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Developments of Cartan Geometry and Related Mathematical Problems

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Developments of Cartan Geometry and Related Mathematical Problems

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Research Institute for Mathematical Sciences

Kyoto University, Kyoto, Japan

Preface

We had a RIMS Symposium entitled "Developments of Cartan Geometry and Related Mathematical Problems" at RIMS Kyoto University from October 24 to October 27, 2005. This is the proceedings of the symposium, in which we collect the papers contributed after the symposium by all the speakers.

The question *what is Cartan geometry?* may be first posed here. As well known by the Erlangen program, in 1870's Lie and Klein opened the vast area of geometries of homogeneous spaces, so-called Klein geometries. After half a century in 1920's Cartan introduced the notion of new geometries, *espaces généralisés*, more general than Klein geometry and Riemannian geometry, and playing with respect to the different homogeneous spaces same role as a Riemannian space plays with respect to an Euclidean space. In modern terminology, *un espace généralisé* is a curved space modeled after a homogeneous space, or a principal fibre bundle with a Cartan connection. This beautiful synthesis that Cartan achieved, however, has been long forgotten except for a few specialists.

Recently there are increasing interests to Cartan connections in studying various geometric structures, in particular, non-integrable distributions, CR-structures, geometries of differential equations, etc., with the viewpoint of nilpotent geometry and parabolic geometry. We might say it is these geometries related with Cartan connections that Cartan geometries mean.

But we would like to use "Cartan geometry" in the broader sense to signify the geometries which have developed after Cartan's geometric spirit, especially after the ideas behind his papers around 1899 - 1911. In fact, the work of Cartan of this period on Pfaff systems, infinite groups, and differential equations seems to be the core of his whole extensive work and containing deep insights and ideas still to be developed.

Our symposium was a good opportunity to bring together in the spirit of Cartan geometry many people hitherto working from different mathematical origins, in different domains, or in different countries. We wish that this volume not only be a testimony of the enthusiasm of exciting discussions in the symposium but also become a source of new developments of Cartan geometry.

Finally, I would like to express my sincere gratitude to the speakers and the participants for their own contributions to make the symposium successful and this volume rich in contents.

June, 2006
Tohru Morimoto

RIMS SYMPOSIUM
Developments of Cartan Geometry and Related Mathematical Problems

October 24 (Monday) - 27 (Thursday), 2005
Research Institute for Mathematical Science, Kyoto University

Program

October 24 (Monday)

9:30–10:20: **Robert Bryant (Duke Univ.)**
The geometry of real hypersurfaces in unimodular complex surfaces

10:30–11:20: **Michail Zhitomirskii (Israel Institute of Technology)**
Cartan prolongation, Legendrization and Monsterization

11:30–12:20: **Jiro Adachi (Hokkaido Univ.)**
Classification of horizontal loops in the standard Engel space

14:00–14:50: **Goo Ishikawa (Hokkaido Univ.)**
Extra singularities in geometric solutions to Monge-Ampère equations of three variables

15:00–15:50: **Nobutada Nakanishi (Gifu Keizai Univ.)**
On Nambu-Lie groups

16:00–16:50: **Hideya Hashimoto (Meijo Univ.)**
On some projective bundles of 6-dimensional submanifolds of octonions

October 25 (Tuesday)

9:30–10:20: **Soji Kaneyuki (Nihon Institute of Technology)**
Causal structures and symmetric spaces

10:30–11:20: **Toshiyuki Kobayashi (RIMS)**
Visible actions on complex manifolds and multiplicity-free representations

11:30–12:20: **Tomoyuki Arakawa (Nara Women's Univ.)**
Infinite-dimensional Lie algebras, vertex algebras and W-algebras

14:00–14:50: **Pawel Nurowski (Warsaw Univ.)**
Differential equations and conformal geometry

15:00–15:50: **Hajime Sato (Nagoya Univ.)**
Differential equations and Schwarzian derivatives

16:00–16:50: **Yoshinori Machida (Numazu Institute of Technology)**
Monge structures, Cartan distributions, and Goursat equations

October 26 (Wednesday)

9:30–10:20: **Peter Greiner (Univ. of Toronto)**
Subelliptic PDE's and subriemannian geometry

10:30–11:20: **Keisuke Ueno (Yamagata Univ.)**
Some asymptotic boundary behavior of a proper harmonic maps between Carnot spaces

11:30–12:20: **Chisato Iwasaki (Himeji Univ. of Technology)**
Symbolic calculus of pseudo-differential operators and curvature of manifolds

14:00–14:50: **Kenro Furutani (Tokyo Science Univ.)**
Heat kernel on nilpotent Lie groups

15:00–15:50: **Takao Akahori (Himeji Univ. of Technology)**
D-branes of A-type from the point of view of CR structures

16:00–16:50: **Sung Ho Wang (KIAS Seoul)**
1-rigidity of CR submanifolds in spheres

October 27 (Thursday)

9:30–10:20: **Andrei Agrachev (SISSA Trieste)**
Differential invariants of control systems: a variational approach

10:30–11:20: **Igor Zelenko (SISSA Trieste)**
On canonical frames for vector distributions

11:30–12:20: **Kazuhiro Shibuya (Hokkaido Univ.)**
Drapeau theorem for differential systems

14:00–14:50: **Reiko Miyaoka (Kyushu Univ.)**
Bryant-Salamon's metric with holonomy G_2

15:00–15:50: **Tohru Morimoto (Nara Women's Univ.)**
Nilpotent analysis and differential equations

Organized by T. Morimoto
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Developments of Cartan Geometry and Related Mathematical Problems

カルタン幾何の進化発展とそれに関連する数学の諸問題

RIMS 研究集会報告集

２００５年１０月２４日～１０月２７日

研究代表者　森本　徹 (Tohru Morimoto)

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