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Real Options, Debt Financing, and Competition [1]

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1 Introduction

The real options approach has become an increasingly standard framework for the investment timing decision in corporate finance (see [2]). Although the early literature on real options investigated the investment decision of a monopolist, recent studies have investigated the problem of several firms competing in the same market from a game theoretic approach (see [1] for an overview). Especially, there are a lot of studies such as [4, 6, 19] that analyze the investment game in duopoly by incorporating equilibrium in a timing game into a real options model. More recently, some studies have been made on incomplete information between firms (e.g., [8, 14]) and agency conflicts in a single firm (e.g., [5, 16]).

On the other hand, one of the most important problems in corporate finance is to derive optimal capital structure. Theory of optimal capital structure by the trade-off between tax advantages and default costs was proposed by [12] in 1950s, and has still developed by many literatures such as [9, 3]. Capital structure and finance naturally have a deep connection with studies on the investment timing decision, though not many real options literatures focus on these matters. Remarkable studies in this area were conducted by Mauer and Sarkar [10] and Sundaresan and Wang [18, 17]. They investigated a firm value, investment timing, debt financing, and endogenous bankruptcy simultaneously in a model where a firm makes a real investment along with issuing debt.

However, existing literatures [10, 18, 17] consider only the monopolistic situation and investigate no competitive situation of several firms. In this paper, we extend the analysis by [18] to the case where several firms try to preempt a market. To put in more concretely, we derive the equilibrium investment strategies in the timing game (cf. [4, 6, 19]) under the competition between among firms that can issue debt at the investment. By this, we clarify the effects of the competition upon the firm value, investment timing, debt financing, and default timing. In order to analytically derive equilibrium, we consider a simple situation where more than one firm is not allowed to receive a profit flow from the market simultaneously[2].

We reveal the effects of debt financing in strategic investment, by deriving equilibrium in the following three types of duopoly:
(i) Competition between two symmetric firms. Both firms can issue debt. This may be interpreted as that each firm has its own lender.
(ii) Competition between two symmetric firms. Only one (called leader) that make an investment prior can issue debt, while the other (called follower) cannot issue debt. This may be interpreted as that there is only one lender for the investment project.
(iii) Competition between two asymmetric firms. One can issue debt, while the other is unlevered for exogenous reasons such as shortage of credit.

[1] This paper is an abbreviated version. All proofs, remarks and some computational results are omitted due to the page restriction.
[2] This assumption is essentially the same as that of [8, 19]

Note that, in the preemptive equilibrium under the competition between unlevered firms the investment is hastened at the zero NPV (Net Present Value) point (i.e., when the NPV of the investment is 0). In contrast, we show that, in equilibrium in Duopoly (i), (ii), and (iii), investment is hastened but later than the zero-NPV point and the firm value is positive. This results from a possibility of leader's bankruptcy. Coupon of debt which the leader issue becomes smaller than that of monopolist, while firms' leverage and credit spread are unchanged from those of a monopolist.

In particular, we show that in (iii) the levered firm always wins the race. That is, the levered firm invest with debt financing prior to the unlevered one and obtain much bigger profit than the unlevered one. We observe that the inequality (ii) < (i) < (iii) holds with respect to both the investment time and the value of the the levered firm.

In addition, we derive the equilibrium strategies in the competitive situation of n symmetric levered firms. As the number of the firms n becomes larger, the investment take places earlier and the coupon and the firm value become smaller. On letting $n \to +\infty$ the investment timing is hastened to the zero-NPV point as well as the firm value decreases to 0. Furthermore, we investigate the social loss due to the preemptive competition among firms by comparing the outcome in the preemptive equilibrium with that of the leader-follower game. We show that, the lager the number of firms, n, the greater the social loss.

The paper is organized as follows. The next section introduces, as a bench mark, the firm values and the investment strategies of unlevered and levered monopolists. In Section 3 we derive the firm value and the investment strategy in equilibrium in the three types of duopoly (i), (ii), and (iii). In Section 4 we derive the equilibrium in oligopoly and then investigate the loss due to the preemptive competition among firms. Section 5 concludes the paper.

2 Monopoly

2.1 Unlevered firm

First, let us explain the setup. This paper follows the one growth option model of [18].[3] Assume that the firm are risk-neutral and behaves in the interests of equityholders. [4] The firm with no initial assets has an option to enter a new market. The firm can choose the investment time, observing the market demand $X(t)$ at time t. The firm collects a profit flow $QX(t)$ by paying a sunk cost I, where $Q(>0)$ and $I(>0)$ are constants. We assume that the firm faces a constant tax rate $\tau \in (0,1)$. For simplicity, we assume that $X(t)$ obeys the following geometric Brownian motion:

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}B(t), X(0) = x(>0),$$

where, μ and $\sigma(>0)$ are constants, $B(t)$ represents the one-dimensional standard Brownian motion. The initial value $X(0) = x$ is a sufficiently small constant so that the firm has to wait for its exercise condition to be met.

