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A Simulation Study on Bayesian Simultaneous Demand and Supply Model with Market-Level Data

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1 Introduction

When predicting consumers' purchasing behavior in a differentiated product market, it is necessary to account for price endogeneity and consumers' heterogeneity. In many situation, the price are endogenously determined within the demand and supply: Based on the market's response, firms set the prices which in turn affect consumers' choices. Ignoring the endogeneity leads to estimation bias with both market-level and consumer-level data

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(Berry, 1994; Villas-Boas & Winner, 1999). Consumers' heterogeneity reflects each consumer's preference in each product characteristics and allows flexible substitution patterns (Berry, Levinsohn & Pakes, 1995; henceforth BLP).

There are two research streams to account for the endogeneity and heterogeneity. One is by BLP (1995), Sudhir (2001), Petrin (2002) and Myojo and Kanazawa (2007) in a frequentist's framework, and the other is by Yang et al. (2003), Jiang et al. (2007), Romeo (2007) and Yonetani et al. (2008) in a Bayesian framework. Both are simultaneous demand and supply models to address the endogeneity. Both incorporate the heterogeneity in the utility functions: The frequentists' models use a random coefficient utility; and the Bayesian models assume that utility coefficients have distributions. To estimate the model parameters, the frequentists' models use the Generalized Method of Moment (GMM) with instruments using market-level data. The Bayesian models use Markov Chain Monte Carlo (MCMC). Yang et al. (2003) develop a full- and limited-information models using consumser purchase incidence data. Romeo (2007) incorporates a GMM objective function in the MCMC using market-level data. Yonetani et al. (2008) extend the Yang et al.'s (2003) full-information model for market-level data while Jiang et al. (2007) extend the limited information model for market-level data.

In this paper, we will present the latest Yonetani et al.'s (2008) model among these simultaneous demand and supply models and perform a simulation study on it. This paper is organized as follows. In Sections 2 and 3, we briefly review Yonetani et al.'s (2008) model and estimation method respectively. Section 4 contains the simulation study. Conclusions and discussions are presented in Section 5.

2 Model specification

2.1 Demand Model

We assume that there are J products indexed by $j = 1, \ldots, J$ in a differentiated product market. Let us denote $j = 0$ as the index of the outside good. A consumer i chooses one of the $J + 1$ alternatives with the highest utility.[4] Researchers observe a $J \times 1$ sales volume vector $\boldsymbol{v}^o = (v_1^o, \ldots, v_J^o)'$ and the overall market size $M = \sum_{j=0}^{J} v_j^o$ where v_0^o is the number of consumers choosing the outside good $j = 0$.

The utility of a consumer i for product j is the log-transformation of a Cobb-Douglass function as

$$u_{ij} = u_{ij}\left(p_j, \boldsymbol{x}_j, \xi_j, y_i, \boldsymbol{\theta}_i, \varepsilon_{ij}\right) = \alpha_i \log\left(y_i - p_j\right) + \boldsymbol{x}_j \boldsymbol{\beta}_i + \xi_j + \varepsilon_{ij}, \qquad (2.1)$$

where y_i is a consumer i's income; p_j is an observed unit price; $\boldsymbol{x}_j$ is a $1 \times (Q-1)$ vector of observed product characteristics; $\boldsymbol{\theta}_i = (\alpha_i, \boldsymbol{\beta}_i')'$ are respectively consumer i's marginal utility for $\log(y_i - p_j)$ and $(Q-1) \times 1$ marginal utility vector for $\boldsymbol{x}_j$; ξ_j is an unobserved product characteristic term; and ε_{ij} is refered to a consumer-level sampling error term. There are four points to be noted. First, α_i and $\boldsymbol{\beta}_i$ reflect consumers' heterogeneity with respect to $\log(y_i - p_j)$ and $\boldsymbol{x}_j$ respectively. Second, the term $\log(y_i - p_j)$ is appropriate when we require to formulate the fact that a higher price affects the utility of a high-income consumer much less than that of a low-income

[4]This assumption is valid for purchasing a durable product.

consumer. Third, the presense of both observed and unobserved product characteristics, $\boldsymbol{x}_j$ and ξ_j, in (2.1) reflects an assumption that researchers observe only some of the product characteristics consumers and producers observe. The presence of unobserved product characteristic ξ_j also allows for a product-level source of sampling error. Fourth, we assume that $p_0 = 0$, $\boldsymbol{x}_0 = \mathbf{0}$ and $\xi_0 = 0$.

We assume that ε_{ij} is independent of $\alpha_i \log(y_i - p_j) + \boldsymbol{x}_j\boldsymbol{\beta}_i + \xi_j$ and that it is also independently and identically Gumbel (type I extreme value) distributed across consumers and products in (2.1). Then we derive the logit choice probability s_{ij} for a consumer i choosing product j as

$$s_{ij} = s_{ij}\left(\boldsymbol{p}, \boldsymbol{X}, \boldsymbol{\xi}, y_i, \boldsymbol{\theta}_i\right) = \frac{\exp\left\{\alpha_i \log\left(y_i - p_j\right) + \boldsymbol{x}_j\boldsymbol{\beta}_i + \xi_j\right\}}{\sum_{k=0}^{J} \exp\left\{\alpha_i \log\left(y_i - p_k\right) + \boldsymbol{x}_k\boldsymbol{\beta}_i + \xi_k\right\}}, \qquad (2.2)$$

where $\boldsymbol{X} = (\boldsymbol{x}'_1, \ldots, \boldsymbol{x}'_J)'$, $\boldsymbol{p} = (p_1, \ldots, p_J)'$ and $\boldsymbol{\xi} = (\xi_1, \ldots, \xi_J)'$.

The market share of product j in the population is

$$s_j^0 = s_j^0\left(\boldsymbol{p}, \boldsymbol{X}, \boldsymbol{\xi}\right) = \int\int s_{ij} f^0\left(y_i\right) f^0\left(\boldsymbol{\theta}_i\right) dy_i d\boldsymbol{\theta}_i, \qquad (2.3)$$

where $f^0(y_i)$ and $f(\boldsymbol{\theta}_i)$ are the population densities of y_i and $\boldsymbol{\theta}_i$ respectively. A sample counterpart of (2.3) is

$$s_j = s_j\left(\boldsymbol{p}, \boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{y}, \boldsymbol{\theta}\right) = \frac{1}{I}\sum_{i=1}^{I} s_{ij}, \qquad (2.4)$$

where $\boldsymbol{y} = (y_1, \ldots, y_I)'$ and $\boldsymbol{\theta} = (\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_I)$. We denote $\boldsymbol{s}$ as a $J \times 1$ market share vector for product $j = 1, \ldots, J$:

$$\boldsymbol{s} = \boldsymbol{s}\left(\boldsymbol{p}, \boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{y}, \boldsymbol{\theta}\right) = \left(s_1, \ldots, s_J\right)'. \qquad (2.5)$$

We define the sales volume v_j in the I consumers as

$$v_j = \text{int}\left(I \cdot \frac{v_j^o}{M} + 0.5\right)$$

for $j = 1, \ldots, J$, where int$(\cdot)$ is the integral part in the expression $(\cdot)$. We also define the number of consumers choosing the outside good $j = 0$ in the I consumers as $v_0 = I - \sum_{j=1}^{J} v_j$. We then denote $\boldsymbol{v}$ as a $J \times 1$ sales volume vector for product $j = 1, \ldots, J$ in the I consumers:

$$\boldsymbol{v} = (v_1, \ldots, v_J)'.$$

2.2 Supply Model

We assume that there are a fixed number F of firms in an oligopolistic market of the J products with Bertrand competition. We also assume that each firm $f = 1, \ldots, F$ produces a subset of the J products and sets price for each of its products according to its pricing strategy that maximizes the total profit function,

$$\Pi_f = \sum_{j \in f} M s_j(\boldsymbol{p})(p_j - c_j), \tag{2.6}$$

where $s_j(\boldsymbol{p}) = s_j(\boldsymbol{p}, \boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{y}, \boldsymbol{\theta})$ in (2.4) and c_j is a unit cost. Let us denote $\boldsymbol{c} = (c_1, \ldots, c_J)'$ and $(\partial \boldsymbol{G}/\partial \boldsymbol{p}) = (\partial \boldsymbol{s}/\partial \boldsymbol{p}) * \boldsymbol{\delta}$ where the sign $*$ represents the element-by-element multiplication of the matrices it connects and the (j, k) element δ_{jk} of $\boldsymbol{\delta}$ is 1 if the products j and k are produced by the same firm and 0 otherwise.[5] Then we obtain the first order conditions for $j = 1, \ldots, J$ from (2.6) as

$$\boldsymbol{p} = -\left\{\left(\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{p}}\right)'\right\}^{-1} \boldsymbol{s} + \boldsymbol{c}, \tag{2.7}$$

[5]We specify the elements of $(\partial \boldsymbol{s}/\partial \boldsymbol{p})$ and $(\partial \boldsymbol{G}/\partial \boldsymbol{p})$ in Appendix A.

assuming the inverse above exists.

