# On the Strategic Behavior of Information Service Company

Tetsuya Furukawa<sup>\*</sup> School of Economics, Tokyo International University 1-13-1 Matoba-Kita, Kawagoe, SAITAMA 350-1197 Japan E-mail:ftetsuya@tiu.ac.jp

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#### Abstract

In this paper, we consider a rating agency for investment objects as a representative type of information service companies. We investigate a phenomenon that the more "authorized" the rating agency's information becomes, the more the profitability of the investment object depends on what the rating agency evaluates, rather than its own nature. We select two types of equilibria and investigate the sufficient conditions for existence of them.

**Keywords**: rating agency, asymmetric information, strategic behavior, externality.

JEL classification: D82, G14

## 1 Introduction

In our daily life, we often make a decision based on various kinds of information provided by information service companies that make a profit on providing beneficial information and levying fees for us. The most famous example of the company is Michelin and its guidebook. Some (or Many) of tourists look for the highly evaluated hotels in it. From hotels' point of view, the ranking affects their profit, and they are obliged to be careful about it.

In this paper, we consider a rating agency of investment objects as a representative type of information service companies. When we consider the rating agency's

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behavior, there is some special feature about the agency. That is, the more "authorized" the rating agency's information becomes, the more the profitability of an investment object depends on what the rating agency evaluates, rather than its own nature.

Suppose that there are two objectives, which have the same profitability under the same size of investment. If the agency sends a message to investors that one is more profitable than the other, the former will make a profit but the latter fails to make a profit. If the agency behaved honestly for investors, he would send a message that "both of them have the same profitability", and both of objects will make a profit.

Note that, the message that one is profitable and the other is not would become "true" in ex post sense. Therefore, the agency can keep reputation after the outcome becomes public. His expectation is always fulfilled when many of the investors employ the agency and obey his message. The agency's message improves the efficiency of an economy if it is true in ex ante sense because it prevents money from being invested in inefficient objects. It is natural, however, that the agency has no incentive to make efforts to provide accurate information with some cost because he makes a profit and keep reputation without cost after being 'authorized'. It might cause inefficiency.

The purpose of this paper is to analyze the 'authorized' agency's behavior and the nature of equilibria. The model considered here is one of the applications of the traditional reputation model (Kreps et al.(1982), Kreps and Wilson(1982), Milgrom and Roberts(1982)). The possible types of the rating agency are H and D. Type H is honest type and type D is dishonest type. 'Honest' means that the agency of type H is eager to make efforts to provide reliable information whenever investors need its information, even when he has already been 'authorized' and its behavior has no effect on his profit. The agency of type D, on the contrary, only makes efforts when the efforts certainly lead to raising profit.

The type D agency would pretend the behavior of type H agency when it affects the belief of investors and raises his profit. The special feature of this paper is that the share (or number) of investors who utilize the information provided by the rating agency affects accuracy of the agency's information. In other words, there exists some external effect on accuracy caused by the number of the agency. Therefore, the relationship between the effort of the agency and the accuracy of information becomes weaker than the case when there is no external effect.

In Furukawa(2000,2005), I analyzed the similar models as a preparation for this paper's analysis. In Furukawa(2000), I analyzed the model that the accuracy of the agency's information does not depend on his effort but depends on the share or number of investors who utilize the information provided by agency. In that model, no strategic nature of the agency's behavior was analyzed. In Furukawa(2005), on the contrary, the accuracy of the agency's information does depend on his effort but does not depends on the share or number of investors who utilize the information provided by the agency. I analyzed the strategic feature of the agency, but did not analyze the nature of externality. In this model, I would integrate them into one model.

In this model, we can find various kinds of equilibria depending on the size of parameter. We attempt to investigate two interesting equilibria and obtain sufficient conditions for existence of them. By investigating them, we realize how the sufficient conditions to exist depend on the initial belief of investors for the agency.

### 2 The Model

We consider a two-period economy with two kinds of agents, investors and a rating agency. Investors make a decision about investment, depending on the information whether they collect or the rating agency provides. The rating agency provides information that affects the investment decision of investors who pay a fee to him.

