

# Insecure Property Rights and Long-Run Growth under Increasing Returns\*

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## Abstract

This paper examines a simple mode of endogenously growing economy in the absence of secure property rights. Unlike the existing studies on growth model without secure property rights that assume constant-returns to scale technologies with fixed labor supply, this paper considers variable labor-leisure choice and increasing returns. As a result of these generalizations, it is shown that growth performance of the economy depends not only on over consumption due to the common-pool problem but also on the scale effect generated by the presence of increasing returns.

## 1 Introduction

The purpose of this paper is to investigate the role of property rights in the process of economic growth. When examining the relationship between property rights and economic growth, it is useful to investigate the behavior of growing economies in which private property rights are insecure. In this respect, a series of papers by Tornell and Velasco (1992), Lane and Tornell (1996), Tornell (1998) and Tornell and Lane (1999) present useful frameworks for analyzing dynamic economies in the absence of secure property rights.<sup>1</sup> These studies construct dynamic games played by multiple interest groups that intend to exploit commonly accessible resources. A distinguished feature

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\*This paper is based on Mino (2006) and Itaya and Mino (2007). A more detailed discussion can be found in Itaya and Mino (2007).

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<sup>1</sup>See also Lindner and Strulik (2002, 2004) and Tornell and Lane (1998).

of this kind of modeling is that due to the common pool problem, a rise in the number of agents participating the exploitation game generally has a negative impact on growth. In addition, there may exist the 'voracity effect': an increase in productivity may yield a larger consumption growth of each agent than the income growth generated by the technological improvement. If this is the case, a higher productivity depresses economic growth, which will not be observed in an economy with secure property rights.

The existing investigations mentioned above use  $Ak$  growth models to obtain analytically tractable solutions of the dynamic games. Namely, they assume that final goods are produced by a commonly owned capital stock alone with a linear technology.<sup>2</sup> In this setting each agent can consume final goods without participating any production activity. One may conjecture that the main findings in the growth models with insecure property rights may be generated not only by the common pool issue but also by such a simple technological specification. To examine this question, we extend the baseline model by assuming that production needs labor as well as capital and hence the agents should participate production activities by supplying their labor force. To keep the tractability of model analysis, we still assume that output linearly depends on capital, which means that the aggregate production technology exhibits increasing returns to scale with respect to capital and labor. As a result of our generalization, we find that there are two opposing factors that affect growth performance of the economy. The first is over consumption caused by the common pool problem and the second is the scale effect generated by the presence of increasing returns. The first factor may yield the voracity effect, while the second one contributes to accelerating growth. The resulting growth performance of the economy with insecure property rights, therefore, depends on which factor dominates in the process of capital accumulation.

This paper is organized as follows. In the next section, we summarize the main results obtained in the simplest  $Ak$  model of endogenous growth without secure property rights. Even though the model is extremely simple, many of the existing studies on growth and property rights have utilized this framework so that it present the baseline consideration on the issue. Section 3 constructs our original model by introducing variable labor supply and increasing returns to scale. Our main results under alternative specifications of utility function are also displayed in this section. Section 4 concludes..

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<sup>2</sup>Lindner and Strulik (2002, 2004) also analyze a model with convex technology under which continuing growth is not sustained in the steady state equilibrium.

## 2 Voracity and Growth: A Simple Example

Before discussing our setting, it would be useful to review the baseline model of endogenous growth without secure property rights. Although the model is extremely simple, the analytical results displayed below present the common background for many existing studies on growth and property rights. Consider an economy consisting of  $n$  ( $\geq 2$ ) interest groups. The production technology of the economy is jointly owned by the groups and it is specified as

$$y = Ak, \quad A > 0, \quad (1)$$

where  $y$  is the aggregate product and  $k$  denotes capital stock. The number of the agents in each group is normalized to one. The objective of each group is to maximize a discounted sum of utilities,

$$U_i = \int_0^{\infty} \frac{c_i^{1-\sigma}}{1-\sigma} e^{-\rho t} dt, \quad \sigma > 0, \quad \sigma \neq 1, \quad \rho > 0, \quad i = 1, 2, \dots, n,$$

where  $c_i$  is consumption of group  $i$ .

