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Mathematical Aspects of the shortest path decision mechanism for plasmodium system of *P. polycephalum*

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This is a survey article of related topics and already done works mainly collaborated with Dr. Tomoyuki Miyaji (D3 student in Dept. of Math. and Life Sci.s, Graduate School of science, Hiroshima University), and partially with Dr. Atsushi Tero and Prof. Toshiyuki Nakagaki (Hokkaido University). Moreover, the author appreciates Prof. Ryo Kobayashi (Hiroshima University) deeply for telling me this interesting problem and giving me a lot of helpful comments and significant discussions. It is noted that, in what follows, the honorifics of professors will be abbreviated.

The plasmodium of true slime mold *Physarum polycephalum* is a large amoeba-like organism. Its body contains a tube network by means of which nutrients and signals circulate through the body in effective manner. When food sources were presented to a starved *P.polycephalum* that was spread over the entire agar surface, it concentrated at every food source, respectively. Almost the entire plasmodium accumulated at the food sources and covered each of them in order to absorb nutrients [3]. Only a few tube remained connecting the quasi-separated components of the plasmodium through the short path. Nakagaki *et al.* showed that this simple organism had the ability to find the minimum-length solution of a maze [4, 5]. The connecting tube traces the shortest path even in a complicated maze. Hydrodynamics theory implies that thick short tubes are in principle the most effective for transportation. And this adaptation process of the tube network is based on an underlying physiological mechanism, that is, a tube becomes thicker as a flux in the tube is larger. This insight might be based on the research on the rhythmic oscillation of *P.polycephalum* [8]. Tero *et al.* made a mathematical model in consideration of the qualitative mechanisms clarified by experiments [9]. In the case of linear adaptive term, the model called Physarum solver. According to their numerical simulations of Physarum solver, the minimum-length solution of a maze can be obtained as an asymptotic steady state of the ODEs model [9, 10].

In 2006, Miyaji and the author have proved that the equilibrium point corresponding to the shortest path in the system is globally asymptotically stable in two kinds of simpler networks, namely, the ring-shaped network and the Wheatstone bridge-shaped network [1]. Moreover, recently, we have proved that Physarum solver must solve the shortest path problem mathematically rigorously from deeper consideration on a general planar graph. See the forthcoming paper [2] in details.

On the other hand, *P.polycephalum* sometimes makes a mistake to solve the maze, for example, when there is a kind of "double-edge" like parallel circuit in a graph. Then, it sometimes chooses a longer path. If the adaptation term has a super-linear form, then the model system of equations can explain this case. In fact, numerical computations show that it sometimes chooses a longer path according to initial data ([9]). In [12], we have explained this fact from the dynamical system point of view. In the consequence, we show not only why *P.polycephalum* makes a mistake, but also show how it makes a mistake. When *P.polycephalum* meets various situations, it cannot always find the shortest path. An example is observed in maze-solving experiment. If a very small food is presented at two exits, multiple paths remain and no more adaptation occurs then. Contrary to this, if food is very big, adaptation proceeds very fast and a single path remains, but the path is not always the shortest one. If the quantity of food is appropriate, then it finds the shortest path. It is an important work how to understand mathematically the whole actual image of such behavior of *P.polycephalum*. In [9], Tero et. al. set adequately the form of the adaptation term of the system of equations, and the coming-in and going-out flux, adequately, to succeed to understand it numerically by the model system of the equations. In fact, if the sigmoidal adaptation term is adopted in the model system, then it can describe that the adaptation of the solution changes according to the initial flux's varying. In [12], although we restrict the adaptation term, fixed as super- linear form, which is corresponding to much food case, we present a dynamical system framework to understand the process of adaptation including the transition process, as we do not only realize the shape of the final state.

The recent study of Miyaji and the author is mentioned finally. In [13], we are interested in the synchronized oscillation property stated and investigated numerically in [8]. There, they considered a 3-component reaction-diffusion system of equations with a kind of conservation law. In [8], they proposed this system to under- stand the periodic oscillation of the body of the plasmodium of *P. polycephalum*. In fact, the system described the time-evolution of the three variables (u, v, w), which may obtain some spatio-temporal oscillation solutions. Here u stands for the sponge part, and v represents the effect of the other ingredients, which let the desirable oscillations occur. w is the new variable which is a clever device of them. We explain the mechanism by which the spatio-temporal oscillation occurs heuristically in the following: We note that if w does not exist, then the system is a coupled oscillators system with diffusion coupling. This system has temporally oscillation solutions, but does not have any spatially structural solution. It is sure that this system is not appropriate for the model system just as it is, but the body of the plasmodium of *Physarum polycephalum* can be separated in the two parts; one is a sponge part, the other is a tubular part. The characteristic property is that the diffusion rates are quite different between the former part and the latter part. Namely, the diffusion coefficient of tubular part is quite larger than the one of sponge part. This is why they have considered the new variable w, which means the tubular part and the diffusion coefficient of w is much larger than those of u, v.. Our objective is that we understand how many structural varieties this system has from the viewpoint of bifurcation of stationary solutions. In details, please refer in [13].

In biological experiment, for example, if you watch a circular plasmodium propagating on a flat ager surface, you can observe an anti-phase oscillation between the peripheral region and the rear of the plasmodium. Such an oscillation pattern is called *peripheral phase inversion.* In [8], they impose the assumption that some of the parameters depend on the space variable and reproduce the peripheral phase inversion by numerical simulation. This is very interesting, and the original system with constant coefficients is also a mathematically attractive object. This is because this system has the mass conservation law, so that a kind of "degree of freedom" of solutions may be less than the usual 3-component system, which undergoes *wave bifurcations*.

Therefore, in [13], we assume that all the coefficients are constant. We investigate behavior of solution orbit of the system near the Hopf bifurcation point of the origin. Especially, *wave instability* is our interest. The wave instability breaks both spatial and temporal symmetries of a homogeneous state while the (uniform) Hopf bifurcation does only temporal symmetry. The wave instability occurs, when a homo- geneous state becomes unstable by a pair of purely imaginary eigenvalues with spatially non-uniform eigenfunctions.

In [13], we can prove mathematically rigorously that the wave instability can occur under natural and appropriate conditions for this system. Moreover, we show some graphs and figures obtained by numerical simulations in which we observe the Hopf critical points' behavior for each Fourier mode and observe the behavior of solutions near the bifurcation points at which two Fourier modes are made unstable at the same time. We especially notice that this system has a preferable cluster size of synchronization of oscillations, which tends to smaller and smaller as ε goes to 0. It may be interesting that, if the effect by which the synchronized oscillation occurs is too much, then the synchro- nized cluster is vanishing and a kind of homogenization happens.

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