In each period, there are two investment objects,  $X_1, X_2$ . The possible outcomes of investment are 'success' and 'failure'. We call the outcome of investment 'success' if an investor gets +1 of net return after one unit of investment, and 'failure' if she gets -1 of net return. Nobody can observe the outcome of investment ex ante because it is determined uncertainly. Both the rating agency and all the investors can observe the outcome ex post.

There are two possible types of the investment objects, G and B. G is an abbreviation of 'good' and B is 'bad'. We suppose that either  $X_1$  or  $X_2$  is type G. The probability at which the outcome would be 'success' depends on the total size of investment on each object, and the probability of success in the type G case is relatively higher than that in the type B case given the size of investment. It is common for both type that the larger the size of investment, the higher the probability of success is. If the size of investment in type B object is much larger than type G object, the probability of success might be higher in type B object than type G object.

Investors and the agency cannot observe the types of the objects ex ante, but the agency can observe the types ex post if he takes some cost to collect information about the object. Though there is the difference between investors and the agency about the ability to observe the types of the objects, both of them can observe the outcome of investment in each period.

Let  $P^G$  be the probability that the object of type G succeeds, and  $P^B$  is defined in the same way. We often use an expression  $P^i(K_i)$  in the following discussion to show that each probability depends on the amount of investment in the object of type  $i \cdot P^i(K_i)$  is twice continuously differentiable, and  $P^i(K_i) = 0$ ,  $d(P^i(K_i))/dK_i > 0$ , the probability is a increasing function of  $K_i$ .

The total sum of investment in the whole economy is  $\overline{K}$ . We suppose that  $1 > P^G(K_G) > P^B(K_B) \ge 0$  for any  $K_i \le \overline{K}$ . This means that the object of type G might end in failure even when all the resource of investment is thrown into it.

The initial belief of investors that the object  $X_1$  (or  $X_2$  ) would be type G is

supposed to be 1/2.

### 2.1 Investors

There is a continuum of investors, and the set of investors is represented by an interval  $[0, \bar{K}]$ . Each investor has one unit of endowment and she makes a decision which and how much she invests in each object. Because each investor's endowment is one unit and the number of investors can be regarded as  $\bar{K}$ , the total sum of investment in the whole economy is  $\bar{K}$ .

Before making an investment decision, each investor chooses whether he employs the agency or not. If an investor decides to employ him, she has to pay fees f > 0.

Investors cannot observe not only the types of the objects but also the type of the agency all the time. The belief of investors that the agency's type would be H in period t is denoted by  $p_t(t = 1, 2)$ .

### 2.2 A Rating Agency

There is only one rating agency in the economy considered here. There are two possibilities about the type of agency, H and D (these are abbreviations of 'Honest' and 'Dishonest'). The types of the agency provide the agency's behavior for collecting information. Type H agency always makes efforts to collect information about objects. Though collecting information needs some cost, c, type H agency does not care the cost and honestly makes efforts. Type D agency, on the contrary to type H, makes efforts only when doing this brings him a benefit. That is, he sends an uncertain message to investors when making efforts does not lead to higher profit for him.

We describe the relationship between accuracy of information provided by the agency and the agency's effort in the following way. Let Q(m|r, e) be the probability that the agency sends the message about the type of an object denoted by m to the investors when the true type of the object is r, and the agency makes efforts, denoted by e = 1, or not, denoted by e = 0. We suppose Assumption 1 about Q(m|r, e).

#### Assumption 1

$$\begin{array}{ll} Q(r|r,1) &= 1 - Q(r'|r,1) = 1, \\ Q(r'|r,0) &= Q(r|r,0) = \frac{1}{2}, \end{array}$$

where  $r \neq r'$ .

### 2.3 Timing of choices

The timing of players' choices in this economy is as follows. At first, the type of the agency is determined by nature. Investors are not able to observe it, and they have the initial belief  $p_1$  that the agency is type H.  $p_1$  is supposed to be exogenously given.

