GROUPS OF MEASURE-PRESERVING HOMEOMORPHISMS AND VOLUME-PRESERVING DIFFEOMORPHISMS OF NONCOMPACT MANIFOLDS AND MASS FLOW TOWARD ENDS

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1. SPACES OF MEASURES AND GROUPS OF MEASURE-PRESERVING HOMEOMORPHISMS

Suppose $M$ is a connected $n$-manifold possibly with boundary. The symbol $\mathcal{B}(M)$ denotes the $\sigma$-algebra of Borel subsets of $M$.

**Definition 1.1.** A Radon measure on $M$ is a Borel measure $\mu$ on $M$ such that $\mu(K) < \infty$ for any compact subset $K$ of $M$. A Radon measure $\mu$ is said to be good if

(i) $\mu(p) = 0$ for any point $p$ of $M$ and
(ii) $\mu(U) > 0$ for any nonempty open subset $U$ of $M$.

**Definition 1.2.**

(1) $\mathcal{M}_g^\partial(M)$ denotes the set of good Radon measures on $M$ with $\mu(\partial M) = 0$.

(2) The weak topology $w$ on $\mathcal{M}_g^\partial(M)$ is the weakest topology such that the function

$$\Phi_f : \mathcal{M}_g^\partial(M) \to \mathbb{R} : \Phi_f(\mu) = \int_M f \, d\mu$$

is continuous for any continuous function $f : M \to \mathbb{R}$ with compact support.

Let $\mathcal{H}(M)$ denote the group of homeomorphisms of $M$ with the compact-open topology. Any subgroup $\mathcal{G}$ of $\mathcal{H}(M)$ is equipped with the subspace topology. $\mathcal{G}_0$ and $\mathcal{G}_1$ denote the connected component and the path-component of the identity in $\mathcal{G}$.

**Definition 1.3.** Suppose $\mu$ is a good Radon measures on $M$. The subgroups $\mathcal{H}(M; \mu) \subset \mathcal{H}(M; \mu\text{-reg}) \subset \mathcal{H}(M)$ are defined as follows:

(1) $h \in \mathcal{H}(M)$ is $\mu$-preserving if $\mu(h(B)) = \mu(B)$ for any $B \in \mathcal{B}(M)$.

$\mathcal{H}(M; \mu)$ denotes the subgroup of $\mathcal{H}(M)$ consisting of $\mu$-preserving homeomorphisms of $M$. 
(2) $h \in \mathcal{H}(M)$ is $\mu$-biregular if \(\mu(h(B)) = 0\) iff $\mu(B) = 0$ for any $B \in \mathcal{B}(M)$.

$\mathcal{H}(M; \mu\text{-reg})$ denotes the subgroup of $\mathcal{H}(M)$ consisting of $\mu$-biregular homeomorphisms of $M$.

The topological group $\mathcal{H}(M)$ acts continuously on the space $\mathcal{M}_{g}^\partial(M)_{w}$ by $h \cdot \mu = h_{*}\mu$, where $h_{*}\mu \in \mathcal{M}_{g}^\partial(M)$ is defined by $(h_{*}\mu)(B) = \mu(h^{-1}(B))$ $(B \in \mathcal{B}(M))$. The subgroup $\mathcal{H}(M; \mu)$ coincides with the stabilizer of $\mu$ under this action.

We also use the following terminologies.

\textbf{Definition 1.4.} Suppose $X$ is a space and $A$ is a subspace of $X$.

1. $A$ is a SDR (strong deformation retract) of $X$ if there exists a homotopy $\varphi_{t} : X \rightarrow X$ such that $\varphi_{0} = id_{X}$, $\varphi_{1}(X) = A$ and $\varphi_{t}|_{A} = id_{A}$ $(0 \leq t \leq 1)$.

2. $A$ is HD (homotopy dense) in $X$ if there exists a homotopy $\varphi_{t} : X \rightarrow X$ such that $\varphi_{0} = id_{X}$ and $\varphi_{t}(X) \subset A$ $(0 < t \leq 1)$.

In both cases the inclusion map $A \subset X$ is a homotopy equivalence with a homotopy inverse $\varphi_{1} : X \rightarrow A$.

\textbf{2. Compact case — Fathi's results}

Suppose $M$ is a compact connected $n$-manifold. The von Neumann-Oxtoby-Ulam theorem [10] asserts that the above action is essentially transitive.