Now, we consider the unlevered firm under all-equity financing. The unlevered firm determines its investment time T by solving the following optimal stopping time problem:

$$V_{ae}(x) = \sup_{T\in\mathcal{T}} \mathbb{E}[\int_T^{+\infty} \mathrm{e}^{-rt}(1-\tau)QX(t)\mathrm{d}t - \mathrm{e}^{-rT}I], \qquad (1)$$

[3]Literature [18] consider the firm with two sequentially ordered growth options in order to investigate debt overhang.

[4]Throughout the paper, we use the terminology "equityholders", following [18]. We do not distinguish equityholders and entrepreneur in the model. Hence, in the remainder of the paper, we can replace equityholders and equity value as entrepreneur and entrepreneur's value, respectively. To put it another way, we may consider that the entrepreneur does not issue equity but has money necessary for the investment project.

where, $\mathcal{T}$ is a set of all $\mathcal{F}_t$ stopping times ($\mathcal{F}_t$ is the usual filtration generated by $B(t)$), and r denotes the risk-free interest rate satisfying $r > \mu$. Problem (1) is reduced to

$$V_{ae}(x) = \sup_{T \in \mathcal{T}} \mathbb{E}[\mathrm{e}^{-rT}(\Pi(X(T)) - I)],$$

where the function $\Pi(X(T))$ is defined by

$$\Pi(X(T)) = \frac{1-\tau}{r-\mu} QX(T). \tag{2}$$

Then the optimal investment time T^i_{ae} and the firm value $V_{ae}(x)$ are easily calculated as

$$T^i_{ae} = \inf\{t > 0 \mid X(t) \geq x^i_{ae}\}, \tag{3}$$

and

$$V_{ae}(x) = \left(\frac{x}{x^i_{ae}}\right)^{\beta} (\Pi(x^i_{ae}) - I). \tag{4}$$

(see, for example, [2, 11]). Here, β is a positive characteristic root defined by

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \ (> 1).$$

and the investment trigger x^i_{ae} is

$$x^i_{ae} = \frac{\beta}{\beta - 1} \frac{I}{\Pi(1)}. \tag{5}$$

As well known, the investment trigger x^i_{ae} is larger than the zero-NPV trigger $x_{NPV} = I/\Pi(1)$.

2.2 Levered firm

This subsection summarizes the results in the one growth option case of [18]. Consider the levered firm that can issue debt with infinite maturity at the investment. In a usual manner, we solve the problem backward.

Assume that the firm has already invested at time s with market demand $X(s)$ along with issuing debt with coupon c. The equityholders (entrepreneur) have incentives to default after debt is in place. They choose the default time T^d to maximize the equity value as follows:

$$\begin{aligned} &E(X(s), c) \\ &= \sup_{\substack{T \in \mathcal{T} \\ T \geq s}} \mathbb{E}[\int_s^T \mathrm{e}^{-r(t-s)}(1-\tau)(QX(t) - c)\mathrm{d}t \mid \mathcal{F}_s]. \end{aligned} \tag{6}$$

The optimal default time T_d is

$$T^d = \inf\{t \geq s \mid X(t) \leq x^d(c)\},$$

where the default trigger $x^d(c)$ is a function of c given by

$$x^d(c) = \frac{\gamma}{\gamma - 1} \frac{r-\mu}{r} \frac{c}{Q}. \tag{7}$$

Here γ denotes a negative characteristic root defined by

$$\gamma = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \ (< 0).$$

Then, at time s the equity value $E(X(s),c)$, the debt value $D(X(s),c)$, and the firm value $V(X(s),c) = E(X(s),c) + D(X(s),c)$ are expressed as

$$E(X(s),c) = \Pi(X(s)) - \frac{(1-\tau)c}{r} - \left(\Pi(x^d(c)) - \frac{(1-\tau)c}{r}\right)\left(\frac{X(s)}{x^d(c)}\right)^{\gamma} \tag{8}$$

$$\begin{aligned} D(X(s),c) &= \mathbb{E}[\int_s^{T^d} \mathrm{e}^{-r(t-s)}c\mathrm{d}t + \mathrm{e}^{-r(T^d-s)}(1-\alpha)\Pi(X(T^d)) \mid \mathcal{F}_s] & (9) \\ &= \frac{c}{r} - \left(\frac{c}{r} - (1-\alpha)\Pi(x^d(c))\right)\left(\frac{X(s)}{x^d(c)}\right)^{\gamma} & (10) \end{aligned}$$

$$V(X(s),c) = \Pi(X(s)) + \frac{\tau c}{r} - \left(\alpha\Pi(x^d(c)) + \frac{\tau c}{r}\right)\left(\frac{X(s)}{x^d(c)}\right)^{\gamma} \tag{11}$$

for $X(s) \geq x^d(c)$, where $\alpha(\geq 0)$ is a given constant representing the default cost. Note that debtholders collects the entire default value, i.e., $(1-\alpha)\Pi(x^d(c))$.