Secondly, the type of the investment object is also determined by nature. Investors cannot observe it. By making efforts to know types with cost c, the agency comes to know it. However, if the agency makes no effort, he knows nothing. The efforts to know the type of the object is executed in the cases when the type of the agency is H, or when the type is D and the investigation leads to the higher profit for the agency.

Thirdly, each investor makes a decision whether she employs the agency or not. If she decides not to employ the agency, she invests a half of her endowment to each of objects,  $X_1$  and  $X_2$ . The agency delivers the information about the types of objects to the investors who decide to utilize the information from agency. The investors employing the agency make a decision depending on the information the agency provide.

Fourthly, outcomes of investment in each object are determined. Investors and the agency observe the outcomes. The agency publishes what his message was to not only the investors who employ him but the ones who did not employ him. All the investors recalculate their belief based on the outcomes and the agency's announcement. Return to ' secondly' situation and the second period starts.

### **3** Investors' Belief and Behavior

### 3.1 Investors' evaluation for the agency

Let  $y_t$  be the probability that the agency of type D honestly collect information about the investment objects. Because of the fact that the agency of type H always makes efforts to collect information and the agency of type D collects information at probability  $y_t$ , the probability that investors receive the *correct* message is,

$$p_t + (1 - p_t)(y_t + (1 - y_t)\frac{1}{2}).$$

The probability that investors receive the *incorrect* message is,

$$(1-p_t)(1-y_t)\frac{1}{2}.$$

Investors cannot observe the types of the objects directly but can observe outcomes of investment in each period. At first, using the above equation, we calculate the probability at which the message from the agency is " $X_1$  is G" and the outcome of  $X_1$  becomes success is

$$P^{G}(K)\left[p_{t}+(1-p_{t})(y_{t}+(1-y_{t})\frac{1}{2})\right]+P^{B}(K)\left[(1-p_{t})(1-y_{t})\frac{1}{2}\right],$$
$$p_{t}P^{G}(K)+(1-p_{t})\left[P^{G}(K)(y_{t}+(1-y_{t})\frac{1}{2})+P^{B}(K)(1-y_{t})\frac{1}{2}\right].$$

or

S denotes the event that an outcome is success. The probability that the agency is H under the event that (1) the message is G and (2) the outcome is success is,

$$P(H|G,S) = \frac{p_t P^G(K)}{p_t P^G(K) + (1-p_t)[P^G(K)(y_t + (1-y_t)\frac{1}{2}) + P^B(K)(1-y_t)\frac{1}{2}]}.$$
 (1)

This is equivalent to the second period belief about the agency when the above events happen,  $p_2$ . In the same manner, we can calculate the other cases as P(H|B,S), P(H|G,F) and so on.

### **3.2** The Expected Profit of Investors

We suppose that the investors who make an investment decision without information from agency invest in each object,  $X_1$  and  $X_2$ , with 1/2 unit. Let  $x_t(t = 1, 2)$  be a share of investors who utilize the agency's information. We calculate the amount of investment in the object which is regarded as 'G' by the agency is  $(x_t + (1/2)(1 - x_t))\bar{K}$  because the agency tells  $x_t$  of investors "the object is 'G', and you should invest in it", and half of the  $(1 - x_t)$  investors who do not employ the agency invest in the object. In the same manner, we derive the amount of investment in the object regarded as 'B' is  $(1/2)(1 - x_t)\bar{K}$ .

We define  $\mu_t$  as the probability in which the object is 'G' when the agency regards it as 'G' in period t.  $\mu_t$  is,

$$\mu_t \equiv p_t + (1 - p_t) \left( y_t + (1 - y_t) \frac{1}{2} \right).$$

The Expected profit of investors who employ the agency is,

$$2[\mu_t P^G + (1 - \mu_t) P^B] - 1 - f.$$
(2)