\textbf{Theorem 2.1. (von Neumann-Oxtoby-Ulam)} Suppose $M$ is compact and $\mu$, $\nu \in \mathcal{M}_{g}^\partial(M)$ with $\nu(M) = \mu(M)$. Then there exists $h \in \mathcal{H}_{\partial}(M)_{0}$ such that $h_{*}\mu = \nu$.

A parametrized version of this theorem was obtained by A. Fathi [6]. Let $\mu \in \mathcal{M}_{g}^\partial(M)$. We need to restrict ourselves to the following subspace of $\mathcal{M}_{g}^\partial(M)$.

\textbf{Definition 2.1.} $\mathcal{M}_{g}^\partial(M; \mu\text{-reg})$ denotes the subset of $\mathcal{M}_{g}^\partial(M)$ consisting of $\nu \in \mathcal{M}_{g}^\partial(M)$ which has the same total mass and the same null sets as $\mu$.

The action of $\mathcal{H}(M)$ on $\mathcal{M}_{g}^\partial(M)$ restricts to the action of the subgroup $\mathcal{H}(M; \mu\text{-reg})$ on the subspace $\mathcal{M}_{g}^\partial(M; \mu\text{-reg})_{w}$. We obtain the orbit map

\[ \pi : \mathcal{H}(M; \mu\text{-reg}) \rightarrow \mathcal{M}_{g}^\partial(M; \mu\text{-reg})_{w} : \pi(h) = h_{*}\mu. \]
Theorem 2.2. (A. Fathi [6], 1980) Suppose $M$ is a compact connected $n$-manifold.

(1) The orbit map $\pi$ admits a section $\sigma : \mathcal{M}_g^\partial(M; \mu-\text{reg}) \to \mathcal{H}(M; \mu-\text{reg}) \subset \mathcal{H}(M; \mu-\text{reg})$.

(2) $\mathcal{H}(M; \mu-\text{reg}) \cong \mathcal{H}(M; \mu) \times \mathcal{M}_g^\partial(M; \mu-\text{reg})$.

(3) $\mathcal{H}(M; \mu) \subset \mathcal{H}(M, \mu-\text{reg}) \subset \mathcal{H}(M)$

(4) $n = 2$

Corollary 2.1. (Yagasaki [13]) $\mathcal{H}(M; \mu)$ is an $\ell_2$-manifold.

Corollary 2.1 easily follows from the next topological characterization of $\ell_2$-manifold.

Theorem 2.3. (T. Dobrowolski - H. Toruńczyk [5])
A topological group $G$ is a $\ell_2$-manifold iff $G$ is a separable, non locally compact, completely metrizable ANR.

3. NON-COMPACT CASE — R. BERLANGA’S RESULTS

Suppose $M$ is a noncompact connected $n$-manifold possibly with boundary. First we introduce some notations on the ends of $M$.

Definition 3.1.

(1) An end $e$ of $M$ is a function which assigns to each compact subset $K$ of $M$ a connected component $e(K)$ of $M - K$ such that $e(K_1) \supset e(K_2)$ if $K_1 \subset K_2$.

(2) $E(M)$ denotes the space of ends of $M$.

$\overline{M} = M \cup E(M)$ denotes the end compactification of $M$.

(3) The topology of $\overline{M}$ is described by the following conditions:

(i) $M$ is an open subspace of $\overline{M}$.

(ii) Fundamental open neighborhoods of $e \in E(M)$ is given by

$$N(e, K) = e(K) \cup \{e' \in E(M) \mid e'(K) = e(K)\} \quad (K \subset M : \text{compact})$$

$\overline{M}$ is a compact metrizable space and $E(M)$ is a 0-dim compact subset of $\overline{M}$.

Let $\mu \in \mathcal{M}_g^\partial(M)$. 
Definition 3.2.

1. $e \in E(M)$ is $\mu$-finite if $\mu(e(K)) < \infty$ for some compact subset $K$ of $M$ (i.e., $e$ has a neighborhood with finite $\mu$-mass).
2. $E_f(M; \mu)$ denotes the subspace of $\mu$-finite ends of $M$.

The von Neumann-Oxtoby-Ulam theorem is extended to the non-compact case in the following form.

