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Asymptotic behavior of solutions for a fluid dynamical model of semiconductor equation

Dedicated to Professor Tai-Ping Liu on his 60th birthday

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1 Introduction

In this short paper, we shall show our recent results on the asymptotic stability of the stationary solution to a simple one-dimensional fluid dynamical model of semiconductor which describes the motion of electrons along a thin chanel connecting the source and drain in a n-type uniporlar transistor like MOS-FET (cf.[1]):

$$(1.1)\qquad \begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + p(\rho))_x = \rho\phi_x - \frac{1}{\tau}\rho u, \quad (x,t) \in (0,1)\times[0,\infty), \\ \phi_{xx} = \rho - D, \end{cases}$$

where $\rho = \rho(t,x) > 0$ is the density of electrons, $u = u(t,x)$ is the velocity, $\phi = \phi(t,x)$ is the electrostatic potential, $D = D(x)$ is a given doping profile (positive charge density of impurities), $p = p(\rho)$ is the pressure, and τ is the relaxation time. We assume the pressure has the form $p(\rho) = \rho^\gamma (\gamma \geq 1)$ and $\tau = 1$ for simplicity. As for the initial boundary value problem to (1.1) with the initial data

$$(1.2)\qquad (\rho, u)(x,0) = (\rho_0, u_0)(x), \quad x \in (0,1),$$

and the Dirichlet type boundary conditions

$$\rho(0,t)=\rho_1,\quad \rho(1,t)=\rho_2,\quad t\geq 0, \tag{1.3}$$
$$\phi(0,t)=0,\quad \phi(1,t)=\Phi_1,\quad t\geq 0,$$

Degond, Markowich('93,[2]) studied the existence and uniqueness of the subsonic stationary solution, then H.-L.Li, P.Markowich and M.Mei('03,[5]) showed the asymptotic stability. However they showed the stability under the various smallness conditions ($|\rho_1-\rho_2|, |\rho_0-\rho_1|, |\Phi_1|, |D-\rho_1| \ll 1$) ,that is, under the situation everything are very close to the constant states $(\hat{\rho},\hat{u},\hat{\phi}) = (\overline{\rho},0,0)(\overline{\rho} := \rho_1=\rho_2)$. In particular, since actual data of semiconductor often read the ratio of the maximum of the doping profile D to the minimum is of order 10^2, a smallness (flatness) condition to D as above seems almost unreal in a physical and engineering point of view. We also note that even in all recent researches on quantum fluid dynamical models(cf.[4][3]), the stability of stationary solutions is studied under similar strict smallness conditions. In this paper, although so far we have not had satisfactory results on the problem (1.1)(1.2)(1.3), we instead treat the problem (1.1) in R with the periodic initial and boundary conditions

$$(\rho,u)(x,0)=(\rho_0,u_0)(x),\quad x\in R \tag{1.4}$$
$$(\rho,u,\phi)(t,x+1)=(\rho,u,\phi)(t,x),\quad x\in R,\ t\geq 0,$$

and investigate the asymptotic stability of the stationary solution under as less smallness conditions in particular on the doping profile D as possible. We assume that

$$(\rho_0,u_0)\in H^2_{period},\quad \inf_{x\in R}\rho_0(x)>0,\quad \int_0^1 \rho_0(x)\,dx=1, \tag{1.5}$$

$$D\in H^1_{period},\quad \inf_{x\in R} D(x)>0,\quad \int_0^1 D(x)\,dx=1. \tag{1.6}$$

Here H^k_{period} is the usual k-th order Sobolev space of the periodic functions with the period 1, and denote the norm by $\|\cdot\|_k$. First we should note that for any doping profile D satisfying (1.6) the unique sationary solution corresponding to the problem (1.1)(1.4) is easily obtained in the form $(\hat{\rho},\hat{u},\hat{\phi})=(\hat{\rho},0,\hat{\phi})(x)$. Then we have

Theorem 1.1. *Any smooth global solution in time of the initial boundary value problem (1.1)(1.4) satisfies the following:*
i) If $\gamma \geq 2$, there exist positive constants ν and E_0 such that

$$\int_0^1 \rho u^2 + (\rho - \hat{\rho})^2 + |(\phi - \hat{\phi})_x|^2 \, dx \leq E_0 \exp\{-\nu t\}, \quad t \geq 0.$$

ii) If $2 > \gamma \geq 1$ and $M := \sup_{x,t} \rho(t,x) < +\infty$, then there exit positive constants $\nu(M)$ and $E_0(M)$ such that

$$\int_0^1 \rho u^2 + (\rho - \hat{\rho})^2 + |(\phi - \hat{\phi})_x|^2 \, dx \leq E_0 \exp\{-\nu t\}, \quad t \geq 0.$$