The Expected profit of investors who do not employ the agency is,

$$\frac{1}{2} \left[ P^{G} \left( x_{t} \bar{K} + \frac{1}{2} (1 - x_{t}) \bar{K} \right) + P^{B} \left( x_{t} \bar{K} + \frac{1}{2} (1 - x_{t}) \bar{K} \right) - 1 \right] + \frac{1}{2} \left[ P^{G} \left( \frac{1}{2} (1 - x_{t}) \bar{K} \right) + P^{B} \left( \frac{1}{2} (1 - x_{t}) \bar{K} \right) - 1 \right].$$
(3)

### **3.3 A Criteria Function**

To analyze the investors' behavior, we define a criteria function. "Criteria" means that the investors' behavior depends on a value of the function. Investors utilize the agency's information when the value of the criteria function is positive, and vise versa.<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>For simplicity, we neglect the case where the value of the criteria function is equal to zero.

We denote the criteria function in period t by  $C^{t}(x_{t}, y_{t})$ , and it is defined by subtracting equation (3) from (2).

$$C^{t}(x_{t}, y_{t}) \equiv 2 \left[ \mu P^{G} \left( x_{t} \bar{K} + \frac{1}{2} (1 - x_{t}) \bar{K} \right) + (1 - \mu) P^{B} \left( x_{t} \bar{K} + \frac{1}{2} (1 - x_{t}) \bar{K} \right) \right] - \frac{1}{2} \left[ P^{G} \left( x_{t} \bar{K} + \frac{1}{2} (1 - x_{t}) \bar{K} \right) + P^{B} \left( x_{t} \bar{K} + \frac{1}{2} (1 - x_{t}) \bar{K} \right) \right] - \frac{1}{2} \left[ P^{G} \left( \frac{1}{2} (1 - x_{t}) \bar{K} \right) + P^{B} \left( \frac{1}{2} (1 - x_{t}) \bar{K} \right) \right] - f$$
(4)

By definition of  $C^t(x_t, y_t)$ , we derive the following lemma.

**Lemma 1** If  $dP^i/dK$  is a increasing function of K, then

$$\partial C^t(x_t, y_t) / \partial x_t > 0$$
, for all  $t, x_t, y_t$ .

### 4 Behavior of the rating agency and equilibria

### 4.1 Behavior of the rating agency

Next we consider the agency's behavior. The payoff for the agency of type D in each period t(t = 1, 2) is  $x_t \bar{K} f - cy_t$ , and the sum of the payoff for two periods is,

$$x_1ar{K}f-cy_1+\delta[x_2ar{K}f-cy_2],$$

where  $\delta$  is a discount factor (  $\delta \in [0, 1]$  ).

We suppose that the alternatives of the agency are only  $y_t = 0$  and 1 for simplicity. We can conclude immediately that  $y_2 = 0$ . There is no benefit for the agency to choose  $y_2 = 1$  with cost c, because he cannot commit himself to choose  $y_2 = 1$ and investors determine to choose  $x_2$  provided that  $y_2 = 0$ .

 $y_1$  and  $x_1$  are simultaneously determined by both the agency and investors. Therefore, the agency strategically choose the level of  $y_1$  with considering the effect it on  $x_2$ .

#### 4.2 Strategy of investors in the second period

We have already known that  $y_2 = 0$ . Therefore,

$$\mu_2 = p_2 + (1 - p_2)\frac{1}{2} = \frac{1}{2}(1 + p_2).$$

The values of the criteria function for each case are,

$$C^{2}(1,0) = 2[\mu_{2}P^{G}(\bar{K}) + (1-\mu_{2})P^{B}(\bar{K})] - \frac{1}{2}[P^{G}(\bar{K}) + P^{B}(\bar{K})] - f$$

$$= \left(p_2 + \frac{1}{2}\right) \left(P^G(\bar{K}) - P^B(\bar{K})\right) + P^B(\bar{K}) - f$$
(5)

$$C^{2}(0,0) = 2\left[\mu_{2}P^{G}\left(\frac{1}{2}\bar{K}\right) + (1-\mu_{2})P^{B}\left(\frac{1}{2}\bar{K}\right)\right] - \frac{1}{2}[2P^{G}(\bar{K}) + 2P^{B}(\bar{K})] - f$$
  
$$= p_{2}\left(P^{G}\left(\frac{1}{2}\bar{K}\right) - P^{B}\left(\frac{1}{2}\bar{K}\right)\right) - f.$$
 (6)

 $C^{2}(1,0) > C^{2}(0,0)$  by lemma 1, we derive Lemma 2 as follows.

