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n人ゲームにおける間接互恵性の周期的・カオス的ダイナミクス

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Abstract

Reciprocity has been widely studied as a mechanism for explaining the evolution of cooperation in various fields. Under reciprocity, cooperation can prevail because a donor of cooperation receives reciprocation from the recipient, called *direct* reciprocity, or from someone else, called *indirect* reciprocity. Nowak & Sigmund demonstrated that evolutionary dynamics of *direct* reciprocity can display oscillation and chaos in 2-person games. Is dynamic cooperation as oscillation or chaos observed only in direct reciprocity? We show that *indirectly* reciprocal cooperation in n-person games can be maintained dynamically as chaos or oscillation. This is, to our knowledge, the first demonstration of chaos in indirect reciprocity. Furthermore, this suggests that oscillatory dynamics are widely observed in the evolution of reciprocity whether it is direct one or indirect one.

1 Introduction

The concept of reciprocity was first proposed by Trivers [1], which claims that cooperation can evolve because a donor of cooperation receives reciprocation from others. Under *direct* reciprocity, a cooperator can be *directly* reciprocated by the recipient of cooperation in repeated interactions [2, 3, 4]. On the other hand, under indirect reciprocity, a cooperator obtains returns from someone else, who knows indirectly through social *reputation* that she is cooperative, in non-repeated interactions [6, 5, 7, 8, 9, 10, 11, 12, 13, 14]. Indirect reciprocity has been applicable to cooperation in a few anonymous interactions such as global markets or on Internet.

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Here, an interesting point in the evolutionary dynamics of direct reciprocity in two-person games is the fact that the dynamics of the shares of strategies and those of the overall level of cooperation display oscillation or chaos [8, 15, 16, 17]. In particular, using two-person iterated prisoners dilemma games, Nowak & Sigmund [16] demonstrated that, depending on the mutation rate, the evolutionary dynamics change from converging to a fixed point to oscillation or chaos. Are these dynamics such as oscillation or chaos limited to the evolutionary dynamics of direct reciprocity in two-person games? Regarding direct reciprocity in n-person games, Eriksson & Lindgren demonstrated that, in n-person games, directly reciprocal cooperation can be maintained as oscillation [18] (but not as chaos).

However, whether dynamic cooperation such as oscillation or chaos is observed or not in indirect reciprocity remains unclear. In the present study, we investigate the evolutionary dynamics of indirect reciprocity in one-shot n-person prisoner's dilemma games including social reputation. Especially, as in the study by Nowak & Sigmund[16], we specifically examine bifurcations into oscillation or chaos depending on the mutation rate.

2 Model

Consider a population comprising an infinite number of individuals. Each individual in the population has a *reputation*, either G *(good)* or B *(bad)*. At the beginning of each generation, the reputation of each individual is presumed as G.

Each *generation* comprises a number of *rounds*. After the first round, each subsequent round occurs with probability w ($0 \ll w < 1$), i.e., the expected value of the number of rounds in a generation is $1/(1-w)$.

In each round, all individuals are divided randomly into groups, each of which comprises n-individuals; all play an n-person prisoner's dilemma game in each group. In this game, each individual chooses either to "cooperate (C)" or "defect (D)". In this study, the cost and the benefit of cooperation are denoted as c and b, respectively, where $b > c > 0$. Suppose that the benefit is shared equally among the $n-1$ other group members. Then, the payoffs for a cooperator, $V(C \mid k)$, and that for a defector, $V(D \mid k)$, where k is the number of opponents cooperating in the group ($0 \leq k \leq n-1$), are

$$V(C \mid k) = \frac{k}{n-1}b - c, \quad V(D \mid k) = \frac{k}{n-1}b. \tag{1}$$

Moreover, the reputation of opponents affects the decision-making process. For this study, we adopt "*image scoring*" as a *reputation criterion*, which prescribes how to judge the reputation of others based on their past actions. Under image scoring [6, 7], the reputation of an individual who has cooperated (defected) in the previous round becomes G (B).