Similar to observed and unobserved product characteristics in (2.1), we may decompose cost c_j into observed and unobserved cost characteristics. However, researchers rarely have even specific observed cost characteristics. Therefore, for the observed cost characteristics, we employ alternative but reasonable variables in the sense that they are expected to be related to the total cost c_j.[6] We often call this observed cost characteristic "cost shifter" which comes from the alternative. Then we assume that c_j is log linear in a $1 \times S$ observed cost shifter vector $\boldsymbol{z}_j$ and an unobserved cost term η_j as

$$\log c_j = \boldsymbol{z}_j \gamma + \eta_j,$$

where $\boldsymbol{\gamma}$ is a $S \times 1$ coefficient vector for $\boldsymbol{z}_j$. Note that each element of $\boldsymbol{\gamma}$ has the interpretation of elasticity of c_j with respect to its corresponding cost shifter with logarithmic form in $\boldsymbol{z}_j$.

Let us denote $\boldsymbol{Z} = (\boldsymbol{z}'_1, \ldots, \boldsymbol{z}'_J)'$ and $\boldsymbol{\eta} = (\eta_1, \ldots, \eta_J)'$. Substituting $\boldsymbol{Z\gamma} + \boldsymbol{\eta}$ for $\boldsymbol{c}$ in (2.7) leads to the following pricing equation:

$$\log \left[\boldsymbol{p} + \left\{ \left(\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{p}} \right)' \right\}^{-1} \boldsymbol{s} \right] = \boldsymbol{Z\gamma} + \boldsymbol{\eta}. \tag{2.8}$$

We can also write $\boldsymbol{p}$ as

$$\boldsymbol{p} = \boldsymbol{p}\left(\boldsymbol{s}, \boldsymbol{X}, \boldsymbol{\xi}, \boldsymbol{\delta}, \boldsymbol{y}, \boldsymbol{\theta}, \boldsymbol{Z}, \boldsymbol{\eta}, \boldsymbol{\gamma}\right). \tag{2.9}$$

[6] Frequently, the alternative valiables include the observed product characteristics $\boldsymbol{x}_j$.

2.3 Simultaneous demand and supply model

From the demand model in Section 2.1 and supply model in Section 2.2, we can see the market share $\boldsymbol{s}$ and price $\boldsymbol{p}$ are simultaneously determined as

$$\begin{aligned} &\boldsymbol{s}|\boldsymbol{p},\boldsymbol{X},\boldsymbol{\xi},\boldsymbol{y},\boldsymbol{\theta}, && (2.5)\\ &\boldsymbol{p}|\boldsymbol{s},\boldsymbol{X},\boldsymbol{\xi},\boldsymbol{\delta},\boldsymbol{y},\boldsymbol{\theta},\boldsymbol{Z},\boldsymbol{\eta},\boldsymbol{\gamma}. && (2.9)\end{aligned}$$

Given the overall market size M, that product j has the market share s_j is equivalent in saying its sales volume is v_j for $j = 1, \ldots, J$ in the I consumers. We thus rewrite the simultaneous demand and supply model as

$$\begin{aligned} &\boldsymbol{v}|\boldsymbol{p},\boldsymbol{X},\boldsymbol{\xi},\boldsymbol{y},\boldsymbol{\theta}, && (2.5)'\\ &\boldsymbol{p}|\boldsymbol{v},\boldsymbol{X},\boldsymbol{\xi},\boldsymbol{\delta},\boldsymbol{y},\boldsymbol{\theta},\boldsymbol{Z},\boldsymbol{\eta},\boldsymbol{\gamma}. && (2.9)'\end{aligned}$$

3 Bayesian Estimation

3.1 Model parameters and their prior distributions

We will extend our simultaneous demand and supply model by using the Bayesian hierarchical modeling. For the marginal utilities $\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_I$ of the I consumers, we assume

$$\boldsymbol{\theta}_i \left| \bar{\boldsymbol{\theta}}, \Sigma_{\boldsymbol{\theta}} \sim MVN\left(\bar{\boldsymbol{\theta}}, \Sigma_{\boldsymbol{\theta}}\right), \right. \tag{3.1}$$

where $\bar{\boldsymbol{\theta}}$ is the $Q \times 1$ mean vector and $\Sigma_{\boldsymbol{\theta}}$ is the $Q \times Q$ covariance matrix. In terms of unobserved product and cost characteristics $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$, we assume

$$\boldsymbol{\xi} \left| \Sigma_d \sim MVN\left(\boldsymbol{0}, \Sigma_d\right), \right. \tag{3.2}$$

$$\boldsymbol{\eta} \left| \Sigma_s \sim MVN\left(\boldsymbol{0}, \Sigma_s\right). \right. \tag{3.3}$$

With these assumptions on $\boldsymbol{\theta}_i$, $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$, the simultaneous demand and supply model is rewritten as

$$\boldsymbol{v}|\boldsymbol{p}, \boldsymbol{\xi}, \boldsymbol{\theta}, \qquad (2.5)'$$

$$\boldsymbol{p}|\boldsymbol{v}, \boldsymbol{\xi}, \boldsymbol{\theta}, \boldsymbol{\eta}, \boldsymbol{\gamma}, \qquad (2.9)'$$

$$\boldsymbol{\theta}_i|\bar{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}}, \qquad (3.1)$$

$$\boldsymbol{\xi}|\boldsymbol{\Sigma}_d, \qquad (3.2)$$

$$\boldsymbol{\eta}|\boldsymbol{\Sigma}_s. \qquad (3.3)$$

Note that the exogenous variables of income $\boldsymbol{y}$, observed product characteristic $\boldsymbol{X}$ and cost shifter $\boldsymbol{Z}$ are left out from (2.5)′ and (2.9)′ for notational simplicity. We call $\boldsymbol{\theta}$, $\bar{\boldsymbol{\theta}}$, $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$, $\boldsymbol{\gamma}$, $\boldsymbol{\Sigma}_d$ and $\boldsymbol{\Sigma}_s$ the model parameters. The joint posterior distribution of the model parameters requires us to hypothesize in addition to (3.1) prior distributions for $\bar{\boldsymbol{\theta}}$, $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$, $\boldsymbol{\gamma}$, $\boldsymbol{\Sigma}_d$ and $\boldsymbol{\Sigma}_s$ respectively as

$$\bar{\boldsymbol{\theta}} \sim MVN\left(\boldsymbol{\mu}_{\bar{\boldsymbol{\theta}}}, \boldsymbol{V}_{\bar{\boldsymbol{\theta}}}\right),$$

$$\boldsymbol{\Sigma}_{\boldsymbol{\theta}} \sim IW_{g_\theta}\left(\boldsymbol{G}_{\boldsymbol{\theta}}\right),$$

$$\boldsymbol{\gamma} \sim MVN\left(\bar{\boldsymbol{\gamma}}, \boldsymbol{V}_{\boldsymbol{\gamma}}\right),$$

$$\boldsymbol{\Sigma}_d \sim IW_{g_d}\left(\boldsymbol{G}_d\right),$$

$$\boldsymbol{\Sigma}_s \sim IW_{g_s}\left(\boldsymbol{G}_s\right).$$

Note that these priors are independent each other. We call $\boldsymbol{\mu}_{\bar{\boldsymbol{\theta}}}$, $\boldsymbol{V}_{\bar{\boldsymbol{\theta}}}$, $g_{\boldsymbol{\theta}}$, $\boldsymbol{G}_{\boldsymbol{\theta}}$, $\bar{\boldsymbol{\gamma}}$, $\boldsymbol{V}_{\boldsymbol{\gamma}}$, g_d, $\boldsymbol{G}_d$, g_s and $\boldsymbol{G}_s$ the hyperparameters.