Lemma 2 The strategy of investors in the second period are,

$$\begin{aligned} x_2 &= 1 \quad if \ C^2(1,0) > C^2(0,0) > 0, \\ x_2 &= 0 \quad if \ 0 > C^2(1,0) > C^2(0,0), \\ x_2 &= 1 \ or \ x_2 = 0 \quad if \ C^2(1,0) > 0 > C^2(0,0). \end{aligned}$$

The third case in Lemma 2 corresponds to the situation when a investor should choose  $x_2 = 1$  if all the other investors choose  $x_2 = 1$ . and vise versa.

### 4.3 Strategies in the first period

In this subsection, we consider the behaviors of both investors and the agency.

In the first period, when the agency sends the message to investors that the type of the object is G, the probability that the object is really G is,

$$\mu_1 = p_1 + (1 - p_1) \frac{1}{2} (1 + y_1).$$

Given  $y_1$ , the values of the criteria function are,

$$C^{1}(1, y_{1}) = 2[\mu_{1}P^{G}(\bar{K}) + (1 - \mu_{1})P^{B}(\bar{K})] - \frac{1}{2}[P^{G}(\bar{K}) + P^{B}(\bar{K})] - f$$
  
$$= \left((1 - p_{1})y_{1} + p_{1} + \frac{1}{2}\right)(P^{G}(\bar{K}) - P^{B}(\bar{K})) + P^{B}(\bar{K}) - f$$
(7)

$$C^{1}(0, y_{1}) = 2\left[\mu_{1}P^{G}\left(\frac{1}{2}\bar{K}\right) + (1-\mu_{1})P^{B}\left(\frac{1}{2}\bar{K}\right)\right] - \frac{1}{2}\left[2P^{G}(\bar{K}) + 2P^{B}(\bar{K})\right] - f$$
  
$$= ((1-p_{1})y_{1} + p_{1})\left(P^{G}\left(\frac{1}{2}\bar{K}\right) - P^{B}\left(\frac{1}{2}\bar{K}\right)\right) - f.$$
(8)

Given  $y_1$ ,  $C(x_1, y_1)$  is an increasing function of  $x_1$ . Therefore,

$$C^{1}(1, y_{1}) > C^{1}(0, y_{1}),$$
, for all  $y_{1}$ .

Connecting the relationship between the value of the criteria function and investors behavior to the above inequality brings about Lemma 3.

$$\begin{aligned} x_1 &= 1 \quad \text{if } C^1(1,y_1) > C^1(0,y_1) > 0, \\ x_1 &= 0, \quad \text{if } 0 > C^1(1,y_1) > C^1(0,y_1), \\ x_1 &= 1 \quad \text{or } x_1 = 0, \quad \text{if } C^1(1,y_1) > 0 > C^1(0,y_1). \end{aligned}$$

The third case in Lemma 3 corresponds to the situation when a investor should choose  $x_1 = 1$  if all the other investors choose  $x_1 = 1$ . and vise versa.

All the cases in Lemma 3 would be possible in any value of  $y_1$  if we suppose some assumptions on the size of parameters and the shape of  $P^i$ . Because we would like to consider the interesting cases, we assume that the value of the criteria function in the first period is,

#### Assumption 2

$$C^{1}(1,1) > 0 > C^{1}(0,0).$$

In order to describe the fact that t  $x_2$  depends on  $x_1, y_1$  through  $p_2$  explicitly, we use a functional form  $x_2(x_1, y_1)$  hereafter. By using this form, the values of  $y_1$  are,

(1) If 
$$\delta(x_2(x_1,1)-x_2(x_1,0))f\bar{K}-c>0$$
, then  $y_1=1$ ,

(2) If 
$$\delta(x_2(x_1,1)-x_2(x_1,0))f\bar{K}-c<0$$
, then  $y_1=0$ .