The equityholders (entrepreneur) choose the investment trigger T^i and coupon $c(X(T^i))$ to maximize the firm value (11). That is, the problem becomes the following :

$$V_{de}(x) = \sup_{\substack{T\in\mathcal{T} \\ c(\geq 0):\mathcal{F}_T-measurable}} \mathbb{E}[\mathrm{e}^{-rT}(V(X(T),c) - I)]. \tag{12}$$

Problem (12) can be interpreted as follows. Assume that debtholders lend K for the debt. Then, the equityholders' (entrepreneur's) value at the investment time T is

$$E(X(T),c) + K - I, \tag{13}$$

while the debtholders' value at T becomes

$$D(X(T),c) - K. \tag{14}$$

Since the sum of (13) and (14) is equal to $V(X(T),c) - I$, the solution of problem (12) is optimal for both the equityholders and the debtholders. The price K determines the asset allocation between equityholders and debtholders, but throughout the paper we do not consider the allocation problem. [5]

Note that $\arg\max_{c\geq 0} V(X(s),c)$ becomes

$$c(X(s)) = \frac{r}{r-\mu}\frac{\gamma-1}{\gamma}\frac{QX(s)}{h} \; (>0), \tag{15}$$

for $X(s) > 0$. Here, h is a constant given by

$$h = \left[1 - \gamma\left(1 - \alpha + \frac{\alpha}{\tau}\right)\right]^{-\frac{1}{\gamma}} > 1.$$

Then, by some calculation we can show

$$V(X(s), c(X(s))) = \psi^{-1}\Pi(X(s)), \tag{16}$$

[5] In [10] agency conflicts between equityholders and debtholders occurs at the investment time because the price K is fixed prior to investment. In contrast, such conflicts do not arise in [18] and this paper because the price K is adjusted by the negotiation at investment timing. The difference between problem (12) and "first best" scenario in [10] is whether the coupon c is controllable.

where the function $\Pi(\cdot)$ is defined by (2) and ψ is a constant given by

$$\psi = \left[1 + \frac{\tau}{(1-\tau)h}\right]^{-1} < 1.$$

As a result, problem (12) can be rewritten as

$$V_{de}(x) = \sup_{T \in \mathcal{T}} \mathbb{E}[e^{-rT}(\psi^{-1}\Pi(X(s)) - I)].$$

Thus, the optimal investment time of (12) is

$$T^i = \inf\{t > 0 \mid X(t) \geq x^i\},$$

and the optimal coupon is $c(x^i)$, where the investment trigger x^i is defined by

$$x^i = \psi x^i_{ae} < x^i_{ae}. \tag{17}$$

Recall that x^i_{ae} is defined by (5). From (7) and (15), we have default trigger

$$x^d(c(x^i)) = \frac{x^i}{h}.$$

The firm value $V_{de}(x)$ at initial time becomes

$$V_{de}(x) = \left(\frac{x}{x^i}\right)^{\beta} (\psi^{-1}\Pi(x^i) - I) > V_{ae}(x). \tag{18}$$

The investment trigger x^i of the levered firm lies between the levered firm's zero-NPV trigger ψx_{NPV} and that of the unlevered one, x^i_{ae}. Note that the unlevered firm's problem (1) corresponds to (12) with $c = 0$. Naturally, the levered firm value (18) is larger than the unlevered one (4). The leverage LV and the credit spread CS at the investment time are calculated as

$$\begin{aligned} LV &= \frac{D(x^i, c(x^i))}{V(x^i, c(x^i))} \\ &= \frac{\gamma - 1}{\gamma}\frac{\psi(1-\tau)(1-\xi)}{h} \end{aligned} \tag{19}$$

and

$$\begin{aligned} CS &= \frac{c(x^i)}{D(x^i, c(x^i))} - r \\ &= r\frac{\xi}{1-\xi}, \end{aligned} \tag{20}$$

respectively, where ξ is defined by

$$\xi = \left(1 - (1-\alpha)(1-\tau)\frac{\gamma}{\gamma - 1}\right) h^{\gamma}.$$

Note that $0 < \xi < 1$ and that both (19) and (20) do not depend on the investment trigger x^i. For further details of the results concerning the levered monopolist, see the one growth option case of [18].

3 Duopoly

This section considers competition between two firms with complete information to focus on strategic investment with debt financing. Assume that each firm receives a cash flow $Q_2X(t)$ when both firms exist in the market. In order to show the essence of the preemptive activities of firms, Section 3.1–3.3 assume $Q_2 = 0$ as in [8, 19]. This means that the market is small enough to be filled with a single firm that entries first. After Section 3.1 describes the bench mark case: competition between unlevered firms, Section 3.2 and 3.3 investigate situation of two symmetric firms that can issue debt, and situation of two asymmetric firms, that is, a levered firm vs. an unlevered firm. Section 3.4 makes a brief comment on a general case of $Q_2 \in (0, Q)$ (negative externalities), though we cannnot make the analytical derivation.