Here we should note that the above *a priori* estimates are not enough to insure the existence of global smooth solution in time. On the other hand, by the arguments in [6]('98), we can not expect the existence of the global smooth solution for general large data. However we still have a possibility to have the global smooth solution around the stationary solution even if the doping profile D is very large. An answer is the following.

Theorem 1.2. *Suppose (1.5) and (1.6). Then there exist positive constants δ, C, and $\beta > 0$ such that if $\|(\rho_0 - \hat{\rho}, u_0)\|_2 \leq \delta$, the initial boundary value problem (1.1)(1.4) has a unique global solution in time satisfying*

$$(\rho, u) \in C^0([0,\infty); H^2_{period}), \quad \phi \in C^0([0,\infty); H^3_{period}),$$

and furthermore

$$\|(\rho - \hat{\rho}, u)(t)\|_2 + \|(\phi - \hat{\phi})(t)\|_3 \leq C\|(\rho_0 - \hat{\rho}, u_0)\|_2 \exp\{-\beta t\}, \quad t \geq 0.$$

2 Stationary solution

In this section, we study the basic properties of the smooth solution of the problems (1.1)(1.4), and the corresponding stationary solution. In what follows, let us assume $\gamma > 1$ for simplicity, since the case $\gamma = 1$ can be treated by properly taking limit $\gamma \to 1$. First, we easily have

Lemma 2.1. *The smooth solution of the problem (1.1)(1.4) satisfies*

$$\int_0^1 \rho(t,x) \, dx = \int_0^1 \rho_0(x) \, dx = 1, \quad t \geq 0, \tag{2.1}$$

$$(2.2) \qquad \rho(t,x) > 0, \quad x \in R, \ t \geq 0,$$

$$(2.3) \qquad \int_0^1 u(t,x)\,dx \leq e^{-t} \int_0^1 u_0(x)\,dx, \quad t \geq 0.$$

Proof The equality (2.1) is shown by integrating the first equation of (1.1) with respect to x. The positivity of the solution (2.2) is proved by a property of the ordinary differential equation for $\rho(t, x(t))$ along the characteristic curve $x'(t) = u(t, x(t))$. For (2.3), we can rewrite the second equation of (1.1) as

$$(2.4) \qquad u_t + (\frac{1}{2}u^2 + \frac{\gamma}{\gamma-1}\rho^{\gamma-1})_x = \phi_x - u$$

then the integration of (2.4) with respect to x easily implies (2.3). Thus the proof is completed.

By the above Lemma 2.1, we can see the natural stationary solution $(\hat{\rho}, \hat{u}, \hat{\phi})(x)$ corresponding to the problem (1.1)(1.4) is described the following system

$$(2.5) \qquad \begin{cases} (\hat{\rho}\hat{u})_x = 0, \\ (\hat{\rho}\hat{u}^2 + \hat{\rho}^\gamma)_x = \hat{\rho}\hat{\phi}_x - \hat{\rho}\hat{u}, \quad x \in R, \\ \hat{\phi}_{xx} = \hat{\rho} - D \end{cases}$$

with the boundary condition

$$(2.6) \qquad (\hat{\rho}, \hat{u}, \hat{\phi})(x+1) = (\hat{\rho}, \hat{u}, \hat{\phi})(x), \qquad x \in R,$$

under the additional conditions

$$(2.7) \qquad \int_0^1 \hat{\rho}\,dx = 1, \quad \inf_x \hat{\rho} > 0, \quad \int_0^1 \hat{u}\,dx = 0.$$

Here we also assume that the doping profile D satisfies (1.6). By the first equation of (2.5) and the conditions (2.7), we can easily have $\hat{u} = 0$ and

$$(2.8) \qquad \begin{cases} (\frac{\gamma}{\gamma-1}\hat{\rho}^{\gamma-1})_x = \hat{\phi}_x, \\ \hat{\phi}_{xx} = \hat{\rho} - D. \end{cases}$$

Then the boundary value problem (2.5)(2.6) is virtually reduced to the problem only for $\hat{\rho}$:

$$(2.9) \qquad \begin{cases} -(\frac{\gamma}{\gamma-1}\hat{\rho}^{\gamma-1})_{xx} + \hat{\rho} = D, \quad x \in R, \\ \hat{\rho}(x) = \hat{\rho}(x+1), \quad x \in R. \end{cases}$$

Thus, by the arguments of the Maximum Principle and the Shauder's Fixed Point Theorem as in [2], we can have the following lemma. We omit the details.

