

which is referred to as a forward-looking specification with rational expectations. This specification is derived by setting $E_t(\pi_{t+1}) = \pi_{t+1}$ in (1).

The second is a purely backward-looking specification:

$$\pi_t = \pi_{t-1} + \beta(c_t - y^e), \tag{3}$$

which is obtained by setting $E_t(\pi_{t+1}) = \pi_{t-1}$ in (1). In Fuhrer (1997) he emphasizes the importance of backward-looking component in the Phillips curve to produce the empirically suitable dynamics.

The third is the core-inflation augmented specification. In this specification we tacitly assume that the government can control people's inflation expectations in the form of $E_t(\pi_{t+1}) = \theta$, where θ is the growth rate of nominal money supply. The specification can be written as

$$\pi_t = \theta + \beta(c_t - y^e). \tag{4}$$

Since the macroeconomic implications under the forward-looking specification has been analyzed by Calvo (1983), we shall focus on the backward-looking specification and the core-inflation augmented specification.

Before discussing the dynamic properties of our model, we shall present the analytical framework in the next section.

3 The model

Here we consider a simple model with three agents: a representative producer; a representative consumer; and the government, and three commodities: physical output; equities; and money.

3.1 The firm

The firm has an efficient level of production, y^e, which is attained when the capital stock is fully utilized. However, there is no guarantee that aggregate demand always equals to the efficient level of output. Generally speaking, the firm controls the utilization rate of capital stock according to the state of economy. If aggregate demand exceeds the efficient output level, the firm produces the corresponding level of output by increasing the utilization rate of capital stock. In the opposite case, the firm meets aggregate demand by decreasing the rate of utilization. We assume that this adjustment process is so rapid that the output market equilibrium is always realized at each moment in time. Thus, the level of output is

$$y_t^s = D_t, \tag{5}$$

where D_t is the level of aggregate demand.

Next, we consider the financial structure of the firm. We assume that the firm issues no additional equities, so that the number of equities is fixed:

$$E_t^s = E = \text{const.}$$

Furthermore, we assume that all profits are paid out as dividends to stockholders in the following form:

$$d_t = y_t^s / E, \tag{6}$$

where d_t is real dividends per equity.

3.2 The consumer

The consumer's lifetime utility depends on consumption c and real money balances m; the representative consumer's utility function is given by

$$\int_0^\infty [\ln(c_t^d) + v(m_t^d)] \exp(-\rho t) dt, \tag{7}$$

where ρ is the subjective rate of time preference. The function $v(m)$ is strictly increasing, strictly concave, differentiable on the open interval $(0, \infty)$, and bounded. Furthermore, it satisfies the Inada conditions: $\lim_{m \to 0} v'(m) = \infty$, $\lim_{m \to \infty} v'(m) = 0$.

The consumer holds two assets: money and equities. The instantaneous budget constraint is expressed in real terms by

$$\dot{a} = ra - c^d + \tau - (r + \pi) m^d, \tag{8}$$

where a is the nominal financial wealth, τ is real lump-sum transfers from the government, and m is real money balances.

To avoid the consumer from running a Ponzi game, the following condition must be obeyed:

$$\lim_{s\to\infty} a_s \exp\left(-\int_0^s r_v dv\right) = 0. \tag{9}$$

This condition is referred to as the no-Ponzi-game condition.

The consumer chooses sequences for c, m, and a so as to maximize (7) subject to (8) and (9). The optimality conditions for the consumer problem are

$$v'(m^d)c^d = r + \pi, \tag{10}$$

$$\dot{c}^d = (r - \rho)c^d. \tag{11}$$

3.3 The government

For simplicity, we assume that the task of the government is limited to maintaining a constant growth rate of nominal money supply:

$$\dot{M}_t^s / M_t^s = \theta, \tag{12}$$

where M^s denotes the nominal money supply. The newly issued money is distributed to the household. The government budget constraint is given by

$$\dot{M}_t^s = p_t \tau_t, \tag{13}$$

or

$$\dot{m}_t^s = \tau_t - \pi_t m_t^s, \tag{14}$$

where $m^s = M^s/p$.