3.1 Competition between unlevered firms

This subsection provides the well known outcome under competition between two unlevered firms (see, for example, [6]). Let $L_{ae}(X(s))$ and $F_{ae}(X(s))$ denote the expected discounted value (at time s) of a firm (called leader) that entries the market first at state $X(s)$ and that of the other (called follower) that responds optimally to the leader. It follows from $Q_2 = 0$ that the follower has no opportunity for investment. Accordingly, the follower's and leader's value become $F_{ae}(X(s)) = 0$ and $L_{ae}(X(s)) = \Pi(X(s)) - I$, respectively. In the situation where neither firms has invested, each firm tries to invest earlier than each other to obtain the leader's payoff $L_{ae}(X(s))$ when the leader's payoff $L_{ae}(X(s))$ is larger than the follower's payoff $F_{ae}(X(s))$. By the preemption, in equilibrium each firm try to invest at the zero-NPV point $X(s) = x_{NPV}$, [6] which is the solution of $\Pi(X(s)) - I = 0$, and as a consequence each firm value becomes zero. There are no equilibrium other than the above one (called preemptive equilibrium). Note that the outcome remains unchanged in the setting where n unlevered firms compete.

3.2 Competition between two symmetric firms

This section considers two types of competition between two symmetric firms with debt financing. First we consider Duopoly (i) in which the firms, whether invest first or not, can issue debt.

As usual, we begin by one of the firms (called leader) has already invested at state $X(s)$. The leader's firm value, denoted $L_{de}(X(s))$, is $L_{de}(X(s)) = \psi^{-1}\Pi(X(s)) - I$ because from the point s the leader can obtain a monopolist's cash flow $QX(t)$ and choose monopolist's default strategy owing to $Q_2 = 0$. Recall that the leader investing at $X(s)$ chooses the optimal coupon (15) and obtains the firm value (16).

On the other hand, the other's (called follower) firm value, denoted $F_{de}(X(s))$, are calculated as follows:

$$
\begin{aligned}
F_{de}(X(s)) &= \left(\frac{X(s)}{x^d(c(X(s)))}\right)^{\gamma} V_{de}(x^d(c(X(s)))) \\
&= \begin{cases} h^{\gamma-\beta}\left(\dfrac{X(s)}{x^i}\right)^{\beta}(\psi^{-1}\Pi(x^i) - I) & (0 < X(s) < hx^i) \\[2ex] h^{\gamma}\left[\psi^{-1}\Pi\left(\dfrac{X(s)}{h}\right) - I\right] & (X(s) \geq hx^i). \end{cases}
\end{aligned}
\tag{1}
$$

Eq. (1) is the value of option to invest after the leader's bankruptcy. Note that the follower chooses the same investment trigger (of course, *not* the same time), coupon, and default trigger

[6]Following [4, 19], this paper assume that one of the firms is chosen as a leader when the firms try to invest at the same threshold. For details of the timing game, see Appendix A.

as those of the monopolist, i.e., x^i, $c(x^i)$, and $x^d(c(x^i))$. Unlike $F_{ae}(X(s)) = 0$ in Section 3.1, $F_{de}(X(s)) > 0$ holds for all $X(s) > 0$. As mentioned in problem (12), the equityholders (entrepreneur) of each firm choose the investment time and coupon to maximize the firm value. Accordingly, the firms attempts to preempt each other when the leader's incentive is positive, i.e., $L_{de}(X(s)) > F_{de}(X(s))$. We have the following proposition.

Proposition 1 There exist a unique x_P satisfying $L_{de}(x_P) = F_{de}(x_P)$ in the interval $(\psi x_{NPV}, x^i)$. In Duopoly (i) there occurs only the following preemptive equilibrium. Each firm try to invest at

$$T_L^i = \inf\{t > 0 \mid X(t) \geq x_P\}$$

and one of the firms executes the investment as a leader at the time T_L^i along with issuing debt with coupon $c(x_P)$. Then the leader defaults at

$$T_L^d = \inf\{t > T_L^i \mid X(t) \leq x_P/h\}.$$

After the leader's bankruptcy, the other, as a follower, invest at

$$T_F^i = \inf\{t > T_L^d \mid X(t) \geq x^i\}$$

along with issuing debt with coupon $c(x^i)$, and then defaults

$$T_F^d = \inf\{t > T_F^i \mid X(t) \leq x^i/h\}.$$

The firm value at initial time becomes

$$h^{\gamma-\beta} V_{de}(x). \tag{2}$$

The preemptive trigger x_P may be smaller than the unlevered firm's zero NPV point x_{NPV}, though of course it is larger than that of the levered case, ψx_{NPV}. For many practical parameter values we observe $x_P < x_{NPV}$. Proposition 1 shows that the leader has smaller investment trigger, coupon, and default trigger, i.e., $x_P < x^i$, $c(x_P) < c(x^i)$ and, $x^d(c(x_P)) = x_P/h < x^d(c(x^i)) = x^i/h$ than those of the monopolist (or follower). Each firm's leverage and credit spared at the investment time remain unchanged from those of the monopolist, i.e., (19) and (20), respectively. This is because the even with fear of preemption by the rival the firm can optimize capital structure. The firm value (2) is $h^{\gamma-\beta}(< 1)$ times of the levered monopolist value (18) due to the preemptive competition.