Lemma 2.2. *The boundary value problem (2.5)(2.6)(2.7) has the unique solution* $(\hat{\rho}, \hat{u}, \hat{\phi})$ *such that* $\hat{\rho} \in H^2_{period}$, $\hat{u} = 0$, $\hat{\phi} \in H^3_{period}$ *and*

$$\underline{D} \leq \hat{\rho}(x) \leq \overline{D}, \quad x \in R \tag{2.10}$$

where

$$\underline{D} := \inf_{x \in R} D(x) > 0, \quad \overline{D} := \sup_{x \in R)} D(x).$$

3 Asymptotic behavior for large initial data

In this section, we show the proof of the Theorem 1.1, that is, show an asymptotic behavior of the smooth solution (ρ, u, ϕ) of the initial boundary value problem (1.1)(1.4) toward the stationary solution $(\hat{\rho}, \hat{u}, \hat{\phi})$, which holds without any smallness conditions both on the initial data and the doping profile. We first note that the equation for u is written in the form

$$u_t + uu_x + \frac{\gamma}{\gamma - 1}(\rho^{\gamma-1} - \hat{\rho}^{\gamma-1})_x - (\phi - \hat{\phi})_x + u = 0 \tag{3.1}$$

which follows from (2.4) and (2.8). Now we multiply the equation (3.1) by ρu and integrate with respect to x. Then using the relation

$$\rho_t = (\phi - \hat{\phi})_{xxt} \tag{3.2}$$

which is derived by the third equations of (1.1), and taking careful integration by parts, we can have the first basic energy estimate

Lemma 3.1. *It holds that*

$$\frac{d}{dt}\left\{ \int_0^1 \frac{1}{2}\rho u^2 + G(\rho, \hat{\rho}) + \frac{1}{2}|(\phi - \hat{\phi})_x|^2 \, dx \right\} + \int_0^1 \rho u^2 \, dx = 0, \quad t \geq 0 \tag{3.3}$$

where

$$G(\rho, \hat{\rho}) := \frac{\gamma}{\gamma - 1} \int_{\hat{\rho}}^{\rho} (\xi^{\gamma-1} - \hat{\rho}^{\gamma-1}) \, d\xi.$$

To obtain the second basic energy estimate, we in turn use the equation for u written in the another form

$$(\rho u)_t + (\rho u^2 + \rho^\gamma - \hat{\rho}^\gamma)_x - (\rho\phi_x - \hat{\rho}\hat{\phi}_x) + \rho u = 0 \tag{3.4}$$

which follows from (1.1) and (2.5) with $\hat{u}=0$. We multiply the equation (3.4) by $-(\phi-\hat{\phi})_x$ and integrate with respect to x. Then using the relation (3.2) again and carrying careful integration by parts, we can have the estimate

$$\frac{d}{dt}\left\{\int_0^1 -(\rho u)(\phi-\hat{\phi})_x+\frac{1}{2}|(\phi-\hat{\phi})_x|^2\,dx\right\}-\int_0^1 |(\phi-\hat{\phi})_{xt}|^2\,dx+ \tag{3.5}$$

$$+\int_0^1 \rho u^2(\rho-\hat{\rho})+(\rho^\gamma-\hat{\rho}^\gamma)(\rho-\hat{\rho})+\frac{1}{2}(\frac{\rho+\hat{\rho}}{2}+D)(\phi-\hat{\phi})_x^2\,dx=0.$$