 $(x_2(x_1,1) - x_2(x_1,0))$  shows that, given the strategy of investors in the *first* period, how the strategy of investors in the *second* period changes depending on whether the agency makes efforts to collect information or not in the first period. If we suppose the size of f and c adequately, case (1) is true when  $x_2(x_1,1) = 1, x_2(x_1,0) = 0$ . We consider this case in the following discussion.

### 5 Two examples of equilibria

The equilibrium strategies of investors and the agency  $(x_1^*, x_2^*, y_1^*, y_2^*)$  are defined as follows.

- (1) The agency's strategy:  $y_1^*$  maximizes his two-period profit defined by  $x_1^*Kf cy_1 + \delta[x_2^*\bar{K}f cy_2^*]$  and  $y_2^* = 0$ , given  $x_1^*, x_2^*$  and  $p_1, f$ .
- (2) Each investor's strategy:  $x_1^*$  maximizes her expected profit in the first period, given  $y_1^*$  and  $p_1, f$ .  $x_2^*$  maximizes her expected profit in the second period, given  $y_2^* = 0, f$  and  $p_2$  which is recalculated after realizing the first period outcome and the agency's announcement.

In this model there are various kinds of equilibria depending on the value of  $p_1, f$  which are exogenously given. We focus on two types of equilibria,

- (1) Equilibrium 1 :  $x_1 = x_2 = 1, y_1 = y_2 = 0,$
- (2) Equilibrium 2 :  $x_1 = x_2 = 1, y_1 = 1, y_2 = 0,$

and investigate the sufficient conditions for existence of them

In Equilibrium 1, the agency of type D never makes efforts to collect information  $(y_1 = y_2 = 0)$ , but all the investors would like to utilize the information of the agency in both periods.

In Equilibrium 2, investor's behavior is the same as Equilibrium 1, but the type D agency collects information in the first period.

### 5.1 Sufficient conditions for existence of Equilibrium 1

The conditions for existence of Equilibrium 1 consist of three inequalities,

$$C^1(1,0) > 0 (9)$$

$$C^2(1,0) > 0 \tag{10}$$

$$\delta(x_2(1,1) - x_2(1,0))f\bar{K} - c < 0.$$
(11)

In order for Inequality (9) to be satisfied, the rearranged inequality

$$C^{1}(1,0) = \left(p_{1} + \frac{1}{2}\right) \left(P^{G}(\bar{K}) - P^{B}(\bar{K})\right) + P^{B}(\bar{K}) - f > 0,$$

has to be satisfied. It is right when  $p_1$  is large enough and f is small enough.

If  $p_1$  is large enough and the belief of the second period satisfies equation (10), that is,

$$C^{2}(1,0) = \left(p_{2} + \frac{1}{2}\right) \left(P^{G}(\bar{K}) - P^{B}(\bar{K})\right) + P^{B}(\bar{K}) - f > 0,$$

then  $x_2(1,0) = 1$ .

We conclude that the equilibrium 1 exists when  $p_1$  is large enough and f is small enough.

### 5.2 Sufficient conditions for existence of Equilibrium 2

The conditions for existence of equilibrium 2 consist of three inequalities,

$$C^1(1,1) > 0 \tag{12}$$

$$C^2(1,0) > 0 \tag{13}$$

$$\delta(x_2(1,1) - x_2(1,0))f\bar{K} - c > 0 \tag{14}$$

Inequality (12) is always satisfied by Assumptions 2. Remember that  $x_2$  depends on the value of  $y_1$  through  $p_2$  which is recalculated after observing the outcome in the first period. Inequality (13) says that for the existence of Equilibrium 2,  $C^2(1,0) > 0$ if  $y_1 = 1$ . By assumption with  $C^2(x_2, y_2)$ ,  $C^2(1, 0) > C^2(0.0)$  is always satisfied. In order for inequality (14) to be satisfied, it has to be satisfied that if  $y_1 = 0$  then  $C^2(0, 0) < 0$ . Therefore, we need that an inequality,

$$C^2(1,0) > 0 > C^2(0.0) \tag{15}$$

has to be satisfied. Rearranging this inequality, we have

$$\frac{f}{P^G(\frac{1}{2}\bar{K}) - P^B(\frac{1}{2}\bar{K})} > p_2 = p_1 > \hat{p}_2 > \frac{f - \frac{1}{2}(P^G(\bar{K}) + P^B(\bar{K}))}{P^G(\bar{K}) - P^B(\bar{K})}.$$

It is possible to find the pair of  $p_1$ , f which satisfies above conditions and Equilibrium 2 exists.