3.2 Distribution of endogenous observed data

The joint posterior of the model parameters also requires us to formulate a joint distirubtion of endogenous observed data of the sales volume $\boldsymbol{v}$ and price $\boldsymbol{p}$. Using the demand (2.5)′ and supply (2.9)′ models, we will specify the joint distribution of $\boldsymbol{v}$ and $\boldsymbol{p}$ with the conditional distribution of $\boldsymbol{v}$ given $\boldsymbol{p}$ and the marginal distribution of $\boldsymbol{p}$. We substitute (2.5)′ for $\boldsymbol{v}$ in (2.9)′, and the the simultaneous demand and supply model becomes

$$\begin{aligned} &\boldsymbol{v}|\boldsymbol{p},\boldsymbol{\xi},\boldsymbol{\theta}, && (2.5)' \\ &\boldsymbol{p}|\boldsymbol{\xi},\boldsymbol{\theta},\boldsymbol{\eta},\boldsymbol{\gamma}, && (2.9)'' \\ &\boldsymbol{\theta}_i|\bar{\boldsymbol{\theta}},\boldsymbol{\Sigma}_{\boldsymbol{\theta}}, && (3.1) \\ &\boldsymbol{\xi}|\boldsymbol{\Sigma}_d, && (3.2) \\ &\boldsymbol{\eta}|\boldsymbol{\Sigma}_s. && (3.3) \end{aligned}$$

In distributional form, we specify the joint distribution of $\boldsymbol{v}$ and $\boldsymbol{p}$ as

$$f\left(\boldsymbol{v},\boldsymbol{p}\,|\boldsymbol{\xi},\boldsymbol{\theta},\boldsymbol{\eta},\boldsymbol{\gamma}\right)=f\left(\boldsymbol{v}\,|\boldsymbol{p},\boldsymbol{\xi},\boldsymbol{\theta}\right)f\left(\boldsymbol{p}\,|\boldsymbol{\xi},\boldsymbol{\theta},\boldsymbol{\eta},\boldsymbol{\gamma}\right).$$

The conditional distribution of $\boldsymbol{v}$ given $\boldsymbol{p}$ is obtained with the market share s_j in (2.4) for $j=0,\ldots,J$ as a multinomial distribution:

$$f\left(\boldsymbol{v}\,|\boldsymbol{p},\boldsymbol{\xi},\boldsymbol{\theta}\right)=\frac{I!}{v_0!\cdots v_J!}s_0^{v_0}\cdots s_J^{v_J}. \tag{3.4}$$

We notice that the pricing equation (2.8) is implicit in price $\boldsymbol{p}$. We thus use the transformation of variable with the pricing equation in (2.8) and the multivariate normal distribution on unobserved cost $\boldsymbol{\eta}$ in (3.3) to obtain the marginal distribution of $\boldsymbol{p}$ as[7]

$$\begin{aligned} &f\left(\boldsymbol{p}\,|\boldsymbol{\xi},\boldsymbol{\theta},\boldsymbol{\gamma},\boldsymbol{\Sigma}_s\right) \\ &=(2\pi)^{-\frac{J}{2}}\,|\boldsymbol{\Sigma}_s|^{-\frac{1}{2}}\left\|\left(\frac{\partial\boldsymbol{\eta}}{\partial\boldsymbol{p}}\right)\right\| \\ &\times\exp\left[-\frac{1}{2}\left[\log\left[\boldsymbol{p}+\left\{\left(\frac{\partial\boldsymbol{G}}{\partial\boldsymbol{p}}\right)'\right\}^{-1}\boldsymbol{s}\right]-\boldsymbol{Z\gamma}\right]'\boldsymbol{\Sigma}_s^{-1}\left[\log\left[\boldsymbol{p}+\left\{\left(\frac{\partial\boldsymbol{G}}{\partial\boldsymbol{p}}\right)'\right\}^{-1}\boldsymbol{s}\right]-\boldsymbol{Z\gamma}\right]\right]. \end{aligned} \tag{3.5}$$

[7]We specify the elements of $(\partial\boldsymbol{\eta}/\partial\boldsymbol{p})$ in Appendix A.

This transformation reduces $\boldsymbol{p}|\boldsymbol{\xi}, \boldsymbol{\theta}, \boldsymbol{\eta}, \boldsymbol{\gamma}$ and $\boldsymbol{\eta}|\Sigma_s$ to $\boldsymbol{p}|\boldsymbol{\xi}, \boldsymbol{\theta}, \boldsymbol{\gamma}, \Sigma_s$ in the simultaneous demand and supply model as

$$\boldsymbol{v}|\boldsymbol{p}, \boldsymbol{\xi}, \boldsymbol{\theta}, \tag{3.4}$$

$$\boldsymbol{p}|\boldsymbol{\xi}, \boldsymbol{\theta}, \boldsymbol{\gamma}, \Sigma_s, \tag{3.5}$$

$$\boldsymbol{\theta}_i|\bar{\boldsymbol{\theta}}, \Sigma_{\boldsymbol{\theta}}. \tag{3.1}$$

$$\boldsymbol{\xi}|\Sigma_d, \tag{3.2}$$

3.3 Specifying the joint posterior of the model parameters

Since unobserved product characteristic $\boldsymbol{\xi}$ is still intricately embeded in the model, it is difficult to obtain the joint posterior of the model parameters by calculating

$$f\left(\boldsymbol{\theta}, \bar{\boldsymbol{\theta}}, \Sigma_{\boldsymbol{\theta}}, \gamma, \Sigma_d, \Sigma_s \,|\boldsymbol{v}, \boldsymbol{p}\right) = \int f\left(\boldsymbol{\xi}, \boldsymbol{\theta}, \bar{\boldsymbol{\theta}}, \Sigma_{\boldsymbol{\theta}}, \gamma, \Sigma_d, \Sigma_s \,|\boldsymbol{v}, \boldsymbol{p}\right) d\boldsymbol{\xi}, \tag{3.6}$$

where, with the distributions obtained so far, we can specify

$$\begin{aligned} & f\left(\boldsymbol{\xi}, \boldsymbol{\theta}, \bar{\boldsymbol{\theta}}, \Sigma_{\boldsymbol{\theta}}, \gamma, \Sigma_d, \Sigma_s \,|\boldsymbol{v}, \boldsymbol{p}\right) \propto f\left(\boldsymbol{v}, \boldsymbol{p}, \boldsymbol{\xi}, \boldsymbol{\theta}, \bar{\boldsymbol{\theta}}, \Sigma_{\boldsymbol{\theta}}, \Sigma_d, \gamma, \Sigma_s\right) \\ & \frac{I!}{v_0! \cdots v_J!} s_0^{v_0} \cdots s_J^{v_J} \\ & \quad \times (2\pi)^{-\frac{I}{2}} \left|\boldsymbol{\Sigma}_s\right|^{-\frac{1}{2}} \left\| \left(\frac{\partial \boldsymbol{\eta}}{\partial \boldsymbol{p}}\right) \right\| \\ & \quad \times \exp\left[-\frac{1}{2}\left[\log\left[\boldsymbol{p} + \left\{\left(\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{p}}\right)'\right\}^{-1} \boldsymbol{s}\right] - \boldsymbol{Z}\gamma\right]' \boldsymbol{\Sigma}_s^{-1} \left[\log\left[\boldsymbol{p} + \left\{\left(\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{p}}\right)'\right\}^{-1} \boldsymbol{s}\right] - \boldsymbol{Z}\gamma\right]\right] \\ & \quad \times MVN\left(\boldsymbol{0}, \boldsymbol{\Sigma}_d\right) \left[\prod_{i=1}^{I} MVN\left(\bar{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}}\right)\right] \\ & \quad \times MVN\left(\boldsymbol{\mu}_{\bar{\boldsymbol{\theta}}}, \boldsymbol{V}_{\bar{\boldsymbol{\theta}}}\right) IW_{g_{\boldsymbol{\theta}}}\left(\boldsymbol{G}_{\boldsymbol{\theta}}\right) IW_{g_d}\left(\boldsymbol{G}_d\right) MVN\left(\bar{\gamma}, \boldsymbol{V}_{\gamma}\right) IW_{g_s}\left(\boldsymbol{G}_s\right). \end{aligned}$$