3.4 Perfect foresight equilibrium

So far, we have considered three markets: the output market, the money market, and the equity market. Because of Walras's law, only two of these markets are independent. Thus, one of them can be eliminated. Henceforth, our analysis focuses on the output market and the money market.

The perfect foresight equilibrium path is defined as a situation in which (i) the planned demands for output and money equal to the corresponding supplies and (ii) all expected values are correctly realized. The output and the money markets clearing conditions require

$$c^d = y^s, \tag{15}$$

$$m^d = m^s. \tag{16}$$

Henceforth, we shall suppress the subscript on the variables c and m since we focus on equilibrium quantities.

The equilibrium dynamics is expressed as the following reduced-form equations:

$$\dot{c} = [(v'(m)c - \pi - \rho]c, \tag{17}$$

$$\dot{m} = (\theta - \pi)m, \tag{18}$$

and the no-Ponzi-game condition.

4 Backward-looking Phillips curve

In this section we study the equilibrium dynamics under the backward-looking Phillips curve. For the sake of later discussion, we rewrite (3) in the continuous-time form:

$$\dot{\pi} = \beta(c - y^e). \tag{19}$$

Equations (17), (18), and (19) constitutes a complete system with three endogenous variables (c, m, π). The long-run steady state is defined as a set of constant variables (c_*, m_*, π_*) satisfying

$$v'(m_*)c_* = \theta + \rho, \tag{20}$$

$$c_* = y^e, \tag{21}$$

$$\pi_* = \theta. \tag{22}$$

From the above equations, we can see the superneutrality of money in the long-run steady state; the steady state level of consumption is independent of money growth. We should furthermore note that the acceleration of money supply raises the inflation rate and thereby lowers the amount of real money balances demanded, namely

$$m_* = m_*(\theta),\ dm_*(\theta)/d\theta = 1/(v''(m_*)c_*) < 0. \tag{23}$$

Linearizing (17), (18), and (19) around the steady state leads to the following Jacobian matrix:

$$\begin{bmatrix} v'c_* & v''c_*^2 & -c_* \\ 0 & 0 & -m \\ \beta & 0 & 0 \end{bmatrix} \tag{24}$$

The corresponding characteristic equation of this system, $P(\lambda) = \lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0$, has the following coefficients:[1]

$$b_1 = -\text{trace}J_1 = -v'(m_*)c_* < 0, \tag{25}$$

[1] The coefficients of the characteristic equation can be computed through the elements of the Jacobian matrix. On this point see, for example, Asada and Semmler (1995, p. 633) and Gandolfo (1997, pp. 247 – 248).

$$\begin{aligned} b_2 &= \text{sum of all second-order principal minors of } J_1 \\ &= \beta c_* > 0, \end{aligned} \tag{26}$$

$$b_3 = -\det J_1 = \beta v''(m_*) m_* {c_*}^2 < 0, \tag{27}$$

$$\Delta := b_1 b_2 - b_3 = v'(m_*)(\eta(m_*) - 1)\beta {c_*}^2, \tag{28}$$

where J_1 is the Jacobian matrix and $\eta(m_*)$ represents the coefficient of relative risk aversion evaluated at the steady state m_*;

$$\eta(m_*) = -\frac{m_* v''(m_*)}{v'(m_*)} > 0. \tag{29}$$

By utilizing $\eta(m_*)$, we can characterize the equilibrium dynamics of our model:

Proposition 1 *If $\eta(m_*) < 1$, then the equilibrium path is the steady state itself. On the other hand, if $\eta(m_*) > 1$, then there exists a continuum of perfect-foresight equilibria.*

However, this assertion is not always the case, because our analysis is limited to the neighborhood of the steady state. Let us now examine further this point, which is our main concern. Concentrating on the parameter θ, we can prove the following proposition.

