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# Real Options Model of R & D

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## 1. Introduction

Real options approaches have become a useful tool for evaluating irreversible investment under uncertainty such as R&D investment. Increasing numbers of the real options literatures such as [3, 5] have investigated strategic interactions of several firms. On the other hand, there are several studies on the decision of a single firm with an option to choose both the type and the timing of the investment projects. In this literature, [2] was the first study to pay attention to the problem and Décamps et al. [1] investigated the problem in more detail.

Despite such active studies on real options, to our knowledge few studies have tried to elucidate how competition between two firms affects their investment decisions in the case where the firms have the option to choose both the type and the timing of the projects. This paper investigates the above problem by extending the R&D model in [5] to a model where the firms can choose the target of the research from two alternative technologies of different standards with the same uncertainty about the market demand. In the model, we show that the competition between the two firms affects not only the firms' investment time, but also their choice of the technology targeted in the project.

We highlight two typical cases that reveal interesting implications. One is the de facto standard case, in which case a firm that completes a technology first can monopolize the profit flow regardless of the standard of the technology. The other is the innovative case, in which case a firm with higher-standard technology can deprive a firm with lower-standard technology of the cash flow by completing the higher-standard technology.

We show that, in the de facto standard case, the competition increases the incentive to develop the lower-standard technology, which is easy to complete, while in the innovative case, the competition increases the incentive to develop the higher-standard technology, which is difficult to complete. In particular, we show that in the de facto standard case the competition is likely to lead the firms to invest in the lower-standard technology, which is never chosen in the single firm situation. This result explains a real problem caused by too bitter R&D competition. Of course, as described in [4], practical R&D management is often much more flexible and complex than the simple model in this paper. However, it is likely that the essence of the results remains unchanged in more practical setups.

The paper is organized as follows. After Section 2 derives the optimal investment timing for the single firm, Section 3 made the general formulation of the competition between two firms. Section 4 derives the firms' strategies in the two typical cases, namely, the de facto standard case and the innovative case.

## 2. Single firm situation

Throughout the paper, we assume all stochastic processes and random variables are defined on the filtered probability space $(\Omega, \mathcal{F}, P; \mathcal{F}_t)$. This paper is based on the model in [5]. This section considers the investment decision of the single firm without fear of preemption. The firm can set up a research project for developing a new technology $i$ (we denote technologies 1 and 2 for the lower-standard and higher-standard technologies, respectively) by paying an indivisible investment cost $k_i$.

In developing technology $i$, from the time of the investment the invention takes place randomly according to a Poisson distribution with constant hazard rate $h_i$. The firm must pay the research expense $l_i$ per unit of time during the research term and can receive the profit flow $D_iY(t)$ from the discovery. Here, $Y(t)$ represents a market demand of the technologies at time $t$. It must be noted that the firm's R&D investment is affected by two different types of uncertainty, i.e., technological uncertainty and product market uncertainty. For simplicity, $Y(t)$ obeys the following geometric Brownian motion, which is independent of the Poisson processes representing technological uncertainty.

$$\mathrm{d}Y(t) = \mu Y(t)\mathrm{d}t + \sigma Y(t)\mathrm{d}B(t) \quad (t > 0), \; Y(0) = y,$$

where $\mu \geq 0, \sigma > 0$ and $y > 0$ are given constants and $B(t)$ denotes the one-dimensional $\mathcal{F}_t$ standard Brownian motion. Quantities $k_i, h_i, D_i$ and $l_i$ are given constants satisfying

$$0 \leq k_1 \leq k_2, \; 0 < h_2 < h_1, \; 0 < D_1 < D_2, \; 0 < l_1 \leq l_2, \tag{1}$$

so that technology 2 is more difficult to develop and generates a higher profit flow from its completion than technology 1.