Alternatively, we may obtain an approximate joint posterior of the model parameters as follows. Denoting $\psi = (\boldsymbol{\theta}, \bar{\boldsymbol{\theta}}, \Sigma_{\boldsymbol{\theta}}, \gamma, \Sigma_d, \Sigma_s)$ for notational convenience, we rewrite the equation (3.6) so that the joint posteior $f(\boldsymbol{\psi}|\boldsymbol{v},\boldsymbol{p})$ appears on both sides as[8]

$$f(\boldsymbol{\psi}|\boldsymbol{v},\boldsymbol{p}) = \int f(\boldsymbol{\psi}|\boldsymbol{\xi},\boldsymbol{v},\boldsymbol{p}) \left[\int f(\boldsymbol{\xi}|\boldsymbol{\psi},\boldsymbol{v},\boldsymbol{p}) f(\boldsymbol{\psi}|\boldsymbol{v},\boldsymbol{p})\, d\boldsymbol{\psi} \right] d\boldsymbol{\xi}. \tag{3.7}$$

This equation suggests an iterative process as follows:

Step A In the brackets, generate $\boldsymbol{\psi}_l$ from $f(\boldsymbol{\psi}|\boldsymbol{v},\boldsymbol{p})$ and then generate $\boldsymbol{\xi}_l$ from $f(\boldsymbol{\xi}|\boldsymbol{\psi}_l,\boldsymbol{v},\boldsymbol{p})$ to obtain $\boldsymbol{\xi}_1,\dots,\boldsymbol{\xi}_L$.

Step B Calculate a Monte Carlo estimator of $f(\boldsymbol{\psi}|\boldsymbol{v},\boldsymbol{p})$ as $\sum_{l=1}^{L} f(\boldsymbol{\psi}|\boldsymbol{\xi}_l,\boldsymbol{v},\boldsymbol{p})/L$ from which we generate $\boldsymbol{\psi}_l$ in **Step A**.

We will next explain how we obtain random draws of $\boldsymbol{\psi}$ and $\boldsymbol{\xi}$ and develop the Markov chain Monte Carlo (MCMC) alrogrithm in Appendix B from the algorithm with **Steps A** and **B** above.

We first consider how to obtain random draws of $\boldsymbol{\psi}$ from the Monte Carlo estimator. We can write $f(\boldsymbol{\psi}|\boldsymbol{\xi}_l,\boldsymbol{v},\boldsymbol{p})$ as

$$f(\boldsymbol{\psi}|\boldsymbol{\xi}_l,\boldsymbol{v},\boldsymbol{p}) = f\left(\boldsymbol{\theta},\bar{\boldsymbol{\theta}},\Sigma_{\boldsymbol{\theta}},\gamma,\Sigma_s|\boldsymbol{\xi}_l,\boldsymbol{v},\boldsymbol{p}\right) f\left(\Sigma_d|\boldsymbol{\xi}_l\right), \tag{3.8}$$

where $f(\boldsymbol{\theta},\bar{\boldsymbol{\theta}},\Sigma_{\boldsymbol{\theta}},\gamma,\Sigma_s|\boldsymbol{\xi}_l,\boldsymbol{v},\boldsymbol{p})$ is a nonstandard parametric form while $f(\Sigma_d|\boldsymbol{\xi}_l)$ is an inverse Wishart distribution. Therefore, the Monte Carlo estimator has a nonstandard parametric form.

One efficient way to obtain random draws of $\boldsymbol{\psi}$ from the Monte Carlo estimator which has a nonstandard parametric form uses the equation (3.8)

[8]This basic idea is known as that of the data augmentation technique (Tanner & Wong, 1987).

without the subscript l which is obtained by setting $L = 1$ in the estimator.[9][10] Then we apply the Gibbs sampler to $f(\boldsymbol{\theta}, \bar{\boldsymbol{\theta}}, \Sigma_{\boldsymbol{\theta}}, \gamma, \Sigma_s | \boldsymbol{\xi}, \boldsymbol{v}, \boldsymbol{p})$. The conditional posteriors of each component of

$$(\boldsymbol{\theta}, \bar{\boldsymbol{\theta}}, \Sigma_{\boldsymbol{\theta}}, \gamma, \Sigma_s) = (\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_I, \bar{\boldsymbol{\theta}}, \Sigma_{\boldsymbol{\theta}}, \gamma, \Sigma_s)$$

given all of the other components in the Gibbs sampler is obtained in Appendix C. Notice that the conditional posterior of $\boldsymbol{\theta}_i$ for $i = 1, \dots, I$ has a nonstandard parametric form while the conditional posteriors of $\bar{\boldsymbol{\theta}}$, $\Sigma_{\boldsymbol{\theta}}$, γ and Σ_s have standard parametric forms from which we can easily obtain their random draws. We thus apply the Metropolis-Hastings algorithm to the conditional posterior of $\boldsymbol{\theta}_i$. We also obtain random draws of Σ_d directly from $f(\Sigma_d | \boldsymbol{\xi}_l)$ which is a inverse Wishrat distribution in Appendix C. As for the generation of random draws of $\boldsymbol{\xi}$ from $f(\boldsymbol{\xi} | \boldsymbol{\psi}, \boldsymbol{v}, \boldsymbol{p})$, we apply the Metropolis-Hastings algorithm $f(\boldsymbol{\xi} | \boldsymbol{\psi}, \boldsymbol{v}, \boldsymbol{p}) = f(\boldsymbol{\xi} | \boldsymbol{\theta}, \gamma, \Sigma_s, \Sigma_d, \boldsymbol{v}, \boldsymbol{p})$ which has a nonstandard parametric form in Appendix C.[11]

4 Simulation study

Using a simulated data from a prespecified set of the model parameters, we test if the proposed method can recover the true model parameters. We

[9] Justification for being able to reduce $L = 1$ is from Tanner & Wong (1987).

[10] We can apply the Metropolis-Hastings algorithm directly to the Monte Carlo estimator. However, it is less efficient because all proposal draws of the model parameters $\boldsymbol{\psi} = (\boldsymbol{\theta}, \bar{\boldsymbol{\theta}}, \Sigma_{\boldsymbol{\theta}}, \gamma, \Sigma_d, \Sigma_s)$ are rejected with 1 minus an acceptance probability at one iteration.

[11] We use random walk Metropolis-Hastings algorithms in Chibs and Greenberg (1995) to generate $\boldsymbol{\theta}_i$ and $\boldsymbol{\xi}$ respectively.

assume that there are 1,000 consumers ($I = 1000$) and three products ($J = 3$) from three different manufacturers in an oligopolistic market of a durable product where a consumer purchases one unit of a product during the course of observation.

We first set the utility u_{ij} of a consumer i for product j in (2.1) and pricing equation in (2.8). We assume the utility u_{ij} to have a consumer i's income y_i, a unit price p_j and one observed product characteristic x_j ($Q = 2$), an unobserved product characteristic term ξ_j and an extreme value error term ε_{ij} as

$$u_{ij} = \alpha_i \log (y_i - p_j) + \beta_i x_j + \xi_j + \varepsilon_{ij}. \tag{4.1}$$

We assume the pricing equation to have one cost shifter z_j ($S = 1$) and an unobserved cost term η_j as

$$\log \left[p_j + \left\{ \left(\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{p}} \right)' \right\}_{j\cdot}^{-1} \boldsymbol{s} \right] = \gamma z_j + \eta_j, \tag{4.2}$$

where $\{(\partial \boldsymbol{G}/\partial \boldsymbol{p})'\}_{j\cdot}^{-1}$ is the jth row of $\{(\partial \boldsymbol{G}/\partial \boldsymbol{p})'\}^{-1}$.