Theorem 3.1. (R. Berlanga [1], 1983)

Suppose $\mu, \nu \in \mathcal{M}_g^\partial(M)$ has same total mass and same finite ends. Then there exists $h \in \mathcal{H}_\partial(M)_1$ with $h_* \mu = \nu$.

A parametrized version of this theorem is obtained recently by R. Berlanga [3]. Simple examples show that the weak topology $w$ on $\mathcal{M}_g^\partial(M; \mu-\text{reg})$ is not enough to extend the section theorem (Theorem 2.2 (1)) to the noncompact case. R. Berlanga introduces a little stronger topology called the finite-end weak topology, which turns out to be the correct topology for this purpose.

Definition 3.3. (Finite-end weak topology) Let $\mu \in \mathcal{M}_g^\partial(M)$.

1. $\mathcal{M}_g^\partial(M; \mu-\text{end-reg})$ denotes the subset of $\nu \in \mathcal{M}_g^\partial(M)$ which has the same total mass, same null sets and same finite ends as $\mu$.
2. Consider the inclusions $M \subset M \cup E_f(M; \mu) \subset \overline{M}$.

The map $\iota$ induces the natural map

$$\iota_* : \mathcal{M}_g^\partial(M; \mu-\text{end-reg}) \to \mathcal{M}_g^\partial(M \cup E_f(M; \mu))_w : \nu \mapsto \overline{\nu} = \iota_* \nu$$

3. The finite-end weak topology $ew$ on $\mathcal{M}_g^\partial(M; \mu-\text{end-reg})$ is the weakest topology such that $\iota_*$ is continuous.

The space $\mathcal{M}_g^\partial(M; \mu-\text{end-reg})_{ew}$ admits the contraction $\varphi_t(\nu) = (1-t)\nu + t\mu$ ($0 \leq t \leq 1$).

Definition 3.4. $\mathcal{H}(M; \mu-\text{end-reg})$ denotes the subgroup of $\mathcal{H}(M)$ consisting of $h \in \mathcal{H}(M)$ which preserves $\mu$-null sets and $\mu$-finite ends of $M$.

The group $\mathcal{H}(M; \mu-\text{end-reg})$ acts continuously on $\mathcal{M}_g^\partial(M; \mu-\text{end-reg})_{ew}$ by $h \cdot \nu = h_* \nu$ and we obtain the orbit map

$$\pi : \mathcal{H}(M; \mu-\text{end-reg}) \to \mathcal{M}_g^\partial(M; \mu-\text{end-reg})_{ew} : \pi(h) = h_* \mu.$$
**Theorem 3.2.** (R. Berlanga [3], 2003)

1. The orbit map $\pi$ has a section

$$\sigma : \mathcal{M}_g^\partial(M; \mu\text{-end-reg})_{ew} \to \mathcal{H}_0(M; \mu\text{-end-reg})_1 \subset \mathcal{H}(M; \mu\text{-end-reg}).$$

2. $\mathcal{H}(M; \mu\text{-end-reg}) \cong \mathcal{H}(M; \mu) \times \mathcal{M}_g^\partial(M; \mu\text{-end-reg})_{ew}$

3. \[ \text{SDR} \]

$$\mathcal{H}(M; \mu) \subset \mathcal{H}(M, \mu\text{-end-reg}) \subset \mathcal{H}(M)$$

The relation between the two groups $\mathcal{H}(M, \mu\text{-end-reg}) \subset \mathcal{H}(M)$ is not known for $n \geq 3$. In $n = 2$ we can apply our results on homeomorphism groups of noncompact 2-manifolds [11, 12] to obtain the following conclusions.

**Theorem 3.3.** (Yagasaki [13])

$$\begin{array}{ccc}
\text{HD} & \text{SDR} & \text{HD} \\
\mathcal{H}(M; \mu)_0 & \subset & \mathcal{H}(M, \mu\text{-end-reg})_0 \subset \mathcal{H}(M)_0 \\
\ell_2\text{-MFD} & \text{ANR} & \text{ANR}
\end{array}$$

The main statement $\mathcal{H}(M, \mu\text{-end-reg})_0 \subset \mathcal{H}(M)_0$ can be derived by the following arguments. When $M$ is a PL $n$-manifold, $\mathcal{H}^{\text{PL}}(M)$ denotes the subgroup of $\mathcal{H}(M)$ consisting of PL-homeomorphisms of $M$.