The firm that monitors the market demand can set up development of either technologies 1 or 2 at the optimal timing maximizing the expected payoff under discount rate $r(> \mu)$. Then, the firm's problem is expressed as the following optimal stopping problem:

$$V_0(y) = \sup_{\tau \in \mathcal{T}} E\left[\max_{i=1,2} E[\int_{\tau+t_i}^{\infty} \mathrm{e}^{-rt} D_i Y(t)\mathrm{d}t - \mathrm{e}^{-r\tau} k_i - \int_{\tau}^{\tau+t_i} \mathrm{e}^{-rt} l_i \mathrm{d}t \mid \mathcal{F}_\tau]\right], \tag{2}$$

where $\mathcal{T}$ is a set of all $\mathcal{F}_t$ stopping times and $t_i$ denotes a random variable representing a Poisson arrival with hazard rate $h_i$ independent of $B(t)$. In problem (2), $\max_{i=1,2} E[\cdots \mid \mathcal{F}_\tau]$ means that the firm can choose the optimal technology at the investment time $\tau$.

Via some calculations, problem (2) can be reduced to

$$V_0(y) = \sup_{\tau \in \mathcal{T}} E[\mathrm{e}^{-r\tau} \max_{i=1,2}(a_{i0}Y(\tau) - I_i)], \tag{3}$$

where $a_{i0}$ and $I_i$ are defined by

$$a_{i0} = \frac{D_i h_i}{(r-\mu)(r+h_i-\mu)} \tag{4}$$

$$I_i = k_i + \frac{l_i}{r+h_i}. \tag{5}$$

Here, $a_{i0}Y(\tau)$ represents the expected discounted value of the future profit generated by technology $i$ at the investment time $\tau$, and $I_i$ represents its total expected discounted cost at time $\tau$. Eq. (1) and (5) imply $I_1 < I_2$, but the inequality $a_{10} < a_{20}$ does not necessarily hold depending upon a trade-off between $h_i$ and $D_i$. Let $V_0(y)$ and $\tau_0^*$ denote the value function and the optimal stopping time in problem (3), respectively. As in most real options literature, we define

$$\beta_{10} = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1, \tag{6}$$

$$\beta_{20} = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0. \tag{7}$$

**Proposition 1** The value function $V_0(y)$ and the optimal stopping time $\tau_0^*$ in the single firm's problem (3) are given as follows:
**Case 1:** $0 < a_{20}/a_{10} \leq 1$

$$V_0(y) = \begin{cases} A_0 y^{\beta_{10}} & (0 < y < y_{10}^*) \\ a_{10}y - I_1 & (y \geq y_{10}^*), \end{cases} \tag{8}$$

$$\tau_0^* = \inf\{t \geq 0 \mid Y(t) \geq y_{10}^*\}. \tag{9}$$

**Case 2:** $1 < (a_{20}/a_{10})^{\beta_{10}/(\beta_{10}-1)} < I_2/I_1$

$$V_0(y) = \begin{cases} A_0 y^{\beta_{10}} & (0 < y < y_{10}^*) \\ a_{10}y - I_1 & (y_{10}^* \leq y \leq y_{20}^*) \\ B_0 y^{\beta_{10}} + C_0 y^{\beta_{20}} & (y_{20}^* < y < y_{30}^*) \\ a_{20}y - I_2 & (y \geq y_{30}^*), \end{cases} \tag{10}$$

$$\tau_0^* = \inf\{t \geq 0 \mid Y(t) \in [y_{10}^*, y_{20}^*] \cup [y_{30}^*, +\infty)\}. \tag{11}$$

**Case 3:** $(a_{20}/a_{10})^{\beta_{10}/(\beta_{10}-1)} \geq I_2/I_1$

$$V_0(y) = \begin{cases} B_0 y^{\beta_{10}} & (0 < y < y_{30}^*) \\ a_{20}y - I_2 & (y \geq y_{30}^*), \end{cases} \tag{12}$$

$$\tau_0^* = \inf\{t \geq 0 \mid Y(t) \geq y_{30}^*\}. \tag{13}$$

Here, constants $A_0, B_0, C_0$ and thresholds $y_{10}^*, y_{20}^*, y_{30}^*$ are determined by imposing value matching and smooth pasting conditions. Note that $I_1 < I_2$ and $\beta_{10} > 1$.