We next set the true model parameters as $\bar{\boldsymbol{\theta}} = (\bar{\alpha}, \bar{\beta})' = (2, 2)'$, $\boldsymbol{\Sigma}_{\boldsymbol{\theta}} = 10^{-1}\boldsymbol{E}_2$, $\boldsymbol{\gamma} = \gamma = 1$, $\boldsymbol{\Sigma}_d = 10^{-4}\boldsymbol{E}_3$ and $\boldsymbol{\Sigma}_s = 10^{-4}\boldsymbol{E}_3$ where $\boldsymbol{E}_2$ and $\boldsymbol{E}_3$ are the 2×2 and 3×3 identity matrices respectively. Then we generate each of $\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_{1000}$ from $MVN(\bar{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}})$ in (3.1), $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3)'$ from $MVN(\boldsymbol{0}, \boldsymbol{\Sigma}_d)$ in (3.2) and $\boldsymbol{\eta} = (\eta_1, \eta_2, \eta_3)'$ from $MVN(\boldsymbol{0}, \boldsymbol{\Sigma}_s)$ in (3.3).

We have the exogenous y_i and x_j in (4.1) and z_j in (4.2). We generate positive values for $y_1, \dots, y_{1000}$ independently from the log normal distribution with mean 1 and standard deviation 0.1. We also generate positive values for x_1, x_2 and x_3 independently from the log normal distribution with mean 0 and standard deviation 0.1. We then set $z_j = \log x_j$ for $j = 1, 2, 3$.

We numerically obtain the endogenous sales volumes $\boldsymbol{v} = (v_1, v_2, v_3)'$ and prices $\boldsymbol{p} = (p_1, p_2, p_3)'$ in the demand and supply system as follows. We first set initial values for $\boldsymbol{p} = (p_1, p_2, p_3)'$ at the level of the 4/5 of the lowest income $\min(y_1, \ldots, y_{1000})$ so that all the 1,000 consumers can afford the three products. Given the initial $\boldsymbol{p}$, we calculate choice probabilities s_{ij} for $i = 1, \ldots, 1000$ and $j = 1, 2, 3$ in (2.2) and the market share s_j for $j = 1, 2, 3$ in (2.4). We then obtain three pairs of (s_j, p_j) for $j = 1, 2, 3$ by the Newton-Raphson method to solve the six dimensional nonlinear simultaneous equations where three of them are from the market share specification (2.4) and the other from the pricing equation (2.8). The sales volumes $\boldsymbol{v} = (v_1, v_2, v_3)'$ are obtained by multiplying the number of consumers 1,000 with the market shares $\boldsymbol{s} = (s_1, s_2, s_3)'$ from the Newton-Raphson method.

Using the simulated data of exogenous $\boldsymbol{y}$, $\boldsymbol{X}$ and $\boldsymbol{Z}$ and endogenous $\boldsymbol{v}$ and $\boldsymbol{p}$, we next estimate the joint posterior of the model parameters $(\boldsymbol{\theta}, \bar{\boldsymbol{\theta}}, \Sigma_{\boldsymbol{\theta}}, \boldsymbol{\gamma}, \Sigma_d, \Sigma_s)$ through the MCMC algorithm in Appendix B. At MCMC0 in the algorithm, we set the hyperparameters as $\boldsymbol{\mu}_{\bar{\boldsymbol{\theta}}} = (2, 2)'$, $\boldsymbol{V}_{\bar{\boldsymbol{\theta}}} = 10^{-3}\boldsymbol{E}_2$, $g_{\boldsymbol{\theta}} = 13$, $\boldsymbol{G}_{\boldsymbol{\theta}} = \boldsymbol{E}_2$, $\bar{\boldsymbol{\gamma}} = \bar{\gamma} = 1$, $\boldsymbol{V}_{\boldsymbol{\gamma}} = V_{\gamma} = 10^{-2}$, $g_d = 7$, $\boldsymbol{G}_d = 3 \times 10^{-4}\boldsymbol{E}_3$, $g_s = 7$ and $\boldsymbol{G}_s = 3 \times 10^{-4}\boldsymbol{E}_3$. We also set the covariance matrices of the proposed distributions of $\boldsymbol{\theta}_i$ and $\boldsymbol{\xi}$ as $\Sigma_{\boldsymbol{\theta}_i^*} = 25^{-3}\boldsymbol{E}_2$ and $\Sigma_{\boldsymbol{\xi}^*} = 25^{-5}\boldsymbol{E}_3$ respectively.

We run 10 MCMC sequences with different initial values of the model parameters for 10,000 iterations. We assess the convergence of the MCMC by inspecting time-series plots of the draws of the model parameters in Figure 1. We then estimate each model parameter using the last 4,000 draws of it. The result in Table 1 shows that the 95% posterior interval of each model

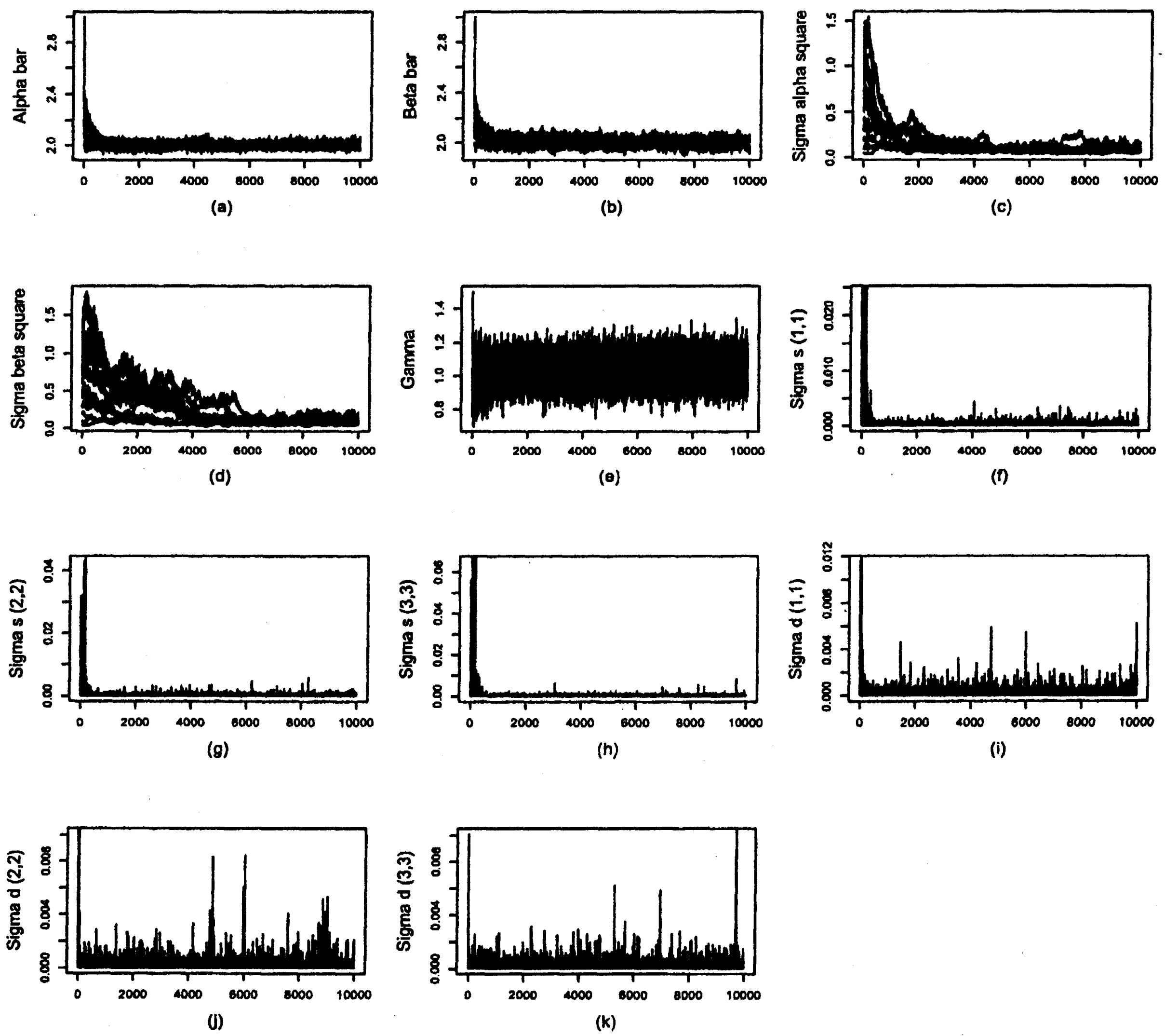
Table 1: Estimated posterior mean, standard deviation (Std.Dev.) and quantiles (2.5%, 50% and 97.5%) for the model parameters in the simulation study.