1. Suppose $M$ is a noncompact connected 2-manifold. Then
   (i) $M$ admits a PL-structure.
   (ii) $\mathcal{H}^{\text{PL}}(M)_0$ is HD in $\mathcal{H}(M)_0$ for any PL-structure on $M$ [12], cf. [7].
2. Suppose $M$ is a PL $n$-manifold and $\mu \in \mathcal{M}_g^\partial(M)$. Then the PL-structure on $M$ can be isotoped to a new PL-structure so that $\mathcal{H}^{\text{PL}}(M) \subset \mathcal{H}(M; \mu\text{-reg})$ [15].

4. **MASS FLOW TOWARD ENDS ON NON-COMPACT $n$-MANIFOLDS**

Suppose $M$ is a noncompact connected $n$-manifold and $\mu \in \mathcal{M}_g^\partial(M)$.

4.1. **Topological Vector Space** $V_\mu(M)$.

First we define a topological vector space $V_\mu(M)$, which parametrizes mass flows toward ends by $\mu$-preserving homeomorphisms.
Definition 4.1.

(1) $\mathcal{B}_{c}(M) = \{B \in \mathcal{B}(M) \mid \text{Fr } B : \text{Compact} \}$

(2) $W(M)$ denotes the space of all functions $a : \mathcal{B}_{c}(M) \to \mathbb{R}$.
   (i) $W(M)$ is a real vector space under the addition and the scalar product of real valued functions.
   (ii) $W(M)$ is equipped with the product topology, i.e., the topology induced by the projections
        
        \[ \pi_C : W(M) \to \mathbb{R} : \pi_C(a) = a(C) \quad (C \in \mathcal{B}_{c}(M)). \]

(3) $V(M) = \{a : \mathcal{B}_{c}(M) \to \mathbb{R} \mid (\ast)_1, (\ast)_2, (\ast)_3\}$
    
    $(\ast)_1 \quad C, D \in \mathcal{B}_{c}(M), \ Cl(C - D), Cl(D - C) : \text{compact} \implies a(C) = a(D)$
    $(\ast)_2 \quad C, D \in \mathcal{B}_{c}(M), \ C \cap D = \emptyset \implies a(C \cup D) = a(C) + a(D)$
    $(\ast)_3 \quad a(M) = 0$
    $V_{\mu}(M) = \{a \in V(M) \mid (\ast)_4\}$
    $(\ast)_4 \quad C \in \mathcal{B}_{c}(M), \ \mu(C) < \infty \implies a(C) = 0$

$V(M)$ and $V_{\mu}(M)$ are linear subspaces of $W(M)$, which are equipped with the subspace topology.

4.2. **Mass flow homomorphism toward ends** $J : \mathcal{H}_{E}(M, \mu) \to V_{\mu}(M)$.

Next we define a continuous group homomorphism $J : \mathcal{H}_{E}(M, \mu) \to V_{\mu}(M)$, which measures a mass moved toward ends by each $h \in \mathcal{H}_{E}(M, \mu)$. Let $E = E(M)$. Each $h \in \mathcal{H}(M)$ has a unique extension $\overline{h} \in \mathcal{H}(\overline{M})$.

Definition 4.2.

(1) $\mathcal{H}_{E}(M, \mu) = \{h \in \mathcal{H}(M, \mu) \mid \overline{h}|_{E} = \text{id}_{E}\}$ (a subgroup of $\mathcal{H}(M, \mu)$)

(2) $J : \mathcal{H}_{E}(M, \mu) \ni h \mapsto J_h \in V_{\mu}(M)$

\[ J_h(C) = \mu(C - h(C)) - \mu(h(C) - C) \quad (C \in \mathcal{B}_{c}(M)). \]

The group $\mathcal{H}_{E}(M, \mu)$ acts continously on $V_{\mu}(M)$ by $h \cdot a = J_h + a$ and the homomorphism $J : \mathcal{H}_{E}(M, \mu) \to V_{\mu}(M)$ coincides with the orbit map at $0 \in V_{\mu}(M)$. 
**Theorem 4.1.** (Yagasaki [14])

1. The map $J$ admits a section $s : V_{\mu}(M) \rightarrow \mathcal{H}_{\partial}(M, \mu)_{1} \subset \mathcal{H}_{E}(M, \mu)$ (i.e., $Js=\text{id}$) with $s(0)=\text{id}_{M}$.