In this study, each individual is assumed to decide an action based on the number of her opponents having reputation G in the group. Such a decision-making rule, called *strategy*, is denoted by an n-dimensional vector $\mathbf{P} = (p_0, \cdots, p_{n-1})$ where $p_k \in \{0, 1\}$, in which p_k indicates the probability that the individual cooperates when the number of opponents having reputation G is k. For example, in 4-person games, strategy $(0,0,0,0)$ called $ALLD$ is the unconditional defector, and strategy $(1,1,1,1)$ called $ALLC$ always cooperates. Moreover, strategy $(0,0,0,1)$ called the strict discriminator ($stDIS$) cooperates only when all the opponents have reputation G, and $(0,0,1,1)$ or $(0,1,1,1)$ called the generous discriminator ($gDIS$) cooperates when at least one or two opponents have reputation G. In all, there are 2^n pure strategies, which we number from 0 to $2^n - 1$. The ith strategy is represented by $\mathbf{P_i}$ (the ith strategy being the binary expression for i). Furthermore, the shares of the respective strategies among the population are denoted respectively as $x_0, \cdots, x_{2^n-1}$. In the example of four-person games, $\mathbf{P_0}$ corresponds to $ALLD$, $\mathbf{P_1}$ to $stDIS$, $\mathbf{P_3}$ or $\mathbf{P_7}$ to $gDIS$, and $\mathbf{P_{15}}$ to $ALLC$.