Parameter	Mean	Std.Dev.	2.5%	50%	97.5%	True value
$\bar{\alpha}$	2.00	0.016	1.97	2.00	2.03	2
$\bar{\beta}$	2.00	0.030	1.94	2.00	2.06	2
σ^2_α	0.087	0.038	0.041	0.077	0.19	0.1
σ^2_β	0.097	0.042	0.044	0.085	0.20	0.1
γ	1.05	0.059	0.93	1.05	1.16	1
$\sigma^2_{s_{11}}$	1.1×10^{-4}	1.2×10^{-4}	2.5×10^{-5}	7.5×10^{-5}	3.8×10^{-4}	10^{-4}
$\sigma^2_{s_{22}}$	1.1×10^{-4}	1.1×10^{-4}	2.6×10^{-5}	7.9×10^{-5}	3.6×10^{-4}	10^{-4}
$\sigma^2_{s_{33}}$	9.4×10^{-5}	1.1×10^{-4}	2.3×10^{-5}	6.8×10^{-5}	3.2×10^{-4}	10^{-4}
$\sigma^2_{d_{11}}$	9.9×10^{-5}	1.1×10^{-4}	2.4×10^{-5}	6.9×10^{-5}	3.5×10^{-4}	10^{-4}
$\sigma^2_{d_{22}}$	1.0×10^{-4}	1.3×10^{-4}	2.3×10^{-5}	6.9×10^{-5}	3.8×10^{-4}	10^{-4}
$\sigma^2_{d_{33}}$	1.0×10^{-4}	1.3×10^{-4}	2.4×10^{-5}	6.9×10^{-5}	3.6×10^{-4}	10^{-4}

Note. We denote σ^2_α, σ^2_β, $\sigma^2_{s_{11}}, \ldots, \sigma^2_{s_{33}}$ and $\sigma^2_{d_{11}}, \ldots, \sigma^2_{d_{33}}$ as the variances of α_i, β_i, $\eta_1, \ldots, \eta_3$ and $\xi_1, \ldots, \xi_3$.

parameter includes the true value. Thus the result provides an evidence for the validity of the proposed method. The algorithm takes about 20 hours, 52 minutes and 19 seconds to complete one sequence using a standard C++ compiler of the Microsoft Visual C++.net Standard Version 2003 on a 2.66 GHz Xeon processor with 2 GB of RAM.

Figure 1: Panels (a) thorugh (k) are plots of 10 parallel sequences corresponding to different starting values of $\bar{\alpha}$, $\bar{\beta}$, σ_α^2, σ_β^2, γ, the $(1,1)$, $(2,2)$ and $(3,3)$ components of Σ_s and those of Σ_d in the simulation study.



5 Conclusion and discusion

In this paper, we presented a Bayesian simultaneous demand and supply model with market-level data by Yonetani et al. (2008). Then we performed a simulation study on the model. We found that, with $1,000$ consumers and three products from three different manufacturers in an oligopolisitic market of durable goods, our proposed method worked reasonably well, but that a considerable amount of computation resources was necessary. In this section, we briefly provide some discussions on the model.

First, unlike Yang et al. (2003), Yonetani et al. (2008) do not model the game between manufacturers and retailers. This model would be suitable if we analyze a market where the retailers are affiliate companies of each parent manufacturer. We can model a game between manufacturers and retailers with market-level data with relatively minor effort.

Yonetani et al. (2008) assume that each coefficient for cost shifters is universal across the manufacturers. This assumption would lack flexibility if a specific cost might be different among the manufacturers given the same amount of cost shifter. We can, however, incorporate this manufacturers' heterogeneity into the cost specification as we did for the coefficients for the demand side product characteristics to reflect consumers' heterogeneity.

The comment "there is the possibility that a given set of exogenous observable and unobservable variables could be associated with a different equilibrium set of prices and quantities, that is, there is no longer a one-to-one map between the unobservables and the endogenous prices" by Berry, Dube and Chintagunta, and Bajari, and rejoinder by Yang et al. (2003) is well taken and further research is needed that addresses these concerns.

A Calculation of the building blocks for matrices used in the supply side specification

The $(\boldsymbol{\partial s}/\boldsymbol{\partial p})$ and $(\boldsymbol{\partial G}/\boldsymbol{\partial p})$ consist of $\partial s_j/\partial p_k$ and $\partial G_j/\partial p_k$ respectively as each (j,k) element:

$$\frac{\partial s_j}{\partial p_j}=\frac{\partial G_j}{\partial p_j}=-\frac{1}{I}\sum_{i=1}^{I}\frac{\alpha_i s_{ij}\left(1-s_{ij}\right)}{y_i-p_j}, \qquad (i=j)$$

$$\frac{\partial s_j}{\partial p_k}=\frac{\partial G_j}{\partial p_k}=\frac{\delta_{jk}}{I}\sum_{i=1}^{I}\frac{\alpha_i s_{ij} s_{ik}}{y_i-p_k}. \qquad (i\neq j)$$

The $(\boldsymbol{\partial \eta}/\boldsymbol{\partial p})$ consists of $\partial \eta_j/\partial p_k$ as the (j,k) element:

$$\frac{\partial \eta_j}{\partial p_j}=\frac{1}{p_j+\left\{(\boldsymbol{\partial G}/\boldsymbol{\partial p})'\right\}_{j\cdot}^{-1}\boldsymbol{s}}\left[1+\left[\frac{\partial}{\partial p_j}\left\{\left(\frac{\boldsymbol{\partial G}}{\boldsymbol{\partial p}}\right)'\right\}_{j\cdot}^{-1}\right]\boldsymbol{s}+\left\{\left(\frac{\boldsymbol{\partial G}}{\boldsymbol{\partial p}}\right)'\right\}_{j\cdot}^{-1}\left(\frac{\boldsymbol{\partial s}}{\boldsymbol{\partial p}}\right)_{\cdot j}\right],$$

$$(i=j)$$

$$\frac{\partial \eta_j}{\partial p_k}=\frac{1}{p_j+\left\{(\boldsymbol{\partial G}/\boldsymbol{\partial p})'\right\}_{j\cdot}^{-1}\boldsymbol{s}}\left[\left[\frac{\partial}{\partial p_k}\left\{\left(\frac{\boldsymbol{\partial G}}{\boldsymbol{\partial p}}\right)'\right\}_{j\cdot}^{-1}\right]\boldsymbol{s}+\left\{\left(\frac{\boldsymbol{\partial G}}{\boldsymbol{\partial p}}\right)'\right\}_{j\cdot}^{-1}\left(\frac{\boldsymbol{\partial s}}{\boldsymbol{\partial p}}\right)_{\cdot k}\right],$$

$$(i\neq j)$$

where $\dfrac{\partial}{\partial p_k}\left\{\left(\dfrac{\boldsymbol{\partial G}}{\boldsymbol{\partial p}}\right)'\right\}^{-1}=-\left\{\left(\dfrac{\boldsymbol{\partial G}}{\boldsymbol{\partial p}}\right)'\right\}^{-1}\left\{\dfrac{\partial}{\partial p_k}\left(\dfrac{\boldsymbol{\partial G}}{\boldsymbol{\partial p}}\right)'\right\}\left\{\left(\dfrac{\boldsymbol{\partial G}}{\boldsymbol{\partial p}}\right)'\right\}^{-1}.$