We have assumed that there is no property right for capital, so that the capital stock is accumulated according to

$$\dot{k} = Ak - \sum_{i=1}^n c_i. \quad (2)$$

In this setting, group  $i$  ( $i = 1, 2, \dots, n$ ) selects the optimal level of  $c_j$  in each moment to maximize  $U_i$  subject to (2) and the initial level of  $k_0$ . Hence, our model is a differential game among  $n$  groups in which the control of each player is  $c_i$  and the state variable of the game is the aggregate capital stock  $k$ . We will focus on the Markov-perfect Nash (feedback Nash) equilibrium of this game.

Defining the value function of group  $i$  as

$$V_i(k) = \max_{\{c_i\}_{t=0}^{\infty}} \left\{ \int_0^{\infty} \frac{c_i^{1-\sigma}}{1-\sigma} e^{-\rho t} dt \quad \text{subject to (2)} \right\},$$

we may set up the following Hamilton-Jacobi-Bellman (HJB) equation for this problem:

$$\rho V_i(k) = \max_c \left\{ \frac{c_i^{1-\sigma}}{1-\sigma} + V_i'(k) (Ak - \sum_{i=1}^n c_i) \right\} \quad \text{for all } t \geq 0, \quad j = 1, 2, \dots, n. \quad (3)$$

The first-order condition for the optimal choice of consumption is

$$c_i^{-\sigma} = V_i'(k). \quad (4)$$

Therefore, the optimal consumption is expressed as

$$c_i = c_i(k).$$

Substituting this into the HJB equation and applying the envelope theorem, we obtain:

$$\rho V'(k) = V'_i(k) A + V''_i(k) (Ak - \sum_{i=1}^n c_i(k)) - V'_i(k) \sum_{j \neq i} c'_j(k), \quad i = 1, 2, \dots, n. \quad (5)$$

Since (4) holds for all  $t \geq 0$ , it gives  $-\sigma c_i^{-\sigma-1} c'_j(k) = V''_i(k)$ . Substituting this and (4) into (5), we find

$$\rho = A - \sigma \frac{c'_j(k)}{c_i(k)} (Ak - \sum_{i=1}^n c_i(k)) - \sum_{j \neq i} c'_j(k), \quad i = 1, 2, \dots, n.$$

We now consider the symmetric equilibrium, i.e,  $c_i(k) = c(k)$  for all  $i$ . In addition, we focus on the balanced-growth equilibrium where  $c$  and  $k$  grow at a common, constant rate. Note that on the balanced-growth path,  $c$  should be proportional to  $k$ . Thus, assuming that  $c_i = c$  and  $c = \phi k$  ( $\phi$  is a constant), equation (3) yields  $\rho\phi = A\phi - \sigma(A - n\phi) - (1 - n)\phi$ , implying that

$$\phi = \frac{\rho + (\sigma - 1)A}{1 + (\sigma - 1)n}. \quad (6)$$

As a consequence, the balanced growth rate,  $g = \dot{k}/k = \dot{c}/c = A - nc/k$ , is written as

$$g = A - n\phi = \frac{A - n\rho}{1 + (\sigma - 1)n}. \quad (7)$$

When  $n = 1$ , the model is reduced to the standard representative agent case where the balanced-growth rate is

$$g = \frac{1}{\sigma} (A - \rho). \quad (8)$$

The balanced-growth rate given by (7) shows that

$$\frac{dg}{dA} = \frac{1}{1 + (\sigma - 1)n}.$$

Therefore,  $dg/dA$  has a negative sign if

$$\sigma < \frac{n - 1}{n}. \quad (9)$$

If this is the case, a rise in the total factor productivity,  $A$ , lowers the balanced-growth rate. In words, when a more efficient technology becomes

available, the interest groups raise their consumption levels more than the income growth due to the technological improvement, and hence the resulting balanced-growth rate decreases. This is a simple example of the 'voracity effect' emphasized by Tonell and Lain (1998): the economy without secure property rights may yield a counter intuitive impact of technological progress that should improve growth performance if property rights are established. In what follows, we extend this simple model and consider the presence of voracity effect as well as the growth effect of a change in the number of players in a more general setting.