2. (i) $\mathcal{H}_{E}(M; \mu) \cong \text{Ker} J \times V_{\mu}(M)$  
   (ii) $\text{Ker} J \subset \mathcal{H}_{E}(M; \mu)$ : a SDR

Ker $J$ contains the subgroup $\mathcal{H}^{c}(M; \mu)$ of $\mu$-preserving homeomorphisms with compact support. Our next aim is the study of relation between these groups.

5. Spaces of volume forms and groups of volume-preserving diffeomorphisms

Suppose $M$ is a connected oriented $C^\infty$ $n$-manifold without boundary.

**Definition 5.1.**

1. $\mathcal{D}^{+}(M)$ denotes the group of orientation-preserving diffeomorphisms of $M$ with the compact-open $C^\infty$-topology.

2. For a positive volume form $\omega$ on $M$,
   $\mathcal{D}(M; \omega)$ denotes the subgroup of $\omega$-preserving diffeomorphisms of $M$.

3. $\mathcal{V}^{+}(M)_{w}$ denotes the space of positive volume forms on $M$ equipped with the weak $C^\infty$ topology.
   For $m \in (0, \infty]$, $\mathcal{V}^{+}(M, m)_{w} = \{\mu \in \mathcal{V}^{+}(M) \mid \mu(M)=m\}$ (the weak $C^\infty$ topology).
   Each $\mu \in \mathcal{V}^{+}(M)$ determines a unique good Radon measure on $M$, which is denoted by the same symbol $\mu$. This defines an inclusion $\mathcal{V}^{+}(M) \subset \mathcal{M}_{g}^{\partial}(M)$.

The topological group $\mathcal{D}^{+}(M)$ acts continuously on $\mathcal{V}^{+}(M)_{w}$ and $\mathcal{V}^{+}(M, m)_{w}$ by $h \cdot \mu = h_{*}\mu = (h^{-1})^{*}\mu$. The subgroup $\mathcal{D}(M; \omega)$ coincides with the stabilizer of $\omega$ under this action.

5.1. **Compact case.**

Suppose $M$ is a compact connected oriented $C^\infty$ $n$-manifold without boundary. Moser's theorem [9] implies the transitivity of this action and its parametrized version.
Theorem 5.1. Suppose $M$ is a compact connected oriented $C^\infty$ n-manifold.

1. (Transitivity) For any $\mu, \nu \in \mathcal{V}^+(M; m)$ there exists $h \in D(M)_1$ such that $h_*\mu = \nu$.

2. (Parametrized version) Let $\omega \in \mathcal{V}^+(M; m)$. Then the orbit map $\pi : D^+(M) \to \mathcal{V}^+(M; m)_w$, $\pi(h) = h_*\omega$, admits a section $\sigma : \mathcal{V}^+(M; m)_w \to D(M)_1 \subset D^+(M)$.

5.2. Non-compact case.

Suppose $M$ is a non-compact connected $C^\infty$ n-manifold without boundary. Recall that $E = E(M)$ is the space of ends of $M$ and $\overline{M} = M \cup E(M)$ is the end compactification of $M$. Each $h \in D(M)$ has a unique extension $\overline{h} \in \mathcal{H}(\overline{M})$.

Definition 5.2. Suppose $F \subset E(M)$ is an open subset.

1. $D^+(M; F) = \{ h \in D^+(M) \mid \overline{h}(F) = F \}$ (a subgroup of $D^+(M)$)

2. $\mathcal{V}^+(M; F) = \{ \mu \in \mathcal{V}^+(M) \mid E_f(M, \mu) = F \}$

3. $\mathcal{M}_g^\partial(M; F) = \{ \mu \in \mathcal{M}_g^\partial(M) \mid E_f(M, \mu) = F \}$

4. (Finite-end weak topology)

The inclusion $M \subset M \cup F (\subset \overline{M})$ induces the injection

$\iota_\# : \mathcal{V}^+(M; m, F) \subset \mathcal{M}_g^\partial(M; F) \to \mathcal{M}_g(M \cup F)_w$.

The finite-end weak topology $\text{ew}$ on $\mathcal{V}^+(M; m, F)$ is the weakest topology such that the maps $\iota_\#$ and $\text{id} : \mathcal{V}^+(M; m, F) \to \mathcal{V}^+(M; m, F)_w$ are continuous.