In Proposition 1, $A_0 y^{\beta_{10}}, B_0 y^{\beta_{10}}$ and $C_0 y^{\beta_{20}}$ correspond to the values of the option to invest in technology 1 at the trigger $y_{10}^*$, the option to invest in technology 2 at the trigger $y_{30}^*$ and the option to invest in technology 1 at the trigger $y_{20}^*$, respectively. In Case 1, where the expected discounted profit of technology 1 is higher than that of technology 2, the firm invests in technology 1 at time (9) independently of $y$. In Case 3, where technology 2 is much superior to technology 1, on the contrary, the firm invests in technology 2 at time (13) regardless of $y$. In Case 2, where both projects has similar values by the trade-off between the profitability and the research term and cost, the firm's optimal investment policy has three thresholds $y_{10}^*, y_{20}^*$ and $y_{30}^*$, and therefore the project chosen by the firm depends on the initial value $y$. Above all, if $y \in (y_{20}^*, y_{30}^*)$, the firm defers not only investment, but also choice of the project type.

## 3. Two firms situation

We turn to a problem of two symmetric firms. We assume that two Poisson processes modeling the two firms' innovation are independent of each other, which means that the progress of the research project by one of the firms does not affect that of its rival. The scenarios of the cash flows into the firms can be classified into four cases. We assume that the cash flows into the firm that has completed a technology first (denoted Firm 1) and the other (denoted Firm

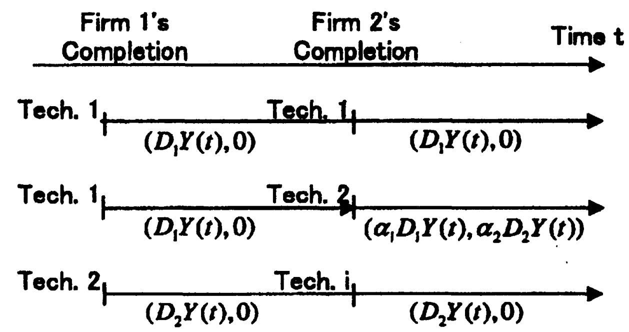


Figure 1: (Firm 1's cash flow, Firm 2's cash flow)

2) follows Figure 1. The quantities $\alpha_i$ are constants satisfying $0 \le \alpha_1, \alpha_2 \le 1$. We consider that the technology's share in the product market determines $\alpha_1$ and $\alpha_2$.

As in [5], we solve the game between two firms backwards. We begin by supposing that one of the firms has already invested, and find the optimal decision of the other. In the remainder of this paper, we call the one who has already invested *leader* and call the other *follower*, though we consider two symmetrical firms. Thereafter, we look at the situation where neither firms has invested, and consider the decision of either as it contemplates whether to go first, knowing that the other will react in the way just calculated as the follower's optimal response. Let $F_i(Y)$ and $\tau^*_{F_i}$ denote the expected discounted payoff (at time $t$) and the investment time of the follower responding optimally to the leader who has invested in technology $i$ at time $t$ satisfying $Y(t) = Y$. We denote by $L_i(Y)$ the expected discounted payoff (at time $t$) of the leader who has invested in technology $i$ at $Y(t) = Y$.

### 3.1. Case where the leader has invested in technology 2

This subsection derives $F_2(Y), \tau^*_{F_2}$ and $L_2(Y)$. Given that the leader has invested in technology 2 at $Y(t) = Y$, the follower's problem can be reduced to

$$F_2(Y) = \sup_{\tau \in \mathcal{T}} E^Y[\mathrm{e}^{-(r+h_2)\tau} \max_{i=1,2}(a_{i2}Y(\tau) - I_i)], \tag{14}$$

where $a_{ij}$ are defined by

$$a_{11} = \frac{D_1 h_1}{(r-\mu)(r+2h_1-\mu)}, \tag{15}$$

$$a_{12} = \frac{D_1 h_1}{(r+h_1+h_2-\mu)(r+h_2-\mu)}\left(1+\frac{\alpha_1 h_2}{r-\mu}\right), \tag{16}$$

$$a_{21} = \frac{D_2 h_2}{(r-\mu)(r+h_1+h_2-\mu)}\left(1+\frac{\alpha_2 h_1}{r+h_2-\mu}\right), \tag{17}$$

$$a_{22} = \frac{D_2 h_2}{(r-\mu)(r+2h_2-\mu)}. \tag{18}$$

The additional discount $\mathrm{e}^{-h_2\tau}$ values the possibility that the follower's option vanishes before its investment by the leader's completion of technology 2. Quantity $a_{ij}Y(\tau)$ represents the expected discounted value of the future cash flow of the firm that invests in technology $i$ at time $\tau$ when its opponent is on the way to development of technology $j$. From the expression (14), we can show the following proposition.