Although the sign of the integral $\int -(\phi-\hat{\phi})_{xt}^2\,dx$ in (3.5) seems a problem, it turns out that the important inequality

$$\int_0^1 \rho^2u^2-|(\phi-\hat{\phi})_{xt}|^2\,dx=(\int_0^1 \rho u\,dx)^2\geq 0 \tag{3.6}$$

holds, which plays an essential role in the estimates in this section. In fact, the inequality (3.6) is proved as follows. By the formula (3.2) and the first equation of (1.1), we have

$$(\rho u+(\phi-\hat{\phi})_{xt})_x=0, \tag{3.7}$$

which shows that we can set $\rho u+(\phi-\hat{\phi})_{xt}=k(t)$ for a function $k(t)$ depending only on t. Then we can see

$$k(t)=\int_0^1 \rho u\,dx,$$

and can prove (3.6) by

$$\begin{aligned}\int_0^1 \rho^2u^2-|(\phi-\hat{\phi})_{xt}|^2\,dx &= \int_0^1 \Big(k(t)-(\phi-\hat{\phi})_{xt}\Big)^2-|(\phi-\hat{\phi})_{xt}|^2\,dx\\ &= k(t)^2-2k(t)\int_0^1(\phi-\hat{\phi})_{xt}\,dx\\ &= k(t)^2.\end{aligned}$$

Hence, applying (2.10) and (3.6) to (3.5), we have the following second basic energy estimate.

Lemma 3.2. *It holds that*

$$\frac{d}{dt}\left\{\int_0^1 -(\rho u)(\phi-\hat{\phi})_x+\frac{1}{2}|(\phi-\hat{\phi})_x|^2\,dx\right\}-\int_0^1 \hat{\rho}\rho u^2\,dx+ \tag{3.8}$$

$$+\int_0^1 (\rho^\gamma - \hat{\rho}^\gamma)(\rho - \hat{\rho}) + \frac{3}{4}\underline{D}\,|(\phi - \hat{\phi})_x|^2\,dx \le 0, \quad t \ge 0.$$

We are now on the stage where we can prove the desired estimates. For any positive constant λ, multiplying (3.3) by λ and adding it to (3.8), we have

$$\frac{d}{dt}E(t) + Q(t) \le 0, \quad t \ge 0 \tag{3.9}$$

where

$$E(t) = \int_0^1 \frac{1}{2}\rho u^2 + G(\rho,\hat{\rho}) + \frac{1+\lambda}{2}|(\phi - \hat{\phi})_x|^2 - \lambda(\rho u)(\phi - \hat{\phi})_x\,dx, \tag{3.10}$$

$$Q(t) = \int_0^1 (1 - \lambda\hat{\rho})\rho u^2 + \lambda(\rho^\gamma - \hat{\rho}^\gamma)(\rho - \hat{\rho}) + \frac{3}{4}\lambda\underline{D}\,|(\phi - \hat{\phi})_x|^2\,dx. \tag{3.11}$$

In order to estimates the terms in $E(t)$ and $Q(t)$, we need another two lemmas.

Lemma 3.3. *It holds that*

$$\left|\int_0^1 \rho u(\phi - \hat{\phi})_x dx\right| \le \int_0^1 (\frac{1}{2}\rho u^2 + \frac{1}{2}(\rho - \hat{\rho})^2)\,dx. \tag{3.12}$$

Proof By the equality (2.1), the Sobolev's lemma and the Poincare's inequality, we have

$$\begin{aligned}
\left|\int_0^1 \rho u(\phi - \hat{\phi})_x\,dx\right| &\le \int_0^1 \frac{1}{2}\rho u^2 + \frac{1}{2}\rho(\phi_x - \hat{\phi}_x)^2\,dx \\
&\le \frac{1}{2}\int_0^1 \rho u^2 dx + \frac{1}{2}\sup_x |\phi_x - \hat{\phi}_x|^2 \\
&\le \frac{1}{2}\int_0^1 \rho u^2 dx + \frac{1}{2}\int_0^1 |\phi_{xx} - \hat{\phi}_{xx}|^2\,dx \\
&= \frac{1}{2}\int_0^1 \rho u^2 dx + \frac{1}{2}\int_0^1 |\rho - \hat{\rho}|^2\,dx.
\end{aligned}$$

Thus the proof of the Lemma 3.3 is completed.