It is also to be noted that (7) presents

$$\frac{dg}{dn} = -\frac{\rho + (\sigma - 1) A}{[1 + (\sigma - 1) n]^2}.$$

To keep  $\phi$  positive,  $\rho + (\sigma - 1) A > 0$  if  $1 + (\sigma - 1) n > 0$ , while  $\rho + (\sigma - 1) A < 0$  if  $1 + (\sigma - 1) n < 0$ . Therefore, in the normal case without voracity effect, we obtain  $dg/dn < 0$ : an increase in the number of interest groups is harmful for growth. In contrast, in the presence of voracity effect, a larger number of players realizes a higher balanced-growth rate. This because, as (6) shows, an increase in  $n$  lowers the consumption share if  $1 + (\sigma - 1) A < 0$ , which contributes to increasing growth.

### 3 A Model with Endogenous Labor Supply

#### 3.1 The Model

Considering endogenous labor supply, we now extend the baseline model in the previous section. The economy produces a homogenous final good and the production technology is specified as

$$y = Akf(L), \quad A > 0, \quad (10)$$

where  $L$  denotes the aggregate labor supply. We assume that function  $f(L)$  is positive, monotonically increasing, strictly concave in  $L$  and satisfies  $f(0) = 0$ . Since we still assume that the production technology is linear in capital stock, the introduction of labor input means that the production technology exhibits increasing return to scale with respect to capital and labor.<sup>3</sup>

Again, we assume that there are  $n$  ( $\geq 2$ ) interest groups. In what follows, we will distinguish the effects of changes in the number of interest groups and

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<sup>3</sup>In the context of the representative agent settings, Benhabib and Farmer (1994) and Pelloni and Waldmann (2000, 2001) utilize the production function (10).

the size of each group. In so doing, we assume that the  $i$ -th group has  $s_i$  members, and hence the total number of agents in the economy at large is  $\sum_{i=1}^n s_i = N$ . All the agents in the same group are identical. While the capital stock  $k$  is jointly owned by the groups, each member supplies its own labor for production. Therefore, when an individual agent supplies  $l_i$  units of labor, the aggregate labor supply is  $L = \sum_{i=1}^n s_i l_i$ . The instantaneous utility of an individual agent in group  $i$  depends positively on consumption,  $c_i$ , and negatively on labor supply,  $l_i$ . Given these assumptions, the objective function of group  $i$  is its discounted-sum of utilities over an infinite-time horizon:

$$U_i = \int_0^{\infty} s_i u(c_i, l_i) e^{-\rho t} dt, \quad \rho > 0, \quad i = 1, 2, \dots, n,$$

where the utility function of each member  $u(c_i, l_i)$  is assumed to be strictly concave in  $(c_i, l_i)$ . The final good is used for consumption and capital formation. Since we have assumed that the aggregate production technology is commonly owned by all the groups, the capital formation is determined by

$$\dot{k} = Akf(\sum_{i=1}^n s_i l_i) - \sum_{i=1}^n s_i c_i. \quad (11)$$

Each group maximizes  $U_i$  by selecting the sequences of  $c_i$  and  $l_i$  subject to (11) together with the given initial level of capital,  $k_0 (> 0)$ .