**Proposition 2** The follower's payoff $F_2(Y)$, investment time $\tau^*_{F_2}$ and the leader's payoff $L_2(Y)$ are given as follows:
**Case 1:** $0 < a_{22}/a_{12} \le 1$

$$F_2(Y) = \begin{cases} A_2 Y^{\beta_{12}} & (0 < Y < y^*_{12}) \\ a_{12}Y - I_1 & (Y \ge y^*_{12}), \end{cases}$$

$$\tau^*_{F_2} = \inf\{s \ge t \mid Y(s) \ge y^*_{12}\},$$

$$L_2(Y) = \begin{cases} a_{20}Y - I_2 - \tilde{A}_2 Y^{\beta_{12}} & (0 < Y < y^*_{12}) \\ a_{21}Y - I_2 & (Y \ge y^*_{12}). \end{cases}$$

**Case 2:** $1 < (a_{22}/a_{12})^{\beta_{12}/(\beta_{12}-1)} < I_2/I_1$

$$\begin{aligned}
F_2(Y) &= \begin{cases} A_2 Y^{\beta_{12}} & (0 < Y < y_{12}^*) \\ a_{12}Y - I_1 & (y_{12}^* \le Y \le y_{22}^*) \\ B_2 Y^{\beta_{12}} + C_2 Y^{\beta_{22}} & (y_{22}^* < Y < y_{32}^*) \\ a_{22}Y - I_2 & (Y \ge y_{32}^*), \end{cases} \\
\tau_{F_2}^* &= \inf\{s \ge t \mid Y(s) \in [y_{12}^*, y_{22}^*] \cup [y_{32}^*, +\infty)\}, \\
L_2(Y) &= \begin{cases} a_{20}Y - I_2 - \tilde{A}_2 Y^{\beta_{12}} & (0 < Y < y_{12}^*) \\ a_{21}Y - I_2 & (y_{12}^* \le Y \le y_{22}^*) \\ a_{20}Y - I_2 - \tilde{B}_2 Y^{\beta_{12}} - \tilde{C}_2 Y^{\beta_{22}} & (y_{22}^* < Y < y_{32}^*) \\ a_{22}Y - I_2 & (Y \ge y_{32}^*). \end{cases}
\end{aligned}$$

**Case 3:** $(a_{22}/a_{12})^{\beta_{12}/(\beta_{12}-1)} \ge I_2/I_1$

$$\begin{aligned}
F_2(Y) &= \begin{cases} B_2 Y^{\beta_{12}} & (0 < Y < y_{32}^*) \\ a_{22}Y - I_2 & (Y \ge y_{32}^*), \end{cases} \\
\tau_{F_2}^* &= \inf\{s \ge t \mid Y(s) \ge y_{32}^*\}, \\
L_2(Y) &= \begin{cases} a_{20}Y - I_2 - \tilde{B}_2 Y^{\beta_{12}} & (0 < Y < y_{32}^*) \\ a_{22}Y - I_2 & (Y \ge y_{32}^*). \end{cases}
\end{aligned}$$

Here, $\beta_{12}$ and $\beta_{22}$ denote (6) and (7) replaced $r$ by $r + h_2$, respectively. Constants $A_2, B_2, C_2$ and thresholds $y_{12}^*, y_{22}^*, y_{32}^*$ are determined by both value matching and smooth pasting conditions, while constants $\tilde{A}_2$, $\tilde{B}_2$ and $\tilde{C}_2$ are determined by the value matching condition alone. Note that $I_1 < I_2$ and $\beta_{12} > 1$.