A firm's endogenous default decision generates a positive firm value in spite of the assumption $Q_2 = 0$. This feature is contrasted with other previous results. In [19, 15] a leader does not always obtain a profit from the market because it takes a random development term from the investment until the completion of the project. The random development term generates a positive value under the competition. In [8] incomplete information about the rival firm's strategy plays a role in generating a positive value under the competition.

Next, let us turn to Duopoly (ii) where only the leader can issue debt. This may be interpreted as that only one lender exists for the investment project. The firm value of the leader who invests at $X(s)$ does not change from $L_{de}(X(s))$. On the other hand, the firm value, denoted F_{ae}^{de}, of the follower who takes the optimal response becomes the following:

$$\begin{aligned} F_{ae}^{de}(X(s)) &= \left(\frac{X(s)}{x^d(c(X(s)))}\right)^{\gamma} V_{ae}(x^d(c(X(s)))) \\ &= \begin{cases} h^{\gamma-\beta}\left(\dfrac{X(s)}{x_{ae}^i}\right)^{\beta}(\Pi(x_{ae}^i) - I) & (0 < X(s) < hx_{ae}^i) \\ h^{\gamma}(\Pi(x_{ae}^i) - I) & (X(s) \geq hx_{ae}^i). \end{cases} \end{aligned} \tag{3}$$

The firms attempt to preempt each other for $X(t)$ satisfying $L_{de}(X(t)) > F_{ae}^{de}(X(t))$. [7] We obtain the following proposition in Duopoly (ii).

Proposition 2 There exists a unique solution of $L_{de}(\hat{x_P}) = F_{ae}^{de}(\hat{x_P})$ in the interval $(\psi x_{NPV}, x^i)$. In Duopoly (ii) there occurs only the following preemptive equilibrium. Each firm try to invest at

$$\hat{T_L^i} = \inf\{t > 0 \mid X(t) \geq \hat{x_P}\}$$

and one of the firms executes the investment as a leader at the time $\hat{T_L^i}$ along with issuing debt with coupon $c(\hat{x_P})$. Then the leader defaults at

$$\hat{T_L^d} = \inf\{t > \hat{T_L^i} \mid X(t) \leq \hat{x_P}/h\}.$$

After the leader's bankruptcy, the other, as a follower, invest at

$$\hat{T_F^i} = \inf\{t > \hat{T_L^d} \mid X(t) \geq x_{ae}^i\}$$

without debt. The firm value at initial time becomes

$$h^{\gamma-\beta} V_{ae}(x). \tag{4}$$

It can be easily checked that $F_{ae}^{de}(X(s)) < F_{de}(X(s))$ for $X(s) > 0$. This implies that the leader has smaller investment trigger, coupon, and default trigger than those of the leader in Duopoly (i), i.e., $\hat{x_P} < x_P$, $c(\hat{x_P}) < c(x_P)$, and $x^d(c(\hat{x_P})) = \hat{x_P}/h < x^d(c(x_P)) = x_P/h$. The firms' leverage and credit spread are the same as (19) and (20) in monopoly. The firm value (4) is $V_{de}(x)/V_{ae}(x)(< 1)$ times of (2) in Duopoly (i). Compared with Duopoly (i), more severe preemptive competition occurs in Duopoly (ii) since the leader enjoys not only the market advantage but also the advantage of capital structure.

3.3 Competition between a levered firm and an unlevered firm

This subsection considers Duopoly (iii): a levered firm vs. an unlevered firm that is not allowed to issue debt for exogenous reasons such as shortage of credit. The firm value of the levered firm that invests as a leader at $X(s)$ agrees with $L_{de}(X(s))$, while the firm value of the unlevered firm that responds optimally as a follower is equal to $F_{ae}^{de}(X(s))$ given by (3). Conversely, the firm value of the unlevered firm that invests as a leader at $X(s)$ becomes $L_{ae}(X(s))$, while the firm value of the levered firm acting as a follower is $F_{ae}(X(s)) = 0$. [8]

The levered firm has an incentive to preempt the unlevered one for $X(s)$ satisfying $L_{de}(X(s)) > 0$, i.e., $X(s) > \psi x_{NPV}$. On the other hand, the unlevered firm tries to become the leader for $X(s)$ satisfying $L_{ae}(X(s)) > F_{ae}^{de}(X(s))$. Taking this into account, we have the following proposition in Duopoly (iii).

Proposition 3 There exists a unique solution $\tilde{x_P}$ of $L_{ae}(\tilde{x_P}) = F_{ae}^{de}(\tilde{x_P})$ in the interval (x_{NPV}, x_{ae}^i). The outcome in Duopoly (iii) is classified into the following two cases.
(a) $\tilde{x_P} < x^i$
Only the following preemption equilibrium occurs. The levered firm invest at

$$\tilde{T_L^i} = \inf\{t > 0 \mid X(t) \geq \hat{x_P}\}$$

[7] This paper considers the model where the equityholders (entrepreneur) tries to maximize the firm value as mentioned in problem (12). This paper does not consider the debtholders' optimal strategy. As will be noted as in Section 6, it is an important future work to analyze how the allocation between equityholders and debtholders changes by the competition among the entrepreneurs.