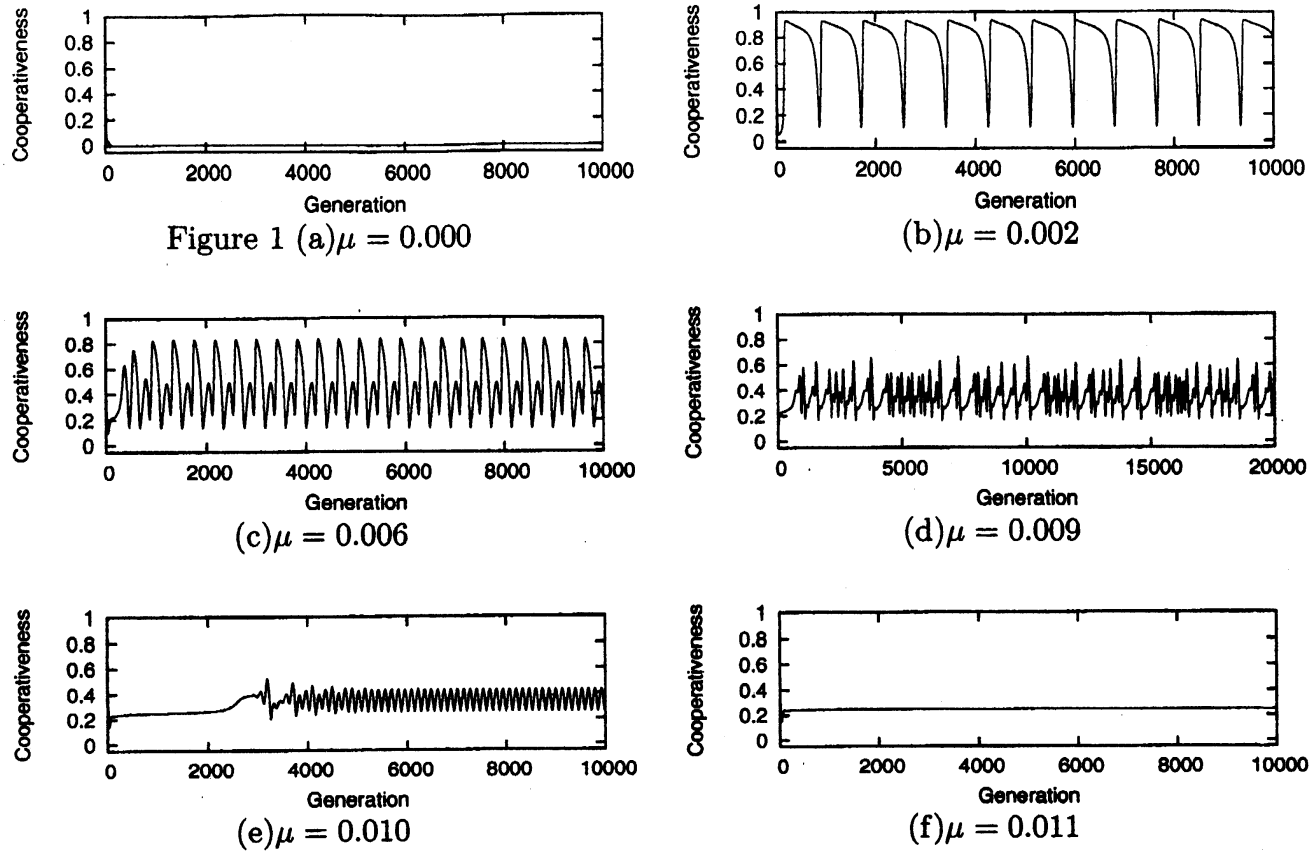

図 1: (a)-(f): Evolutionary dynamics of the overall level of cooperation (the initial state is that $x_0 = \cdots = x_{15} = 1/16$). (a) mutation rate, $\mu = 0.000$, (b) $\mu = 0.002$, (c) $\mu = 0.006$, (d) $\mu = 0.009$, (e) $\mu = 0.010$ and (f) $\mu = 0.011$.

In addition, we assume that, with the probability ϵ ($0 < \epsilon \ll 1$), an individual who intends to cooperate fails to cooperate because of a mistake *etc* [19]. Therefore, we replace 1 in elements of the strategy vectors with $1 - \epsilon$. For instance, strategy $(1, 0, 1, 0)$ is replaced with $(1 - \epsilon, 0, 1 - \epsilon, 0)$.

To investigate the evolution of the shares among the population of the strategies, we consider the natural selection process by which strategies that achieve higher fitness, defined as the expected total payoff, increase their shares among the population. Furthermore, we include mutation, the rate of which is measured using μ, in the evolutionary dynamics. (The derivation of the fitness for each strategy and the mathematical formulation of the evolutionary dynamics are described in [22].)

3 Result & Discussion

Let us introduce the evolutionary dynamics for the group size of $n = 4$: the benefit of cooperation is $b = 10$; the cost of cooperation is $c = 1$; the probability that each subsequent round occurs is $w = 0.9$; and the probability of implementation error, $\epsilon = 0.01$. No evolutionarily stable strategy exists for this parameter setting. We also examined the cases of $n = 3, 5$ and confirmed that the result does not change qualitatively [20]. (When n is extremely large, it is expected that the frequency of cooperation is lower [21, 23].)

Figure 1 illustrates the numerical calculation of the evolutionary dynamics of the overall level of cooperation under various mutation rates μ. At the initial state of the evolutionary dynamics given in each figure, all strategies are present in equal shares: $x_0 = \cdots = x_{15} = 1/16$. Numerical simulations of the evolutionary dynamics starting from 50 random initial states and also from the population of $ALLD$ (i.e., $x_0 = 1$, $x_1 = \cdots = x_{15} = 0$) showed that the resulting attractors in the figure do not depend on the initial states as far as the mutation rate, μ, is not extremely small (under the parameter setting in the figure, $\mu > 0.0002$), which suggests that these attractors are global unless μ is extremely small.

As shown in Fig. 1, the evolutionary dynamics change qualitatively as the mutation rate increases. In the case of no mutation ($\mu = 0$), the resulting dynamics converges to a fixed point at which the overall level of cooperation is very low. Moreover, as the mutation rate increases, the dynamics displays quasi-periodic oscillation. The larger the mutation rate, the more complicated the dynamics become. The dynamics bifurcate into chaos, and subsequently revert to oscillation, and to convergence to a fixed point when the mutation rate increases further.