Constants $A_2, B_2, C_2$ and thresholds $y_{12}^*, y_{22}^*, y_{32}^*$ in Proposition 2 correspond to constants $A_0, B_0, C_0$ and thresholds $y_{10}^*, y_{20}^*, y_{30}^*$ in Proposition 1, respectively. Constants $\tilde{A}_2$, $\tilde{B}_2$ and $\tilde{C}_2$ value the possibility that $Y$ rises above $y_{12}^*$ prior to the leader's completion, the possibility that $Y$ rises above $y_{32}^*$ prior to the leader's completion, and the possibility that $Y$ falls bellow $y_{22}^*$ prior to the leader's completion, respectively.

### 3.2. Case where the leader has invested in technology 1

In this subsection, unlike in the previous subsection, there remains the follower's option to invest in technology 2 after the leader's invention of technology 1 if the follower has not invested yet. Due to this option value, we need more complicated discussion in this subsection.

Let $f_1(Y)$ and $\tau_{f_1}^*$ be the expected discounted payoff and the optimal stopping time of the follower responding optimally to the leader who has already succeeded in development of technology 1 at $Y(t) = Y$. In other words, $f_1(Y)$ represents the remaining option value to invest in technology 2 after the leader's completion of technology 1. We need to derive $f_1(Y)$ and $\tau_{f_1}^*$ before analyzing $F_1(Y)$ and $\tau_{F_1}^*$. Given that the leader has already completed technology 1 at $Y(t) = Y$, the follower's problem becomes

$$f_1(Y) = \sup_{\tau \in \mathcal{T}} E^Y[\mathrm{e}^{-r\tau}(\alpha_2 a_{20} Y(t) - I_2)]. \tag{19}$$

It is easy to obtain the value function $f_1(Y)$ and the optimal stopping time $\tau_{f_1}^*$ in problem (19). If $\alpha_2 > 0$, then

$$f_1(Y) = \begin{cases} B' Y^{\beta_{10}} & (0 < Y < y') \\ \alpha_2 a_{20} Y - I_2 & (Y \ge y'), \end{cases} \tag{20}$$

$$\tau_{f_1}^* = \inf\{s \ge t \mid Y(s) \ge y'\}, \tag{21}$$

where $B'$ and $y'$ are constants determined by the value matching and smooth pasting conditions (we omit the explicit solutions to avoid cluttering). If $\alpha_2 = 0$, then we have $f_1(Y) = 0$ and $\tau^*_{f_1} = +\infty$.

Assuming that the leader has begun developing technology 1 at $Y(t) = Y$, the follower's problem can be expressed as follows:

$$F_1(Y) = \sup_{\tau\in\mathcal{T}} E^Y[e^{-(r+h_1)\tau} \max_{i=1,2}(a_{i1}Y(\tau) - I_i) + 1_{\{\tau\geq s_1\}}e^{-rs_1} f_1(Y(s_1))], \qquad (22)$$

where $s_1$ denotes the time when the leader completes technology 1. Compared with the follower's problem (14), problem (22) has the additional term $E^Y[1_{\{\tau\geq s_1\}}e^{-rs_1}f_1(Y(s_1))]$. This term corresponds to the value of the option for the inactive follower to invest in technology 2. Generally, problem (22), unlike (14), is difficult to solve analytically because of the additional term. In the next section, we overcome the difficulty by focusing on two typical cases, namely, the de fact standard case, where $(\alpha_1, \alpha_2) = (1, 0)$, and the innovative case, where $(\alpha_1, \alpha_2) = (0, 1)$.

## 4. Analysis in two typical cases

In order to exclude a situation where both firms mistakenly invest simultaneously, we assume that the initial value $y$ is small enough, that is, $\max_{i=1,2}(a_{i0}y - I_i) < 0$ (**Assumption A**), as in [5], when we discuss the preemption equilibrium. We moreover restrict our attention to the case where the firm always chooses the higher-standard technology 2 in the single firm situation to contrast the competitive situation with the single firm situation. To put it more concretely, we assume $(a_{20}/a_{10})^{\beta_{10}/(\beta_{10}-1)} \geq I_2/I_1$ (**Assumption B**), so that Case 3 follows in Proposition 1.