In this differential game, each player's strategies are its consumption and labor supply and the state variable of the game is the aggregate stock of capital. As well as in the previous section, we focus on the Markov-perfect Nash equilibrium. We thus assume that each group's strategies,  $c_i$  and  $l_i$ , are functions of the current level of the aggregate capital  $k$  alone. As a consequence, that the value function of the  $i$ -th group's optimization problem at time  $t$  can be written as

$$V_i(k(t)) \equiv \max \int_t^{\infty} s_i e^{-\rho(\tau-t)} u(c_i(\tau), l_i(\tau)) d\tau.$$

This function satisfies the Hamilton-Jacobi-Bellman (HJB) equation such that

$$\rho V_i(k) = \max_{c_i, l_i} \{s_i u(c_i, l_i) + V_i'(k) [Akf(\sum_{i=1}^n s_i l_i) - \sum_{i=1}^n s_i c_i]\} \quad (12)$$

for all  $t \geq 0$ . In solving the maximization problem defined in the right-hand-side of (12), the  $i$ -th group takes the other players' strategies,  $\{c_j, l_j\}_{j \neq i}$  ( $j = 1, 2, \dots, n$ ), as given. The first-order conditions for maximization are:

$$u_c(c_i, l_i) - V_i'(k) = 0, \quad i = 1, 2, \dots, n, \quad (13)$$

$$u_l(c_i, l_i) + V_i'(k) Ak f'(L) = 0, \quad i = 1, 2, \dots, n. \quad (14)$$

Equations (13) and (14) yield a set of the Markov-perfect Nash solutions that are expressed as  $\{c_i(k), l_i(k)\}$  ( $i = 1, 2, \dots, n$ ). Note that due to our assumption of strict concavity of  $u(c_i, l_i)$  and  $f(L)$ , the above maximization problem satisfies the second-order conditions as well.

### 3.2 Balanced-Growth Equilibrium

The Markov-perfect Nash strategies,  $\{c_i(k), l_i(k)\}$  ( $i = 1, 2, \dots, n$ ), simultaneously satisfy (13) and (14). Substituting these optimal solutions into the HJB equation (12), we obtain

$$\rho V_i(k) = s_i u(c_i(k), l_i(k)) + V_i'(k) [Ak f(\sum_{j=1}^n s_j l_j(k)) - \sum_{j=1}^n s_j c_j(k)]. \quad (15)$$

Using the envelop theorem, we find that differentiation of both sides of (15) with respect to  $k$  yields:

$$\begin{aligned} \rho V_i'(k) &= V_i'(k) [Af(\sum_{j=1}^n s_j l_j(k)) + Ak(\sum_{j \neq i}^n s_j l_j'(k)) f'(\sum_{j=1}^n s_j l_j(k))] \\ &\quad - V_i'(k) \sum_{i \neq j}^n s_j c_j'(k) + V_i''(k) [Ak f(\sum_{j=1}^n l_i(k)) - \sum_{j=1}^n s_i c_j(k)], \\ i &= 1, 2, \dots, n. \end{aligned} \quad (16)$$

In the following analysis, we restrict our attention to the symmetric equilibrium in which  $s_i = s$ ,  $c_i(k) = c(k)$  and  $l_i(k) = l(k)$  for all  $i$ . We also focus on the balanced-growth equilibrium where  $c$ ,  $k$  and  $y$  grow at a positive, common rate. Since the production technology has the  $Ak$  property, these restrictions require that the optimal consumption of each agent is proportional to the aggregate capital stock and that the optimal labor supply is constant over time. Hence, we can set

$$c_i(k) = \phi k, \quad l_i(k) = l, \quad i = 1, 2, \dots, n, \quad (17)$$

where  $\phi$  and  $l$  are unknown, positive constants.

To be more specific, we now assume that the instantaneous utility function takes the following form:

$$u(c_i, l_i) = \frac{c_i^{1-\sigma}}{1-\sigma} h(l_i), \quad 0 < \sigma < 1.$$

Here, we assume:

$$0 < \sigma < 1, \quad h'(l) < 0, \quad h''(l) < 0,$$

which ensure that  $u(c_i, l_i)$  is strictly concave in  $(c_i, l_i)$ . Then, in view of (17), condition (13) becomes  $V'(k) = \phi^{-\sigma} k^{-\sigma} h(l)$ , implying that

$$V''(k) = -\sigma \phi^{-\sigma} k^{-\sigma-1} h(l). \quad (18)$$

Since (17) means that  $c'_i(k) = \phi$  and  $l'_i(k) = 0$ , from (17), (??) and (18) we find that (16) can be written as