The $(\partial/\partial p_j)(\boldsymbol{\partial G}/\boldsymbol{\partial p})$ in $(\boldsymbol{\partial \eta}/\boldsymbol{\partial p})$ consists of $\partial^2 G_k/\partial p_j\partial p_l$ as its (k,l) element:

$$\frac{\partial^2 G_j}{\partial p_j\partial p_j}=-\frac{1}{I}\sum_{i=1}^{I}\frac{\alpha_i s_{ij}\left(1-s_{ij}\right)\left(2\alpha_i s_{ij}-\alpha_i+1\right)}{\left(y_i-p_j\right)^2}, \qquad (j=k=l)$$

$$\frac{\partial^2 G_k}{\partial p_j\partial p_k}=\frac{1}{I}\sum_{i=1}^{I}\frac{\alpha_i^2 s_{ij}s_{ik}\left(2s_{ik}-1\right)}{\left(y_i-p_j\right)\left(y_i-p_k\right)}, \qquad (j\neq k=l)$$

$$\frac{\partial^2 G_j}{\partial p_j\partial p_l}=\frac{\delta_{jl}}{I}\sum_{i=1}^{I}\frac{\alpha_i^2 s_{ij}s_{il}\left(2s_{ij}-1\right)}{\left(y_i-p_j\right)\left(y_i-p_l\right)}, \qquad (j=k\neq l)$$

$$\frac{\partial^2 G_k}{\partial p_j \partial p_j} = \frac{\delta_{kj}}{I} \sum_{i=1}^{I} \frac{\alpha_i s_{ij} s_{ik} (2\alpha_i s_{ij} - \alpha_i + 1)}{(y_i - p_j)^2}, \qquad (j = l \neq k)$$

$$\frac{\partial^2 G_k}{\partial p_j \partial p_l} = \frac{2\delta_{kl}}{I} \sum_{i=1}^{I} \frac{\alpha_i^2 s_{ij} s_{ik} s_{il}}{(y_i - p_j)(y_i - p_l)}. \qquad (j \neq k \neq l)$$

B MCMC algorithm

We estimate the model parameters using the following MCMC algorithm.

MCMC0 Set the hyperparameters $\boldsymbol{\mu}_{\bar{\boldsymbol{\theta}}}$, $\boldsymbol{V}_{\bar{\boldsymbol{\theta}}}$, $g_{\boldsymbol{\theta}}$, $G_{\boldsymbol{\theta}}$, $\bar{\gamma}$, $\boldsymbol{V}_{\gamma}$, g_d, $\boldsymbol{G}_d$, g_s and $\boldsymbol{G}_s$, the covariances of the dumping distributions, $\Sigma_{\boldsymbol{\xi}^*}$ and $\Sigma_{\boldsymbol{\theta}_i^*}$, and initial values $\boldsymbol{\xi}^{(0)}$, $\boldsymbol{\theta}^{(0)}$, $\bar{\boldsymbol{\theta}}^{(0)}$, $\Sigma_{\boldsymbol{\theta}}^{(0)}$, $\gamma^{(0)}$ and $\Sigma_s^{(0)}$.

For $t = 1, \ldots,$

MCMC1 Generate a proposal $\boldsymbol{\xi}^*$ from $MVN(\boldsymbol{\xi}^{(t-1)}, \Sigma_{\boldsymbol{\xi}^*})$.

MCMC2 Calculate

$$R_{\boldsymbol{\xi}^{(t)}} = \min\left(\frac{f\left(\boldsymbol{\xi}^* \,\middle|\, \boldsymbol{\theta}^{(t-1)}, \gamma^{(t-1)}, \Sigma_s^{(t-1)}, \Sigma_d^{(t-1)}, \boldsymbol{v}, \boldsymbol{p}\right)}{f\left(\boldsymbol{\xi}^{(t-1)} \,\middle|\, \boldsymbol{\theta}^{(t-1)}, \gamma^{(t-1)}, \Sigma_s^{(t-1)}, \Sigma_d^{(t-1)}, \boldsymbol{v}, \boldsymbol{p}\right)}, 1 \right).$$

MCMC3 Set $\boldsymbol{\xi}^{(t)} = \boldsymbol{\xi}^*$ with probability $R_{\boldsymbol{\xi}^{(t)}}$ or $\boldsymbol{\xi}^{(t)} = \boldsymbol{\xi}^{(t-1)}$ with probability $1 - R_{\boldsymbol{\xi}^{(t)}}$.

For $i = 1, \ldots, I$

MCMC4 Generate a proposal $\boldsymbol{\theta}_i^*$ from $MVN(\boldsymbol{\theta}_i^{(t-1)}, \Sigma_{\boldsymbol{\theta}_i^*})$.

MCMC5 Calculate

$$R_{\boldsymbol{\theta}_i^{(t)}} = \min\left(\frac{f\left(\boldsymbol{\theta}_i^* \,\middle|\, \boldsymbol{\theta}_1^{(t)}, \ldots, \boldsymbol{\theta}_{i-1}^{(t)}, \boldsymbol{\theta}_{i+1}^{(t-1)}, \ldots, \boldsymbol{\theta}_I^{(t-1)}, \boldsymbol{\omega}^{(t-1)}, \boldsymbol{\xi}^{(t)}, \boldsymbol{v}, \boldsymbol{p}\right)}{f\left(\boldsymbol{\theta}_i^{(t-1)} \,\middle|\, \boldsymbol{\theta}_1^{(t)}, \ldots, \boldsymbol{\theta}_{i-1}^{(t)}, \boldsymbol{\theta}_{i+1}^{(t-1)}, \ldots, \boldsymbol{\theta}_I^{(t-1)}, \boldsymbol{\omega}^{(t-1)}, \boldsymbol{\xi}^{(t)}, \boldsymbol{v}, \boldsymbol{p}\right)}, 1 \right),$$

where $\boldsymbol{\omega}^{(t-1)} = (\bar{\boldsymbol{\theta}}^{(t-1)}, \Sigma_{\boldsymbol{\theta}}^{(t-1)}, \gamma^{(t-1)}, \Sigma_s^{(t-1)})$.

MCMC6 Set $\boldsymbol{\theta}_i^{(t)} = \boldsymbol{\theta}_i^*$ with probability $R_{\boldsymbol{\theta}_i^{(t)}}$ or $\boldsymbol{\theta}_i^{(t)} = \boldsymbol{\theta}_i^{(t-1)}$ with probability $1 - R_{\boldsymbol{\theta}_i^{(t)}}$.

MCMC7 Return to **MCMC4** until we obtain $\boldsymbol{\theta}_1^{(t)}, \dots, \boldsymbol{\theta}_I^{(t)}$.

MCMC8 Generate $\bar{\boldsymbol{\theta}}^{(t)}$ from $f(\bar{\boldsymbol{\theta}}|\boldsymbol{\theta}^{(t)}, \Sigma_{\boldsymbol{\theta}}^{(t-1)})$.

MCMC9 Generate $\Sigma_{\boldsymbol{\theta}}^{(t)}$ from $f(\Sigma_{\boldsymbol{\theta}}|\boldsymbol{\theta}^{(t)}, \bar{\boldsymbol{\theta}}^{(t)})$.

MCMC10 Generate $\boldsymbol{\gamma}^{(t)}$ from $f(\boldsymbol{\gamma}|\boldsymbol{\theta}^{(t)}, \Sigma_s^{(t-1)}, \boldsymbol{\xi}^{(t)}, \boldsymbol{p})$.

MCMC11 Generate $\Sigma_s^{(t)}$ from $f(\Sigma_s|\boldsymbol{\theta}^{(t)}, \boldsymbol{\gamma}^{(t)}, \boldsymbol{\xi}^{(t)}, \boldsymbol{p})$.

MCMC12 Generate $\Sigma_d^{(t)}$ from $f(\Sigma_d|\boldsymbol{\xi}^{(t)})$.