In the first place, we analytically derive the follower's payoff $F_1(Y)$ and the leader's payoff $L_1(Y)$ in both the de fact standard and innovative cases. Note that the results on $F_2(Y)$ and $L_2(Y)$ in Proposition 2 hold true by substituting $(\alpha_1, \alpha_2) = (1, 0)$ and $(\alpha_1, \alpha_2) = (0, 1)$ into (16) and (17). Then, using $L_1(Y)$ and $L_2(Y)$, we define

$$\begin{aligned} L(Y) &= \max_{i=1,2} L_i(Y), \\ F(Y) &= \begin{cases} F_1(Y) & (L_1(Y) > L_2(Y)) \\ F_2(Y) & (L_1(Y) \leq L_2(Y)). \end{cases} \end{aligned}$$

Comparing $L(Y)$ with $F(Y)$, we examine the situation where both firm try to preempt each other.

### 4.1. De facto standard case

Since $\alpha_2 = 0$ holds in this case, the follower's option value $f_1(Y)$ vanishes just like in Subsection 3.2. Thus, we can solve the follower's problem (22) in the same way as problem (14). Indeed, $F_1(Y)$ and $\tau^*_{F_1}$ agree with $F_2(Y)$ and $\tau^*_{F_2}$ replaced $a_{i2}, \beta_{i2}$ with $a_{i1}, \beta_{i1}$, respectively in Proposition 2, where $\beta_{11}(> 1)$ and $\beta_{21}(< 0)$ denote (6) and (7) replaced discount rate $r$ with $r + h_1$, respectively. In this case, we denote three thresholds corresponding to $y^*_{12}, y^*_{22}$ and $y^*_{32}$ in Proposition 2 by $y^*_{11}, y^*_{21}$ and $y^*_{31}$, respectively. Then, the payoff $L_1(Y)$ of the leader who has invested in technology 1 at $Y(t) = Y$ coincides with $L_2(Y)$ replaced $a_{2i}, I_2, \beta_{i2}$ and $y^*_{i2}$ by $a_{1i}, I_1, \beta_{i1}$ and $y^*_{i1}$, respectively in Proposition 2.

Let us compare the follower's decision in the de facto standard case with the monopolist's decision derived in Section 2. Using $r - \mu > 0$ and $h_1 > h_2 > 0$, we have

$$\frac{a_{21}}{a_{11}} < \frac{a_{22}}{a_{12}} < \frac{a_{20}}{a_{10}}. \qquad (23)$$

Eq. (23) states that the relative expected profit of technology 2 to technology 1 is smaller than that in the single firm case. Using $1 < \beta_{10} < \beta_{12} < \beta_{11}$, we also obtain

$$1 < \frac{\beta_{11}}{\beta_{11}-1} < \frac{\beta_{12}}{\beta_{12}-1} < \frac{\beta_{10}}{\beta_{10}-1}. \tag{24}$$

Eq. (23) and (24) suggest a possibility that $(a_{2i}/a_{1i})^{\beta_{1i}/(\beta_{1i}-1)}$ is smaller than $I_2/I_1$ and 1 even under Assumption B. Then the follower's optimal choice could be technology 1. In consequence, the presence of the leader increases the follower's incentive to choose the lower-standard technology 1, which is easy to complete, compared with in the single firm situation. It can be shown that $F_1(Y) < F_2(Y) \quad (Y > 0)$. That is, from the follower's viewpoint, the case where the leader has chosen technology 2 is preferable to the case where the leader has chosen technology 1. This is due to that the leader who has invested in technology 1 is more likely to preempt the follower because of its short research term.

Finally, we consider the situation where neither firm has invested. Let us see that there exists a possibility that technology 1 can be developed due to the competition, even if technology 2 generates much more profit than technology 1 at its completion. Although, as has been pointed out, $(a_{2i}/a_{1i})^{\beta_{1i}/(\beta_{1i}-1)}$ could be smaller than $I_2/I_1$ and 1 under Assumption B, we now consider the case where

$$\left(\frac{a_{2i}}{a_{1i}}\right)^{\frac{\beta_{1i}}{\beta_{1i}-1}} \geq \frac{I_2}{I_1} \tag{25}$$

holds, which means that a cash flow resulting from technology 2 is expected to be much greater than that of technology 1.