$$\rho = Af(Nl) - s(n-1)\phi - \sigma[Af(Nl) - N\phi],$$

implying that

$$\phi = \frac{\rho + (\sigma - 1)Af(Nl)}{N/n + (\sigma - 1)N}, \quad (19)$$

where  $N = sn$ . Note that under the symmetric condition, (13) and (14) give  $\phi h'(l) = (\sigma - 1)Af'(Nl)$ . Hence, from (19) we obtain

$$\frac{\rho/A + (\sigma - 1)f(Nl)}{N/n + (\sigma - 1)N} = \frac{(\sigma - 1)f'(Nl)}{h'(l)}. \quad (20)$$

This equation determines the equilibrium level of individual labor supply,  $l$ , on the balanced-growth path.<sup>4</sup> Letting the steady-state level of  $l$  be  $l^*$ , the relation between consumption and the aggregate capital stock is thus given by

$$c = \frac{\rho + (\sigma - 1)Af(Nl^*)}{N/n + (\sigma - 1)N} k,$$

where  $l^*$  is the solution of (20). Consequently, the balanced-growth rate, which is given by  $g = \dot{k}/k = Af(Nl^*) - Nc/k$ , can be expressed as

$$g = \frac{Af(Nl^*) - n\rho}{1 + (\sigma - 1)n}. \quad (21)$$

### 3.3 Voracity Effect

In our setting, the steady-state levels of  $g$  and  $l$  are determined by (20) and (21). Equation (21) shows that the effect of a change in productivity on growth is:

$$\frac{dg}{dA} = \frac{1}{1 + (\sigma - 1)n} \left[ f(Nl^*) + Af'N \frac{dl^*}{dA} \right], \quad (22)$$

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<sup>4</sup>Suppose that  $f(0) = 0$ ,  $f'(0) = +\infty$  and  $h'(0) = 0$ . Then it is easy to see that if  $1 + (\sigma - 1)n < 0$ , (20) has a unique solution. If  $1 + (\sigma - 1)n > 0$ , there may exist multiple balanced-growth paths. In this paper we do not consider the case of nonunique long-run equilibrium.

where from (20)  $dl^*/dA$  is given by

$$\frac{dl^*}{dA} = \frac{\rho h'^2}{A^2 (\sigma - 1) [h'^2 f' N - s (1 + (\sigma - 1) n) (N f'' h' - f' h'')]}$$

This has a negative sign, if  $1 - (1 - \sigma) n < 0$  and  $N f'' h' > f' h''$ . Hence, if a rise in  $A$  yields a sufficient reduction of individual labor supply, then (??) shows that  $dg/dA$  has a positive value even under  $1 + (\sigma - 1) n < 0$ . This means that introducing endogenous labor supply and increasing returns to scale reduces the possibility that the voracity effect prevails. However, note that  $dl^*/dA$  can be negative for the case of  $1 + (\sigma - 1) n > 0$  as well. Thus in our generalized setting, the voracity effect may be present even though  $1 + (\sigma - 1) n > 0$ , under which a higher  $A$  always increase the long-term growth rate in the standard  $Ak$  technology. In a similar vein, we may confirm that the growth of an increase in the number of interest groups cannot be uniquely determined without imposing further specification on the functional forms and the magnitudes of parameters involved in the model. Notice that This is true regardless whether a rise in the number of group  $n$  is associated with a rise in the total population  $N$  : neither the signs of  $dg/dn|_{N=\text{constant}}$  nor  $dg/dn|_{s=\text{constant}}$  are determined under our assumptions.