Proposition 2 *Suppose that there exists a critical value, $\theta = \theta_H$, such that $\eta(m_*(\theta_H)) = 1$. Then the dynamic system undergoes a Hopf bifurcation, which generates persistent fluctuations.*

Proposition 2 establishes the existence of limit cycles through the Hopf bifurcation theorem.

5 Core-inflation augmented Phillips curve

In this section we analyze a model with the core-inflation augmented Phillips curve. Using (4), (17), and (18), we can obtain the following two-dimensional system:

$$\dot{c} = [(v'(m)c - \beta(c - y^e) - \theta - \rho]c, \tag{30}$$

$$\dot{m} = -\beta(c - y^e)m. \tag{31}$$

The long-run steady state ($\dot{m} = \dot{c} = 0$) of the above dynamics is given by the intersection of the locus $\dot{c} = 0$:

$$c = \frac{(\beta y^e - \theta - \rho)}{\beta - v'(m)} := h(m), \tag{32}$$

and the locus $\dot{m} = 0$:

$$c = y^e. \tag{33}$$

By solving (32) and (33), we can obtain the same long-run steady state as in the backward looking specification.

$$v'(m^*)y^e = \theta + \rho, \tag{34}$$

$$c^* = y^e. \tag{35}$$

In the following, we assume that

$$\beta y^e > \theta + \rho > 0.$$

Before turning to the intensive investigations of the equilibrium dynamics, it will be helpful to define a critical point $m = m^c$ such that $v'(m^c) = \beta$.

The locus $\dot{c} = 0$ depicted by $c = h(m)$ has negative values for $m < m^c$. Thus our attention is restricted to the region $m > m^c$ when $c = h(m)$ is considered. The function $c = h(m) = (\beta y^e - \theta - \rho)y^e/(\beta - v'(m))$ has the following properties for $m > m^c$: (i) the derivative $h'(m)$ is negative: $h'(m) = (\beta y^e - \theta - \rho)v''(m)/[\beta - v'(m)]^2 < 0$, (ii) $\lim_{m \to m^{c+}} h(m) = +\infty$, and (iii) $\lim_{m \to +\infty} h(m) = y^e - (\theta + \rho)/\beta (:= \tilde{c}) < y^e$. These properties assert that the steady state m^* is uniquely determined.

We can find that there exist two types of dynamic trajectories. First, we can find the full-utilization path where the level of consumption converges to the full-utilization level y^e and the level of real money balances approaches m^*. Secondly, we can observe the recession paths where the level of consumption approaches $\tilde{c}$ asymptotically and the level of real money balances diverges to infinity. In this case the nominal interest rate declines to zero, since $r + \pi = v'(m)c$. It is obvious that a liquidity trap arises in this monetary economy. Furthermore the economy experiences deflation; $\pi = -\rho < 0$.

By checking the no-Ponzi-game condition, we have come to the following proposition:

Proposition 3 *Suppose that $\rho + \theta < \beta y^e$. Then:*

(i) *If $\theta > 0$, then the full-utilization path, which converges to the steady state point (m^*, y^e), is the unique perfect foresight path.*

(ii) *If $\theta < 0$, then the perfect foresight path exhibits indeterminacy: any recession path as well as the full-utilization path is a perfect foresight path. On the recession paths, we can observe liquidity traps with deflation.*

6 Concluding remarks

In this paper, we have presented two monetary optimizing model with sticky prices. First, we examine the model with the backward-looking Phillips curve. Our main finding is the existence of perpetual and endogenous fluctuations around the long-run steady state. Secondly, we consider the model with the core-inflation augmented Phillips curve. In this case we show that the emergence of recessionary paths with liquidity traps.

Finally I touch the possibility of chaotic fluctuations in the model with the backward-looking Phillips curve. If we prove the existence of the trajectory which, leaving the steady state through the unstable manifold, is bent backward near the boundary and eventually enters the stable manifold of the steady state, we can apply the Silnikov theorem to our model. Although this study would be difficult, trying to solve this problem by means of numerical simulations seems to be a promising topic for future research.

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