The group $D^+(M; F)$ acts continuously on $\mathcal{V}^+(M; m, F)_w$ by $h \cdot \mu = h_*\mu$ and the stabilizer of $\omega \in \mathcal{V}^+(M; m, F)_w$ coincides with the subgroup $D(M; \omega)$. Transitivity of this action was verified by R. E. Greene - K. Shiohama [8].

Theorem 5.2. (R. E. Greene - K. Shiohama [8])

For any $\mu, \nu \in \mathcal{V}^+(M; m, F)$ there exists $h \in D(M)_1$ such that $h_*\mu = \nu$.

A $C^\infty$-modification of R. Berlanga's argument [3] leads to the parametrized version of this theorem.
Theorem 5.3. (Yagasaki [15])

Suppose $P$ is a paracompact Hausdorff space and $\mu, \nu : P \to V^+(M; F)_{ew}$ are maps such that $\mu_p(M) = \nu_p(M) \ (p \in P)$. Then there exists a map $h : P \to D(M)_1$ such that

(i) $h_{p*}\mu_p = \nu_p \ (p \in P)$ and (ii) if $p \in P$ and $\mu_p = \nu_p$, then $h_p = id_M$.

Corollary 5.1. Let $\omega \in V^+(M; m, F)$.

1. The orbit map $\pi : D^+(M; F) \to V^+(M; m, F)_{ew}, \pi(h) = h_*\omega$, admits a section $\sigma : V^+(M; m, F)_{ew} \to D(M)_1 \subset D^+(M; F)$.
2. (i) $D^+(M; F) \cong V^+(M; m, F)_{ew} \times D(M; \omega)$ (ii) $D(M; \omega) \subset D^+(M; F)$:

3. Mass flow toward ends on non-compact $C^\infty n$-manifolds.

Suppose $M$ is a non-compact connected $C^\infty n$-manifold without boundary and $\omega \in V^+(M)$. The topological vector space $V(M), V_\omega(M)$ and a continuous group homomorphism $J^\omega : D_E(M, \omega) \to V_\omega(M)$ are defined as in §4.1 and §4.2. For $h \in D_E(M; \omega)$

$$J^\omega_h : B_c(M) \to \mathbb{R} : \ J^\omega_h(C) = \omega(C - h(C)) - \omega(h(C) - C) \ (C \in B_c(M)).$$

The group $D_E(M, \omega)$ acts continuously on $V_\omega(M)$ by

$$h \cdot a = J^\omega_h + a \quad (h \in D_E(M, \omega), a \in V_\omega(M)).$$

The map $J^\omega : D_E(M, \omega) \to V_\omega(M)$ coincides with the orbit map at $0 \in V_\omega(M)$.

Definition 5.3. For two maps $\mu, \nu : P \to V^+(M)$ we write as $\mu \sim c \nu$ if

for any $p \in P$ there exists a neighborhood $U$ of $p$ in $P$ and a compact subset $K \subset M$ such that $\mu_q = \nu_q$ on $M - K$ for any $q \in U$.

Theorem 5.4. Suppose $P$ is a paracompact Hausdorff space and $\mu, \nu : P \to V^+(M)_{ew}, a : P \to V(M)$ are maps such that $\mu \sim c \nu, \ (\mu - \nu)(M) = 0$ and $a_p \in V_{\mu_p}(M) \ (p \in P)$.

Then there exists a map $h : P \to D(M)_1$ such that

(i) $h_{p*}\mu_p = \nu_p \ (p \in P)$ and (ii) if $p \in P$ and $\mu_p = \nu_p$, then $J^\mu_h = a_p$.

Corollary 5.2. Let $\omega \in V^+(M)$.

1. The map $J^\omega : D_E(M, \omega) \to V_\omega(M)$ admits a section $s : V_\omega(M) \to D(M, \omega)_1 \subset D_E(M, \omega)$ $(J^\omega s = id_{V_\omega(M)})$ with $s(0) = id_M$.
2. (i) $D_E(M; \omega) \cong \ker J^\omega \times V_\omega(M)$ (ii) $\ker J^\omega \subset D_E(M; \omega)$ : a SDR

Our next aim is to study the relation between two groups $D^c(M; \omega) \subset \ker J^\omega$. 
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