As an example, let us assume:

$$h(l) \equiv \frac{1}{\chi} (1-l)^{\chi(1-\sigma)}, \quad 0 < \chi < 1, \quad (23)$$

$$f(L) = L^\beta, \quad 0 < \beta < 1,$$

which is defined on  $l \in [0, 1]$ . Since  $1-l$  represents leisure, (23) means that the instantaneous felicity of the household is expressed as the standard Cobb-Douglas function of consumption and leisure in such a way that

$$u(c, l) \equiv \frac{\chi^{-1} (c(1-l)^\chi)^{1-\sigma}}{1-\sigma}. \quad (24)$$

Under these specifications, we see that (20) becomes

$$\frac{\rho/A + (\sigma - 1) (Nl)^\beta}{N/n + (\sigma - 1) N} = \frac{\beta (Nl)^{\beta-1}}{(1-l)^{\chi(1-\sigma)-1}}. \quad (25)$$

It is easy to see that under our restrictions,  $0 < \sigma < 1$  and  $0 < \beta < 1$ , equation (25) has a unique solution of  $l^* \in (0, 1)$ . Thus the balanced-growth rate given by

$$g = \frac{AN^\beta l^{*\beta} - n\rho}{1 + (\sigma - 1) n}$$

is also uniquely determined as well.<sup>5</sup>

In this situation, the effect of a change in the total factor productivity,  $A$ , on the equilibrium level of employment,  $l^*$ , is given by:

$$\frac{dl^*}{dA} = \frac{\rho(1-l^*)^{-\chi(1-\sigma)-1}}{D},$$

where

$$D = A^2(\sigma-1)\beta(Nl^*)^{\beta-1}[\rho(1-l^*)^{-\chi(1-\sigma)-1} - s(1+(\sigma-1)n)(N(\beta-1)(Nl^*)^{-1} - (1-l^*)^{-\chi(1-\sigma)-1})].$$

Since  $0 < \sigma < 1$  and  $0 < \beta < 1$ , the above shows that  $dl^*/dA$  has a negative sign if  $1 + (\sigma - 1)n < 0$ . Therefore, the impact of a rise in  $A$  on the balanced-growth rate determined by (22) is ambiguous. That is, even in the presence of voracity-effect condition,  $1 + (1 - \sigma)n < 0$ , the reduction of  $l^*$  caused by a rise in  $A$  may reduce the aggregate output-capital ratio,  $A(Nl^*)^\beta$ . If this is the case, a higher  $A$  does not increase labor input, which may prevent the agents from over consumption and thus the voracity effect will not be observed.

### 3.4 Separable Utility

We first consider a simpler case where the instantaneous utility function is additively separable the utility function is given by

$$u(c_i, l_i) = \log c_i + \Lambda(l_i), \quad \Lambda' < 0, \quad \Lambda'' < 0, \quad (26)$$

conditions (??) and (18) respectively become  $V'(k) = 1/\phi k$  and  $V''(k) = -1/\phi k^2$ . Thus (16) shows that  $\phi = \rho$ . As a result, (13) and (14) yield

$$-\rho\Lambda'(l) = Af'(Nl). \quad (27)$$

This equation determines the steady-state level of individual labor supply for the case of additively separable utility. The balanced-growth rate in this case is

$$g = Af(Nl^*) - n\rho. \quad (28)$$

First, consider the effect of a rise in  $A$ . Using (27) and (28), we find:

$$\frac{dg}{dA} = f(Nl^*) - \frac{AN(f')^2}{Af''N + \rho\Lambda''} > 0.$$

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<sup>5</sup>As pointed out in footnote (), the steady-state level of  $l^*$  (so the balanced growth rate) may not be unique when  $1 + (1 - \sigma)n > 0$ . Itaya and Mino (2007) consider the case of multiple balanced-growth paths as well.

Therefore, if the agents utility functions are additively separable between consumption and labor, there is no voracity effect.

Next, consider a change in the number of interest groups. If the total population,  $N (= sn)$ , is constant, a rise in  $n$  (so a decrease in the number of agents in each group,  $s$ ) unambiguously lowers the balanced growth rate, because from (27) the steady state level of  $l$  does not depend on  $n$  but on  $N$ . In contrast, if the number of groups stays constant but the number of members of each group increases (so the total population  $N$  rises), we obtain

$$\left. \frac{dg}{ds} \right|_{n=\text{constant}} = \frac{\rho n l^* A f' \Lambda''}{AN f'' + \rho \Lambda''} > 0.$$

Thus if the number of players is constant, a rise in population stimulates growth because there is only scale effect. However, if the population rises due to an increase in the number of groups, we obtain:

$$\left. \frac{dg}{dn} \right|_{s=\text{constant}} = \frac{\rho s l^* A f' \Lambda''}{AN f'' + \rho \Lambda''} - \rho.$$

The sign of this effect is ambiguous.