[8] As shown in Proposition 3, in equilibrium, the levered firm always become a leader, and therefore the order is never realized.

along with issuing debt of coupon $c(\tilde{x_P})$, and then defaults

$$\tilde{T_L^d} = \inf\{t > \tilde{T_L^i} \mid X(t) \leq \tilde{x_P}/h\}.$$

After the levered firm's bankruptcy the unlevered firm invests at

$$\tilde{T_{Fa}^i} = \inf\{t > \tilde{T_L^d} \mid X(t) \geq x_{ae}^i\}.$$

The levered firm value at initial time is equal to

$$\left(\frac{x}{\tilde{x_P}}\right)^{\beta} (V(\tilde{x_P}, c(\tilde{x_P})) - I). \tag{5}$$

The unlevered firm value agrees with (4).

(b) $\tilde{x_P} \geq x^i$

Only the following equilibrium (called dominant leader type) occurs. The levered firm invest at

$$T^i = \inf\{t > 0 \mid X(t) \geq x^i\}$$

along with issuing debt of coupon $c(x^i)$, and then defaults

$$T^d = \inf\{t > T^i \mid X(t) \leq x^i/h\}.$$

After the levered firm's bankruptcy the unlevered firm invests at

$$\tilde{T_{Fb}^i} = \inf\{t > T^d \mid X(t) \geq x_{ae}^i\}.$$

The levered firm value at initial time is the same as that of the monopolist, $V_{de}(x)$, given by (18). The unlevered firm value at initial time is (4).

As explained in [6, 7], there may arise three types of equilibrium, namely preemption, dominant type, and joint investment. In (a) in Proposition 3 the preemption equilibrium occurs, while the dominant leader type equilibrium occurs in (b). In both cases, the levered firm that enjoys optimal capital structure becomes the leader. The result is realistically intuitive. For quite a large τ, which leads a small x^i, condition (b) is satisfied. In (b) the levered firm is dominant owing to the great tax advantage over the unlevered one.

Let us take a look at the investment strategies in Proposition 3. Note that the unlevered firm's investment trigger is the same as that of the unlevered monopolist. With respect to the levered firm's strategy in (a), we can show inequalities $\hat{x_P} < \tilde{x_P} < x^i$, $c(\hat{x_P}) < c(\tilde{x_P}) < c(x^i)$, and $x^d(c(\hat{x_P})) = \hat{x_P}/h < x^d(c(\tilde{x_P})) = \tilde{x_P}/h < x^d(c(x^i)) = x^i/h$. As in the previous propositions, the firm's leverage and credit spread at the investment time are unchanged from (19) and (20) of the monopolist. Note that the trigger $\tilde{x_P}$, unlike x_P, always is larger than the unlevered firm's zero NPV trigger x_{NPV}. The inequality $\tilde{x_P} > x_P$ holds for most parameter values, though it can not be theoretically proved. In (b), the levered firm can take the best strategy. i.e., the monopolist' strategy because of the strong tax effect.

We now consider the firm value in Proposition 3. In both cases, the unlevered firm must wait for the leader's bankruptcy. Due to the waiting time the unlevered firm value (4) becomes $h^{\gamma-\beta}(< 1)$ times of the monopolist's value. Note that in the unlevered firm value is the same in both cases in spite of $\tilde{T_{Fa}^i} \neq \tilde{T_{Fb}^i}$. The levered firm value in (a) is also reduced from that of monopoly due to the suboptimal investment timing. By $\tilde{x_P} > x_P > \hat{x_P}$, the levered firm value (5) becomes larger than (4) and (2) in Duopoly (i) and (ii). Not to mention, the levered firm value in (b) agrees with that of the monopolist because it can take the optimal strategy. To sum up, the fact that the rival changes from levered to unlevered means a decline in the rival's competition power and therefore it increases the levered firm value. Note that in both cases the levered firm value exceeds (4) of the unlevered one.

3.4 Case of $Q_2 > 0$

This subsection makes a brief explanation about the results in the general case such that $0 < Q_2 < Q$, although the setting does not allow us to show clear results. As a bench mark, we consider the competition between two unlevered firms. By $Q_2 > 0$ the follower can entry the market where the leader survives when the market demand $X(s)$ is sufficiently great. The leader's profit is reduced from $QX(t)$ to $Q_2X(t)$ after the follower's entry. Since the leader's incentive is smaller than that in the case of $Q_2 = 0$, the preemption trigger becomes larger than the zero-NPV trigger x_{NPV} in Section 3.1, which generates a positive firm value in equilibrium.