Lemma 3.4. *It holds the following:*
i) If $\gamma \ge 1$, there exists a constant $c > 0$ such that for any $\rho > 0$ and $\hat{\rho} \in [\underline{D}, \overline{D}]$,

$$G(\rho,\hat{\rho}) \le c(\rho^\gamma - \hat{\rho}^\gamma)(\rho - \hat{\rho}), \quad \frac{1}{2}(\rho - \hat{\rho})^2 \le c(\rho^\gamma - \hat{\rho}^\gamma)(\rho - \hat{\rho}). \tag{3.13}$$

ii) If $\gamma \geq 2$, there exists a constant $c > 0$ such that for any $\rho > 0$ and $\hat{\rho} \in [\underline{D}, \overline{D}]$,

$$G(\rho, \hat{\rho}) \geq c^{-1}(\rho - \hat{\rho})^2. \tag{3.14}$$

iii) If $\gamma \geq 1$ and $\sup_{x \in R, t \geq 0} \rho(t,x) =: M < +\infty$, then there exists a constant $c(M) > 0$ such that for any $\rho > 0$ and $\hat{\rho} \in [\underline{D}, \overline{D}]$,

$$G(\rho, \hat{\rho}) \geq c(M)^{-1}(\rho - \hat{\rho})^2. \tag{3.15}$$

The proof of the Lemma 3.4 is given by standard arguments of the Tayler's formula, so we omit it.

Now suppose that $\gamma \geq 2$ for simplicity, because the case where $\gamma \geq 1$ can be treated on the same line. Applying the Lemma 3.3 and 3.4 to (3.10) and (3.11), and choosing λ suitably small, we have

$$\int_0^1 \rho u^2 + (\rho - \hat{\rho})^2 + |(\phi - \hat{\phi})_x|^2 \, dx \leq C\, E(t), \quad t \geq 0 \tag{3.16}$$

and also

$$E(t) \leq \nu^{-1} Q(t), \quad t \geq 0 \tag{3.17}$$

for some positive constants C and ν. So we finally obtain

$$\frac{d}{dt} E(t) + \nu E(t) \leq 0, \quad t \geq 0. \tag{3.18}$$

Thus, by (3.16) and (3.18), we complete the proof of Theorem 1.1.

4 Asymptotic stability of stationary solution

In this section, we give a rough sketch of the proof of Theorem 1.2, that is, show the asymptotic stability of the stationary solution $(\hat{\rho}, 0, \hat{\phi})$. First, for any fixed stationary solution $(\hat{\rho}, 0, \hat{\phi})$ constructed in the section 2, we set

$$\psi = \rho - \hat{\rho}, \quad e = \phi - \hat{\phi}, \tag{4.1}$$

and rewrite the initial boundary value problem (1.1)(1.4) in terms of (ψ, u, e) as

$$\begin{cases} \psi_t + (\hat{\rho} + \psi) u_x + (\hat{\rho}_x + \psi_x) u = 0, \\ u_t + u u_x + \frac{\gamma}{\gamma - 1}((\hat{\rho} + \psi)^{\gamma - 1} - \hat{\rho}^{\gamma - 1})_x - e_x + u = 0, \\ e_{xx} = \psi, \quad (x, t) \in R \times [0, \infty), \end{cases} \tag{4.2}$$

with the initial data and boundary conditions

$$(4.3)\qquad \begin{aligned} &(\psi, u)(0,x) = (\psi_0, u_0)(x) := (\rho_0 - \hat{\rho}, u_0)(x), \quad x \in R, \\ &(\psi, u, e)(t, x+1) = (\psi, u, e)(t, x), \quad x \in R,\ t \geq 0. \end{aligned}$$

Then, for suitably small $(\psi_0, u_0) \in H^2_{period}$, we look for the global solution in time $(\psi, u) \in C^0([0,\infty); H^2_{period}), e \in C^0([0,\infty); H^3_{period})$ which stays in a neighborhood of

$$(4.4)\qquad \|(\psi, u)\|_2 \leq \bar{\epsilon}_0 := \underline{D}/4$$

and exponentially decays as the time goes to infinity. To do that, we use the typical arguments to construct the global solution by combining the local existence theorem together with the *a priori* estimate. As for the local existence, we omit the arguments, because the principal part of the system (4.2) for (ψ, u) is a symmetrizable hyperbolic one and so standard arguments on the Cauchy problem for quasilinear symmetric hyperbolic system are applicable to our case. The *a priori* estimate we need is as follows.