As for an example, let us specify  $f$  and  $h$  functions in such a way that  $f(L) = L^\beta$  and  $\Lambda(l) = -l^{1+\chi}/(1+\chi)$ , where  $0 < \beta < 1$  and  $\chi > 0$ . Then from (27) the steady-state level of  $l$  is given by

$$l^* = \left[ \frac{(1-\sigma) A \beta}{\rho} \right]^{\frac{1}{\chi+1-\beta}} N^{\frac{\beta-1}{\chi+1-\beta}}.$$

Hence, the balanced growth rate given by (28) is written as

$$g = A^{\frac{\chi+1}{\chi+1-\beta}} \left[ \frac{(1-\sigma) \beta}{\rho} \right]^{\frac{\beta}{\chi+1-\beta}} (sn)^{\frac{\beta\chi}{\chi+1-\beta}} - \rho n.$$

Since the right-hand side of the above is strictly concave in  $n$ , the the growth effect of a change in the number of agents is:

$$\left. \frac{dg}{dn} \right|_{s=\text{constant}} > 0 \text{ for } n < \hat{n}, \quad \left. \frac{dg}{dn} \right|_{s=\text{constant}} < 0 \text{ for } n > \hat{n},$$

where  $\hat{n}$  satisfies

$$\left( \frac{\beta\chi}{\chi+1-\beta} \right) A^{\frac{\chi+1}{\chi+1-\beta}} \left[ \frac{(1-\sigma) \beta}{\rho} \right]^{\frac{\beta}{\chi+1-\beta}} s^{\frac{\beta\chi}{\chi+1-\beta}} \hat{n}^{\frac{(\beta-1)(\chi+1)}{1+\chi-\beta}} = \rho.$$

Namely, when the number of interest groups is smaller than  $\hat{n}$ , the scale effect due to the presence of increasing returns dominates the negative effect of common pool problem caused by an increase in the number of players. However, if  $n$  exceeds  $\hat{n}$ , the scale effect is not large enough to cancel the common pool effect and thus a larger number of interest groups depresses the long-term growth. This example demonstrates that, unlike the representative-agent economy with increasing returns, a rise in the scale of economy may have a negative impact on growth if property rights are insecure.

The presence of scale effect, i.e. a larger economy grows faster, has been often criticized because it does not fit well to the empirical reality. Our example presents a reply to such a question: if property rights are not well established, then the presence of scale effect supported by increasing returns does not always produce better performance in long-term growth. This may explain poor growth performances in some developing countries with large population and less established property rights.

## 4 Conclusion

This paper has examined a growth model with insecure private property rights. We introduce endogenous labor supply into the standard  $Ak$  growth model that has been commonly used in the existing literature on property rights and growth. Since our model allows the presence of increasing returns to scale, the growth performance of the economy depends not only on the negative effect of common pool problem but also on the positive scale effect generated by the presence of increasing returns. In particular, the effects of a rise in productivity and the population of economic agents would be significantly different from the results obtained in the standard setting. Our discussion indicates that examining economies without secure property rights under alternative environments would be useful to shed further light on the relationship between property rights and growth.

In this paper we have focused on the balanced-growth equilibrium. An extended version of this paper, Itaya and Mino (2007), explores dynamic properties, determinacy of equilibrium of the model economy and find that there is a close connection between equilibrium indeterminacy and the presence of voracity effect. In addition to dynamic analysis, the welfare implication and policy recommendations in our setting would be worth exploring. We are planning to conduct further investigation on these topics.<sup>6</sup>

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<sup>6</sup>See also Long and Sorger (2006) for a generalized modelling of Tornell and Lane (1998, 1999).

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