We now consider Duopoly (i) – (iii). In every case, the follower may invest for large $X(s)$ prior to the leader's default. Note that the follower in (ii) and (iii) never defaults. The fact changes the leader's default trigger in the market where both are active from $x^d(c)$ to $x^d(c)Q/Q_2$. Thus, in (ii) and (iii) both the equity and debt values of the leader are reduced from (8) and (10). Expecting the possibility of the follower's interception, the leader issues debt with smaller coupon than $c(X(s))$. On the other hand, because of the decrease in the leader's value and the increase in the follower's value the preemption triggers, denoted $\hat{x}_P'$ and $\tilde{x}_P'$ in (ii) and (iii), become larger than $\hat{x}_P$ and $\tilde{x}_P$, respectively. By the trade-off between these two effects, it is ambiguous whether the leader's coupon in the investment time in (ii) and (iii) are smaller than $c(\hat{x}_P)$ and $c(\tilde{x}_P)$. The leverage and credit spread may also change from those of the monopolist.

In Duopoly (i), the analysis is more complicated. The follower can choose its coupon taking account of the outcome of the exit timing game discussed in [13], when it enters the market where the leader survives. The follower is likely to choose smaller coupon than that of the leader so that it can win the exit timing game (i.e., it can collect a monopolistic profit flow $QX(t)$ after the leader's bankruptcy). In this case, the leader's default trigger change from $x^d(c)$ to $x^d(c)Q/Q_2$, which implies the similar outcome to those of (ii) and (iii). The inequality $\hat{x}_P' < x'_P, \tilde{x}_P'$ is unchanged, where x'_P denotes the preemption trigger in (ii) with $Q_2 \in (0, Q)$.

4 Oligopoly

4.1 Competition among n levered firms

Throughout this section, we assume that the market is small enough to be filled with a single firm, i.e., $Q_2 = 0$. In this section we generalize Duopoly (i) to the situation of n firms that can issue debt. We obtain the following proposition:

Proposition 4 Under the competition among n firms, only the following preemptive equilibrium occurs.
Each firm tries to invest at

$$T^i_{(n)} = \inf\{t > 0 \mid X(t) \geq x^i_{(n)}\}$$

and one of the firms (denoted Firm n) executes the investment at the time [9] along with issuing debt with coupon $c(x^i_{(n)})$. Then the firm defaults at

$$T^d_{(n)} = \inf\{t > T^i_{(n)} \mid X(t) \leq x^i_{(n)}/h\}.$$

After the Firm n's default, the remainders ($n - 1$ firms) try to invest at

$$T^i_{(n-1)} = \inf\{t > T^d_{(n)} \mid X(t) \geq x^i_{(n-1)}\}$$

[9]As in Proposition 1, we assume that one of the firms is chosen at fair probability, i.e., $1/n$.

and one of the firms (denoted Firm $n-1$) executes the investment at the time along with issuing debt with coupon $c(x^i_{(n-1)})$. Then the firm defaults at

$$T^d_{(n-1)} = \inf\{t > T^i_{(n-1)} \mid X(t) \le x^i_{(n-1)}/h\}.$$

$$\vdots$$

After Firm 2' default, the last firm (denoted Firm 1) invest at

$$T^i_{(1)} = \inf\{t > T^d_{(2)} \mid X(t) \ge x^i_{(1)}\}$$

along with issuing debt with coupon $c(x^i_{(1)})$, and then defaults at

$$T^d_{(1)} = \inf\{t > T^i_{(1)} \mid X(t) \le x^i_{(1)}/h\}.$$

Here the investment trigger $x^i_{(k)}$ of Firm k is defined by the unique solution of

$$\psi^{-1}\Pi(x^i_{(k)}) - I = h^{(k-1)(\gamma-\beta)} \left(\frac{x^i_{(k)}}{x^i}\right)^{\beta} (\psi^{-1}\Pi(x^i) - I) \quad (\psi x_{NPV} < x^i_{(k)} \le x^i). \tag{1}$$

The investment triggers $x^i_{(k)}$ satisfy

$$\psi x_{NPV} < x^i_{(n)} < x^i_{(n-1)} < \ldots < x^i_{(2)} = x_P < x^i_{(1)} = x^i. \tag{2}$$

In equilibrium the firm value is equal to

$$h^{(n-1)(\gamma-\beta)} V_{de}(x). \tag{3}$$

As $n \to +\infty$, the firm value (3) and the preemption trigger $x^i_{(n)}$ converges to 0 and ψx_{NPV}, respectively.

From Proposition 4 we have the following inequalities $x^i_{(k+1)} < x^i_{(k)}$, $c(x^i_{(k+1)}) < c(x^i_{(k)})$, and $x^d(c(x^i_{(k+1)})) = x^i_{(k+1)}/h < x^d(c(x^i_{(k)})) = x^i_{(k)}/h$ (see Table 1). As in the previous propositions, the leverage and credit spread at the investment time do not change those of the monopolist. The firm value (3) is $h^{(\gamma-\beta)(n-1)}(< 1)$ times of the monopolist value $V_{de}(x)$. The firm value monotonically decreases to 0 as the number of the firm, n, increase. This can be interpreted that a positive excess profit that arises in oligopoly (finite n) vanishes in the competitive market (infinite n). In the competitive market where infinite firms compete, every firm attempts to invest at the zero-NPV trigger ψx_{NPV}. Our results in the limiting case are similar to the results obtained in the model by [8].