### 5.3 The characteristics of equilibrium 1 and 2

According to the sufficient conditions for the existence of Equilibrium 1, if  $p_1$  is large enough, investors employ the agency in both periods even though  $y_1 = 0$ . Though the type D agent chooses a dishonest behavior,  $y_1 = 0$ , the probability for the agency to be type D is small, and all the investors employ the agency.

It should be noted that the probability of success with the object regarded as 'G' by the agency is higher than the case when there is no rating agency, because all the investors employ the agency and the investors' endowments avoid dispersing. The role of the agency to prevent the investors' endowments from dispersing decreases the cost of employing the agency in effect. That is one of the reasons why all the investors employ the agency though he makes no effort.

The role of the agency explained above still exists in Equilibrium 2, but  $y_1 = 1$ is an equilibrium behavior here. As the discussion about Equilibrium 1, if  $p_1$  is too large, then  $y_1 = 0$  is better than  $y_1 = 1$  for the agency. Likewise, if  $p_1$  is small enough, regardless of the outcome in the first period, investors do not exploit the agency. In that case, the agency chooses  $y_1 = 0$ . Therefore, there exists the upper and the lower bound of  $p_2$  for Equilibrium 2 to exist.

# 6 Conclusion

In this paper, we have investigated a situation where there is uncertainty about the types of investment objects and asymmetric information about the agency which provides information with investment for investors. As the traditional reputation model analyzed, there is a possibility that a dishonest agent pretends an honest agent's behavior if it makes the dishonest agent's profit higher than he does not.

In the model considered here, there are various kinds of equilibria depending on parameter values. We have selected two equilibria from them and investigated the sufficient conditions for existence of them. The one of the equilibria has the feature that all investors employ the agency in both periods even though the dishonest agency never makes efforts to collect information. This equilibrium happens when the initial belief of investors that the agency is honest, denoted by  $p_1$ , is large enough and the employing fee of the agency, f, is small enough.

The other has the feature that all the investors employ the agency in both periods and the dishonest agency makes efforts to collect information in the first period. This is the case when  $p_1$  is between some upper and lower bound and f is small enough.

A notable feature of the model is that there is some kind of externality in the relationship between accuracy of the agency's information and the number of investors who employ the agency. The larger the number of investors employing the agency becomes, the more accurate the agency's information is. The reason why it happens is that the outcome of investment depends on the number of investors who invest in. In some sense, the role of the agency is not only providing information but coordinating investor's behavior. It is unnecessary for the agency to make efforts to collect information in order to keep reputation after he has established reputation and all the investors employ him. It may be a source of inefficiency in an economy, and I believe that it is worth exploring to analyze the nature of this point in detail.

# References

- Furukawa, T.(2000) "The Rating Agency's Effects on Investors Behavior", The Journal of Tokyo International University, The school of Economics, 23.(In Japanese)
- [2] Furukawa, T.(2005) "An Reputation Model Approach to Information Service Company", unpublished.(In Japanese)
- [3] Kreps, D.M., P.Milgrom, J.Roberts and R.Wilson (1982) Rational Cooperation in the Finitely Repeated Prisoners' Dilemma, Journal of Economic Theory, 27,245-252.
- [4] Kreps, D. and R. Wilson (1982) Reputation and Imperfect Information, Journal of Economic Theory, 27, 253-279.
- [5] Milgrom, P.R. and D.J. Roberts (1982) Predation, Reputation, and Entry Deterrence, Journal of Economic Theory, 27, 280-312.