MCMC13 If the Gibbs sampler for $f(\boldsymbol{\theta}, \bar{\boldsymbol{\theta}}, \Sigma_{\boldsymbol{\theta}}, \boldsymbol{\gamma}, \Sigma_s|\boldsymbol{\xi}, \boldsymbol{v}, \boldsymbol{p})$ converges when using

$$f\left(\boldsymbol{\theta}_i \middle| \boldsymbol{\theta}_1^{(t)}, \dots, \boldsymbol{\theta}_{i-1}^{(t)}, \boldsymbol{\theta}_{i+1}^{(t)}, \dots, \boldsymbol{\theta}_I^{(t)}, \boldsymbol{\omega}^{(t-1)}, \boldsymbol{\xi}^{(t)}, \boldsymbol{v}, \boldsymbol{p}\right)$$

for $i = 1, \dots, I$ at the end of **MCMC7**, $f(\bar{\boldsymbol{\theta}}|\boldsymbol{\theta}^{(t)}, \Sigma_{\boldsymbol{\theta}}^{(t-1)})$ in **MCMC8**, $f(\Sigma_{\boldsymbol{\theta}}|\boldsymbol{\theta}^{(t)}, \bar{\boldsymbol{\theta}}^{(t)})$ in **MCMC9**, $f(\boldsymbol{\gamma}|\boldsymbol{\theta}^{(t)}, \Sigma_s^{(t-1)}, \boldsymbol{\xi}^{(t)}, \boldsymbol{p})$ in **MCMC10** and $f(\Sigma_s|\boldsymbol{\theta}^{(t)}, \boldsymbol{\gamma}^{(t)}, \boldsymbol{\xi}^{(t)}, \boldsymbol{p})$ in **MCMC11**, and if $f(\Sigma_d|\boldsymbol{\xi}^{(t)})$ in **MCMC12** converges as well, the standard MCMC argument guarantees that the stationary distribution is $f(\boldsymbol{\psi}|\boldsymbol{v}, \boldsymbol{p}) = f(\boldsymbol{\theta}, \bar{\boldsymbol{\theta}}, \Sigma_{\boldsymbol{\theta}}, \boldsymbol{\gamma}, \Sigma_s|\boldsymbol{\xi}, \boldsymbol{v}, \boldsymbol{p}) f(\Sigma_d|\boldsymbol{\xi})$. Hence, stop the iteration. Otherwise increase t by one and return to **MCMC1**.

C Conditional posteriors

We obtain the conditional posterior distributions in the MCMC as follows.

$f(\boldsymbol{\xi}|\boldsymbol{\theta}, \boldsymbol{\gamma}, \Sigma_d, \Sigma_s, \boldsymbol{v}, \boldsymbol{p})$

$$
\begin{aligned}
&\propto s_0^{v_0} \cdots s_J^{v_J} \cdot \\
&\quad \times |\boldsymbol{\Sigma}_s|^{-\frac{1}{2}} \left\| \left(\frac{\partial \boldsymbol{\eta}}{\partial \boldsymbol{p}} \right) \right\| \\
&\quad \times \exp \left[-\frac{1}{2} \left[\log \left[\boldsymbol{p} + \left\{ \left(\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{p}} \right)' \right\}^{-1} \boldsymbol{s} \right] - \boldsymbol{Z\gamma} \right]' \boldsymbol{\Sigma}_s^{-1} \left[\log \left[\boldsymbol{p} + \left\{ \left(\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{p}} \right)' \right\}^{-1} \boldsymbol{s} \right] - \boldsymbol{Z\gamma} \right] \right] \\
&\quad \times |\boldsymbol{\Sigma}_d|^{-\frac{1}{2}} \exp \left(-\frac{1}{2} \boldsymbol{\xi}' \boldsymbol{\Sigma}_d^{-1} \boldsymbol{\xi} \right),
\end{aligned}
$$

$$
\begin{aligned}
&f\left(\boldsymbol{\theta}_i \,|\boldsymbol{\theta}_{-i}, \bar{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}}, \boldsymbol{\gamma}, \boldsymbol{\Sigma}_s, \boldsymbol{\xi}, \boldsymbol{v}, \boldsymbol{p}\right) \\
&\propto \frac{M!}{v_0! \cdots v_J!} s_0^{v_0} \cdots s_J^{v_J} \cdot \\
&\quad \times |\boldsymbol{\Sigma}_s|^{-\frac{1}{2}} \left\| \left(\frac{\partial \boldsymbol{\eta}}{\partial \boldsymbol{p}} \right) \right\| \\
&\quad \times \exp \left[-\frac{1}{2} \left[\log \left[\boldsymbol{p} + \left\{ \left(\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{p}} \right)' \right\}^{-1} \boldsymbol{s} \right] - \boldsymbol{Z\gamma} \right]' \boldsymbol{\Sigma}_s^{-1} \left[\log \left[\boldsymbol{p} + \left\{ \left(\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{p}} \right)' \right\}^{-1} \boldsymbol{s} \right] - \boldsymbol{Z\gamma} \right] \right] \\
&\quad \times |\boldsymbol{\Sigma}_{\boldsymbol{\theta}}|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} \left(\boldsymbol{\theta}_i - \bar{\boldsymbol{\theta}}\right)' \boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{-1} \left(\boldsymbol{\theta}_i - \bar{\boldsymbol{\theta}}\right) \right\},
\end{aligned}
$$

$$
\begin{aligned}
&\bar{\boldsymbol{\theta}} \,|\boldsymbol{\theta}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}} \sim N\left(\left(I\boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{-1} + \boldsymbol{V}_{\bar{\boldsymbol{\theta}}}^{-1} \right)^{-1} \left(I\boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{-1} \boldsymbol{\nu} + \boldsymbol{V}_{\bar{\boldsymbol{\theta}}}^{-1} \boldsymbol{\mu}_{\bar{\boldsymbol{\theta}}} \right), \left(I\boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{-1} + \boldsymbol{V}_{\bar{\boldsymbol{\theta}}}^{-1} \right)^{-1} \right) \\
&\quad \text{where } \boldsymbol{\nu} = \frac{1}{I} \sum_{i=1}^{I} \boldsymbol{\theta}_i,
\end{aligned}
$$

$$
\boldsymbol{\Sigma}_{\boldsymbol{\theta}} \,|\boldsymbol{\theta}, \bar{\boldsymbol{\theta}} \sim IW_{g_{\boldsymbol{\theta}}+I} \left(\sum_{i=1}^{I} \left(\boldsymbol{\theta}_i - \bar{\boldsymbol{\theta}}\right) \left(\boldsymbol{\theta}_i - \bar{\boldsymbol{\theta}}\right)' + \boldsymbol{G}_{\boldsymbol{\theta}} \right),
$$

$$
\begin{aligned}
&\boldsymbol{\gamma} \,|\boldsymbol{\theta}, \boldsymbol{\Sigma}_s, \boldsymbol{\xi}, \boldsymbol{p} \sim N\left(\left(\boldsymbol{\Sigma}_{s*}^{-1} + \boldsymbol{V}_{\boldsymbol{\gamma}}^{-1} \right)^{-1} \left(\boldsymbol{\mu} + \boldsymbol{V}_{\boldsymbol{\gamma}}^{-1} \bar{\boldsymbol{\gamma}} \right), \left(\boldsymbol{\Sigma}_{s*}^{-1} + \boldsymbol{V}_{\boldsymbol{\gamma}}^{-1} \right)^{-1} \right), \\
&\quad \text{where } \boldsymbol{\mu} = \boldsymbol{Z}' \boldsymbol{\Sigma}_s^{-1} \left[\log \left[\boldsymbol{p} + \left\{ \left(\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{p}} \right)' \right\}^{-1} \boldsymbol{s} \right] \right] \text{ and } \boldsymbol{\Sigma}_{s*}^{-1} = \boldsymbol{Z}' \boldsymbol{\Sigma}_s^{-1} \boldsymbol{Z},
\end{aligned}
$$

$$
\begin{aligned}
&\boldsymbol{\Sigma}_s \,|\boldsymbol{\theta}, \boldsymbol{\gamma}, \boldsymbol{\xi}, \boldsymbol{p} \\
&\sim IW_{g_s+1} \left(\left(\log \left[\boldsymbol{p} + \left\{ \left(\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{p}} \right)' \right\}^{-1} \boldsymbol{s} \right] - \boldsymbol{Z\gamma} \right) \left(\log \left[\boldsymbol{p} + \left\{ \left(\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{p}} \right)' \right\}^{-1} \boldsymbol{s} \right] - \boldsymbol{Z\gamma} \right)' + \boldsymbol{G}_s \right),
\end{aligned}
$$

$$
\boldsymbol{\Sigma}_d \,|\boldsymbol{\xi} \sim IW_{g_d+1} \left(\boldsymbol{\xi}\boldsymbol{\xi}' + \boldsymbol{G}_d \right).
$$

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