Proposition 4.1. *There exist positive constants $\epsilon_0(< \bar{\epsilon}_0)$, C, and β such that if $(\psi, u, e) \in C^0([0,T]; H^2_{period})^2 \times C^0([0,T]; H^3_{period})$ for some $T > 0$ is the solution of the initial boundary value problem (4.2)(4.3) and*

$$(4.5)\qquad \sup_{0\leq t\leq T} \|(\psi, u)(t)\|_2 \leq \epsilon_0,$$

then it holds

$$(4.6)\qquad \|(\psi, u)(t)\|_2 + \|e(t)\|_3 \leq C\|(\psi_0, u_0)\|_2 \exp\{-\beta t\}, \quad 0 \leq t \leq T.$$

To show the Proposition 4.1, in what follows, we assume that the initial boundary value problem (4.2)(4.3) has the solution

$$(\psi, u, e) \in C^0([0,T]; H^2_{period})^2 \times C^0([0,T]; H^3_{period})$$

for some $T > 0$ and also assume

$$\sup_{0\leq t\leq T} \|(\psi, u)(t)\|_2 \leq \overline{\epsilon_0}.$$

Then, using the equation (4.2) and (4.4), we can first show

Lemma 4.2. *There exist positive constants $\epsilon(< \bar{\epsilon}_0)$ and C such that if*

$$\sup_{0\leq t\leq T} \|(\psi, u)(t)\|_2 \leq \epsilon,$$

then it holds

$$\|(\psi_t, u_t)\|_1 \le C\|(\psi, u)\|_2, \quad 0 \le t \le T \tag{4.7}$$

and

$$\|(\psi, u)\|_2 + \|e\|_3 \le C(\|(\psi, u)\| + \|(\psi_t, u_t)\| + \|(\psi_{tt}, u_{tt})\|), \quad 0 \le t \le T. \tag{4.8}$$

By (4.7), we may assume $\|(\psi_t, u_t)\|_1$ also as small as we want in the process of *a priori* estimates, and by (4.8), it turns out that it is enough to obtain the exponential decay estimates for $\|(\psi, u)\| + \|(\psi_t, u_t)\| + \|(\psi_{tt}, u_{tt})\|$ to show the Proposition 4.1.

Next, by the arguments in the proof of Theorem 1.1, we can easily show that there exist positive constants C and β such that

$$\|(\psi, u, e_x)(t)\|^2 \le C\|(\psi_0, u_0)\|^2 \exp\{-\beta t\}, \quad 0 \le t \le T. \tag{4.9}$$

In order to obtain the estimates for the higher derivatives, we differentiate the system (4.1) with respect to t to have

$$\begin{cases} \psi_{tt} + ((\hat{\rho} + \psi)u_t)_x + (\psi_t u)_x = 0, \\ u_{tt} + u_t u_x + u u_{tx} + \gamma((\hat{\rho} + \psi)^{\gamma-2}\psi_t)_x - e_{xt} + u_t = 0, \\ e_{txx} = \psi_t. \end{cases} \tag{4.10}$$

As in the arguments in the proof of Theorem 1.1, we multiply the second equation of (4.10) by $((\hat{\rho} + \psi)u_t - \lambda e_{xt})$ and integrate it with respect to x. Noting the relation

$$\int_0^1 (\hat{\rho} + \psi)u_t((\hat{\rho} + \psi)^{\gamma-2}\psi_t)_x \, dx = \int_0^1 (\psi_{tt} + (\psi_t u)_x)(\hat{\rho} + \psi)^{\gamma-2}\psi_t \, dx,$$

we may carefully carry the integration by parts. Then, choosing the λ and ϵ properly small, we can prove

$$\|(\psi_t, u_t)(t)\|^2 \le C\|(\psi_0, u_0)\|_1^2 \exp\{-\beta t\}, \quad 0 \le t \le T \tag{4.11}$$

for some positive constants β and C in the same way as in the section 3. Similarly, after differentiating the system (4.10) with respect to t again and multiplying it by $((\hat{\rho} + \psi)u_{tt} - \lambda e_{xtt})$, we also can prove

$$\|(\psi_{tt}, u_{tt})(t)\|^2 \le C\|(\psi_0, u_0)\|_2^2 \exp\{-\beta t\}, \quad 0 \le t \le T \tag{4.12}$$

for some positive constants β and C. Thus, combining the estimates (4.8)(4.9)(4.11)(4.12), we can complete the proof of the Proposition 4.1, and then Theorem 1.2.

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