Since the initial value $Y(0) = y$ is small enough (Assumption A), in the single firm situation the firm invests in technology 2 (Assumption B) as soon as the market demand $Y(t)$ rises to the level $y_{30}^*$. Development of technology 1 is meaningless because the firm without fear of preemption can defer the investment sufficiently. However, the firm with fear of preemption by its rival will attempt to obtain the leader's payoff by investing a slight bit earlier than its rival when the leader's payoff $L(Y)$ is larger than the follower's payoff $F(Y)$. Repeating this process causes the investment trigger to fall to the point (denoted, $y_P$)where $L(Y)$ is equal to $F(Y)$. At the point the firms are indifferent between the two roles, and then one of the firms invests at time $\inf\{t \geq 0 \mid Y(t) \geq y_P\}$ as leader, while the other invests at time $\tau_{F_i}^*$ (if there remains the option to invest) as follower. This asymmetric outcome is called *preemption equilibrium*. If the fear of preemption hastens the investment time sufficiently, the preemption trigger $y_P$ is much smaller than $y_{30}^*$, and becomes the intersection of $L_1(Y)$ and $F_1(Y)$ rather than that of $L_2(Y)$ and $F_2(Y)$. It suggests a possibility that in the preemption equilibrium the leader invests in technology 1, even if (25) is satisfied.

### 4.2. Innovative case

This subsection examines the innovative case. We consider the follower's optimal response assuming that the leader has invested in technology 1 at $Y(t) = Y$. We can show that in the innovative case the follower's best response $\tau_{F_1}^*$ coincides with $\tau_{f_1}^*$ and that $F_1(Y) = f_1(Y) = V_0(Y)$ hold. That is, the follower behaves as if there were no leader. Using the follower's investment time, we have the leader's payoff $L_1(Y)$ as $L_2(Y)$ replaced $a_{2i}, I_2, \beta_{12}$ and $y_{32}^*$ by $a_{1i}, I_1, \beta_{11}$ and $y_{30}^*$, respectively in Case 3 in Proposition 2.

Next, we compare the follower's decision in the innovative case with the monopolist's decision. We can easily show

$$1 < \frac{a_{20}}{a_{10}} < \frac{a_{2i}}{a_{1i}} \quad (i = 1, 2). \tag{26}$$

Eq. (26) means that the relative expected profit of technology 2 to technology 1 is greater than that in the single firm case, contrary to (23) in the de facto standard case. Since (24) remains true, the relationship between $(a_{22}/a_{12})^{\beta_{12}/(\beta_{12}-1)}$ and $I_2/I_1$ depends on the parameters even under Assumption B. However, in most cases the effect of (26) dominates the effect of (24). To sum up, the presence of the leader, unlike in the de facto standard case, tends to decrease the incentive of the lower-standard technology 1, which is easy to complete. By definition of the follower's problem (14), we can show $F_2(Y) < V_0(Y) = F_1(Y) \quad (Y > 0)$, contrary to the de facto standard case. With respect to the preemption equilibrium, we obtain the following proposition.

**Proposition 3** The inequality

$$L_1(Y) < F_1(Y) \quad (Y > 0) \tag{27}$$

holds, and therefore in the preemption equilibrium the leader always chooses technology 2. Furthermore, if $(a_{22}/a_{12})^{\beta_{12}/(\beta_{12}-1)} > I_2/I_1$, in the preemption equilibrium the follower, also, always chooses technology 2.

Table 1 summarizes the comparison results between the two cases.

Table 1: Comparison between the de facto standard and innovative cases.

| | De facto standard | Innovative |
|---|---|---|
| Relative expected profit | $a_{2i}/a_{1i} < a_{20}/a_{10}$ | $a_{2i}/a_{1i} > a_{20}/a_{10}$ |
| Follower's value function | $F_1(Y) < F_2(Y)$ | $F_1(Y) > F_2(Y)$ |
| Preemption equilibrium | Both firms: likely to choose Tech. 1 | Leader: Tech. 2, Follower: Tech. 2 |

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