Here, how are the oscillation and chaos formed and maintained? We describe the dynamics of the shares of the strategies resulting in the oscillation from the population of unconditional defectors ($ALLD$) to answer this question. First, $(0,1,0,0)$ and $(0,0,1,0)$ strategists invade the $ALLD$ population. The $(0,1,0,0)$ and $(0,0,1,0)$ strategists earn almost equal fitness to that for $ALLD$ because they do not cooperate both in the first round where all individuals have good reputation and after the first round where all $ALLD$ opponents have a bad reputation (note that, in the population almost entirely consisting of $ALLD$, most opponents are $ALLD$ strategists whose reputation is good at the first round and bad after the first round). For that reason, they are not exploited by $ALLD$, whereas the other strategists are exploited by $ALLD$. Therefore, by the effect of mutation, $(0,1,0,0)$ and $(0,0,1,0)$ strategists increase, whereas $ALLD$ strategists decrease. (When slight differences arise in fitness, mutation draws the evolutionary dynamics to the center of the state space where strategy shares are almost equivalent.) Next, once the shares of $(0,1,0,0)$ and $(0,0,1,0)$ strategists increase to some degree, the strictest discriminators, $stDIS$, who cooperate only when all the opponents have good reputation, can increase their share by exploiting the $(0,1,0,0)$ and $(0,0,1,0)$ strategists. When the share of $stDIS$ strategists rise to a certain degree, they can overcome the $ALLD$ strategists and become predominant because the $stDIS$ strategists punish defectors strictly. That is, although $stDIS$ can not invade the $ALLD$ population alone [23], they can invade the $ALLD$ population by exploiting other strategies which obtain almost the same fitness as that for $ALLD$ in the population mostly consisting of $ALLD$.

However, $stDIS$ strategists, once they become predominant, harm each other as a result of triggering error defections because of their excessive strictness. For that reason, they can not earn the high level fitness. Therefore, the $stDIS$ strategists are vulnerable to invasion by more generous strategists, generous discriminators ($gDIS$), who cooperate when at least one or two opponents have a good reputation, or by unconditional cooperators ($ALLC$). Finally, once the generous strategists prevail over the population, $ALLD$ strategists invade the population again. This mechanism explains the emergence of the oscillation from the population of $ALLD$. Moreover, once the evolutionary dynamics reach the oscillatory attractor, the share of $ALLD$ strategists is at most 40%, so $stDIS$ strategists can invade the $ALLD$ population without the assistance of $(0,1,0,0)$ and $(0,0,1,0)$ strategists.

The mechanism for the formation of the chaos from the $ALLD$ population is almost the same as that for the oscillation. In the chaotic attractor, in addition to the generous discriminators, $(0,1,0,1)$ and $(1,0,1,0)$ strategists invade the population predominated by the strictest discriminators, which makes the cycle irregular (chaos).

So far, we have investigated the evolutionary phenomena of indirect reciprocity in n-person games. The analyses have shown that indirectly reciprocal cooperation in n-person games can be formed and maintained dynamically as oscillation or chaos. This is, to our knowledge, the first demonstration of chaos in indirect reciprocity. Furthermore, the evo-

lutionary dynamics bifurcate as the mutation rate increases. Similar phenomena have been observed in the evolution of the other type of reciprocity, i.e., direct reciprocity [16, 18]. Considering these results, the result of this paper suggests that dynamics as oscillation or chaos are widely observed in the evolution of reciprocal cooperation under dilemma games with strategic mutation whether it is direct one or indirect one [24].

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[19] We, as in [11], do not consider errors which cause unintentional cooperation. We confirmed that the resulting dynamics do not change appreciably even if errors causing unintentional cooperation are included.

[20] Concerning the benefit of cooperation, b (the cost c is fixed at 1), similar evolutionary phenomena are observed if b becomes larger (but chaos is hardly observed when b is extremely large). On the other hand, if b becomes smaller ($b < 8$), cooperation hardly forms. For w and ϵ, the observed evolutionary dynamics do not essentially change as far as w and $1 - \epsilon$ are sufficiently close to 1.

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[24] Regarding the evolution of indirect reciprocity in two-person games, dynamic cooperation has not been reported [8] except for the specific case in which the number of rounds in a generation is constant and where implementation error is not included [7]. Few studies have investigated the evolutionary dynamics of indirect reciprocity in a population of various strategies (most studies have specifically examined the evolutionary dynamics of only three strategies[7, 8, 11] or dealt only with evolutionary stability[10]).