4.2 Social loss due to preemption

This subsection focuses on the social loss due to the preemptive competition among firms. First we consider the outcome of the leader-follower game, in which the order of the firms is exogenously given in advance. Without fear of preemption by the others, every firm chooses the monopolist's strategy, i.e., investment trigger x^i, coupon $c(x^i)$, and default trigger $x^d(c(x^i)) = x^i/h$ (see Table 2). The firm value of Firm k, which invests after $n-k$ firms' defaults, is

$$h^{(n-k)(\gamma-\beta)} V_{de}(x) \tag{4}$$

By comparing Table 1 with Table 2, we can see inefficiency caused by preemption. From Table 1 and 2, we see that the value of Firm 1, which is given the worst role in the leader-follower game, agrees with the value of all firms in the preemptive equilibrium. The total sum of values of n firms is

$$nh^{(\gamma-\beta)(n-1)}V_{de}(x) \downarrow 0 \quad (n \to +\infty) \tag{5}$$

in the preemptive equilibrium, while the sum in the leader-follower game is

$$\sum_{k=1}^{n} h^{(\gamma-\beta)(k-1)}V_{de}(x) = \frac{1-h^{(\gamma-\beta)n}}{1-h^{\gamma-\beta}}V_{de}(x) \uparrow \frac{V_{de}(x)}{1-h^{\gamma-\beta}} \quad (n \to +\infty). \tag{6}$$

We define the (relative) social loss due to preemption by n firms, denoted $Loss(n)$, by $Loss(n) =$ 1− (5)/(6). Then we have

$$Loss(n) = 1 - \frac{nh^{(\gamma-\beta)(n-1)}(1-h^{\gamma-\beta})}{1-h^{(\gamma-\beta)n}} \uparrow 1 \quad (n \to +\infty). \tag{7}$$

From (7) we can state that an increase in the number of firms, n, causes the severe preemptive competition and the inefficient outcome with great social loss.

Table 1: Preemption game.

	Firm n	Firm $n-1$	$\cdots$	Firm 2	Firm 1
Investment	$x^i_{(n)} <$	$x^i_{(n-1)} <$	$\cdots <$	$x^i_{(2)} = x_P <$	$x^i_{(1)} = x^i$
Coupon	$c(x^i_{(n)}) <$	$c(x^i_{(n-1)}) <$	$\cdots <$	$c(x^i_{(2)}) <$	$c(x^i_{(1)})$
Default	$x^i_{(n)}/h <$	$x^i_{(n-1)}/h <$	$\cdots <$	$x^i_{(2)}/h <$	$x^i_{(1)}/h$
Value	$h^{(n-1)(\gamma-\beta)}V_{de}(x)$	$h^{(n-1)(\gamma-\beta)}V_{de}(x)$	$\cdots$	$h^{(n-1)(\gamma-\beta)}V_{de}(x)$	$h^{(n-1)(\gamma-\beta)}V_{de}(x)$

Table 2: Leader-follower game.

	Firm n	Firm $n-1$	$\cdots$	Firm 2	Firm 1
Investment	x^i	x^i	$\cdots$	x^i	x^i
Coupon	$c(x^i)$	$c(x^i)$	$\cdots$	$c(x^i)$	$c(x^i)$
Default	x^i/h	x^i/h	$\cdots$	x^i/h	x^i/h
Value	$V_{de}(x) >$	$h^{\gamma-\beta}V_{de}(x) >$	$\cdots >$	$h^{(n-2)(\gamma-\beta)}V_{de}(x) >$	$h^{(n-1)(\gamma-\beta)}V_{de}(x)$

5 Conclusion

This paper has investigated strategic investment with debt financing by extending the monopolist's one growth option case in [18] to the case allowing preemptive activities of several firms. We analyzed three types of duopoly, namely, (i) two symmetric firms that, whether leader or follower, can issue debt, (ii) two symmetric firms of which only the leader can issue debt, and (iii) a levered firm vs. an unlevered firm. The main results in duopoly can be summarized as follows.

Unlike in the competition between unlevered firms, the possibility of the leader's default generates a positive excess profit to the firms in equilibrium. In (iii) the levered firm always invests first and overwhelms the unlevered one. The order of hardness in the preemptive competition is (ii), (i), (iii).

Moreover, we have derived the equilibrium in oligopoly of n levered firms, and have shown that the social loss due to preemption increases with the number of firms.

We provide some interesting issues of future research. Following [18], the model in this paper does not impose any exogenous restriction between the investment cost I and the amount $K(< D(X(T), c))$ which the entrepreneur borrows by means of debt financing. In the real world, a small entrepreneurial firm that cannot issue equity is likely to be imposed a hard restriction such that a part of I must be financed by debt. In a model with such a restriction we may know the effects of the competition on the leverage, although analytical discussion seems difficult. This paper does not consider the debtholders' optimal strategy. In a model where debtholders are regarded as an independent player of the investment game, we may understand how the competition among several entrepreneurs changes the allocation between the entrepreneurs and the debtholders.

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